THREE-DIMENSIONAL OPTIMAL KALMAN ALGORITHM FOR GPS-BASED POSITIONING ESTIMATION OF THE STATIONARY OBJECT

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INTRODUCTION

Navigation is defined as the science of getting a craft or person from one place to another [1]. Some navigation aids are very complex and transmit electronic signals, that are referred to as radionavigation aids. Various types of radionavigation aids exist, which can be either ground-based or space-based. The Global Positioning System (GPS) was created in the early 1960s by the National Aeronautics and Space Administration (NASA), developing satellite systems for positioning determination. GPS is now being used to provide positioning and timing information for a number of applications where it is essential that the accuracy and reliability of the GPS information can be assured [2]. The susceptibility of the GPS signals to interference is of concern to the GPS user community. Because of the low receiver power of the GPS signals, outages can easily occur due to unintentional interference.

Theoretically, the Kalman filter is an estimator [3] for what is called the *linear-quadratic problem*, which is the problem of estimating the instantaneous "state" of a linear dynamic system perturbed by white noise. The resulting estimator is statistically optimal with respect to any quadratic function of estimation error. R. E. Kalman's paper describing a recursive solution of the discrete-data linear filtering problem was published in 1960 [4].

This project presents the design and development of a multidimensional Kalman filter with the purpose to estimate the tri-dimensional position of a stationary object based on GPS measurements. Because this is not the only filtering algorithm available, a comparison with other four types of filters (one-dimensional optimal Kalman algorithm, quasi-optimal stationary Kalman algorithm, simple moving average algorithm and optimally unbiased moving average algorithm) is also developed.

METHODOLOGY

Consider a discrete observation signal ξ_ν that is a linear combination of a discrete signal λ_ν and a white noise $n_{o\nu}$. The observation and state equations are [5]:

$$\xi_\nu = \mathbf{H}_\nu \lambda_\nu + \mathbf{u}_\nu + \mathbf{n}_{o\nu}, \tag{1}$$

$$\lambda_\nu = \mathbf{A}_{\nu-1} \lambda_{\nu-1} + \mathbf{n}_{\lambda\nu}, \tag{2}$$

where ν=0,1,2,... corresponds to discrete-time t_ν and measuring time interval $\Delta = t_\nu - t_{\nu-1}$, ξ_ν is m-dimensional observation vector formed by the reference short-term noisy GPS signals, λ_ν is n-dimensional three-dimensional position state vector (latitude, longitude, altitude), $\mathbf{H}_\nu$ is mxn dimensional measurement matrix, $\mathbf{u}_\nu$ is m-dimensional vector that contains the control signals (which for the filtering task is taken as zero), $\mathbf{A}_\nu$ is nxn dimensional state transition matrix, $\mathbf{n}_{ov}$ and $\mathbf{n}_{\lambda v}$ are jointly independent vector white Gaussian noises with mean-zero and covariance matrixes $\mathbf{V}_v$ and Ψ_v which are of *mxm* and *nxn* dimensions, respectively. As usually, we will deal with single observations and estimate some states. It means that if $m<n$ then one may use the following algorithm of lineal Kalman filtering based on (1) and (2):

$v=v+1$

$$\tilde{\mathbf{R}}_v = \mathbf{A}^T_{v-1}\mathbf{R}_{v-1}\mathbf{A}_{v-1} + \Psi_v \quad (3)$$

$$\mathbf{K}_v = \tilde{\mathbf{R}}_v\mathbf{H}^T_v\left(\mathbf{H}_v\tilde{\mathbf{R}}_v\mathbf{H}^T_v + \mathbf{V}_v\right)^{-1} \quad (4)$$

$$\hat{\lambda}_v = \mathbf{A}_{v-1}\hat{\lambda}_{v-1} + \mathbf{K}_v\left(\xi_v - \mathbf{u}_v - \mathbf{H}_v\mathbf{A}_{v-1}\hat{\lambda}_{v-1}\right) \quad (5)$$

$$\mathbf{R}_v = \left(\mathbf{I} - \mathbf{K}_v\mathbf{H}_v\right)\tilde{\mathbf{R}}_v, \quad (6)$$

where $\hat{\lambda}_v$ is a vector of three-dimensional positioning estimates, **I** is unit matrix, and $\mathbf{R}_v$ is the error covariance matrix. Solution (3) is justified for a common case and may not be simplified, as a rule. Errors between estimates and original signals will be determined using:

$$e = \hat{\lambda}_v - \lambda_v \ . \quad (7)$$

From the equation (7), the following statistical parameters will be also determined:

a. Mean value of the Error ($\bar{e}$).
b. Root Mean Square Deviation (**RMSD**, σ_e).
c. Root Mean Square Error (**RMSE**, $\sqrt{\bar{e}^2 + \sigma_e^2}$).
d. Maximal Error (maximal value of e).

RESULTS

Now, we will compare the 3-Dimensional Kalman estimates with the One-dimensional Optimal Kalman algorithm [6], the Quasi-Optimal stationary Kalman algorithm [6], the Simple Moving Average (MA) algorithm [6] and the Optimally Unbiased Moving Average (OMA) algorithm [6]. For this case, we use the same conditions and the information from the Observation ξ_v. Tables 1, 2 and 3 show the statistical data obtained from the errors for the three signals, respectively. Also, Figures 1, 2 and 3 show the comparison between the different filters for the three signals, respectively.

