# A Fixed Time Observer for Flux and Load in Induction Motors $\star$

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**Abstract:** This paper presents a second order sliding mode observer