Table 1 – Statistical values obtained from the different algorithms for the signal x .

	Estimate's mean	Bias of Error	RMSD	RMSE	Maximal Value
3-D Kalman	4054.923	31.358	349.359	350.764	1460.309
1-D Kalman	4054.320	31.433	335.451	336.920	1365.408
1-D Stationary	4055.245	31.638	304.002	305.644	1153.318
Simple MA	3818.170	29.475	241.340	243.133	498.533
Optimal MA	3824.408	36.128	303.026	305.172	712.668

Table 2 – Statistical values obtained from the different algorithms for the signal y .

	Estimate's mean	Bias of Error	RMSD	RMSE	Maximal Value
1	2	3	4	5	6
3-D Kalman	3008.725	71.779	285.029	293.929	1152.731
1-D Kalman	3009.500	71.753	274.267	283.497	1081.556

1	2	3	4	5	6
1-D Stationary	3009.305	71.684	249.958	260.034	922.489
Simple MA	2834.164	67.212	211.452	221.877	633.800
Optimal MA	2837.085	70.322	255.231	264.742	592.001

Table 3 – Statistical values obtained from the different algorithms for the signal z .

	Estimate's mean	Bias of Error	RMSD	RMSE	Maximal Value
3-D Kalman	5017.712	-14.174	469.401	469.615	1767.886
1-D Kalman	5021.790	-14.298	451.979	452.206	1649.259
1-D Stationary	5021.343	-14.648	432.605	432.865	1384.148
Simple MA	4710.869	-32.100	355.820	357.265	974.333
Optimal MA	4713.880	-29.027	436.138	437.103	833.336

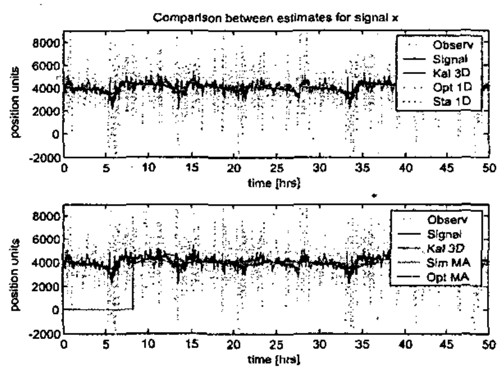

Figure 1 – Estimates for the Signal x .

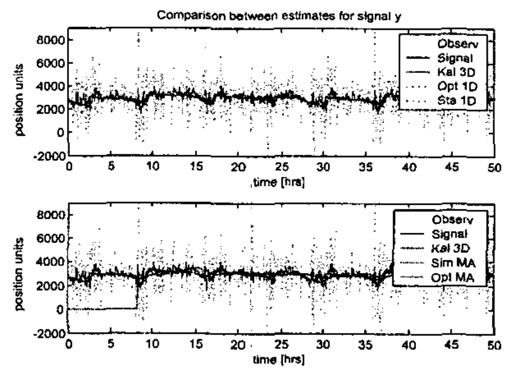

Figure 2 – Estimates for the Signal y .

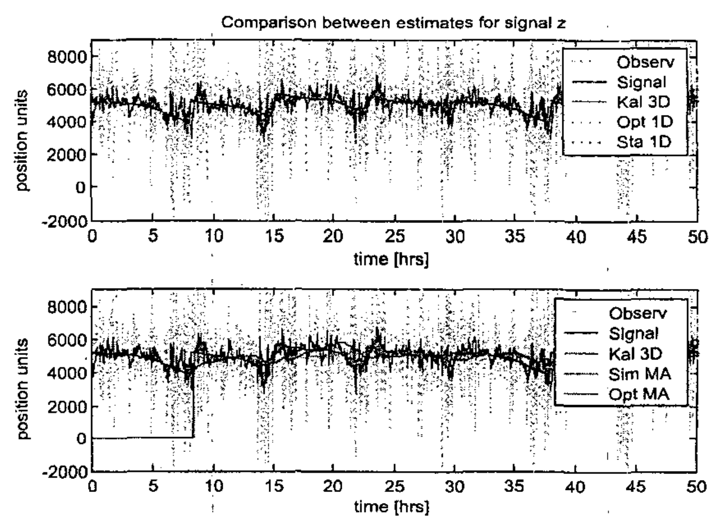

Figure 3 – Estimates for the Signal z .

CONCLUSIONS

The comparison between the three different Kalman algorithms showed that the general behavior of the estimates are similar, only with a very minimal difference that is not possible to be seen on the Figures, only with the statistical data. For the comparison with the simple MA and the Optimally Unbiased MA estimates, it was possible to see that the filters can provide accurate estimates of the signals, but their problems are the transients. If we want to obtain reduced transients, the bias are going to increment, on the other hand, to obtain a reduced bias, the transients will increase. For the stationary object, this algorithms, specially the simple MA, can provide a good result in a very easy way (the filter is very simple)

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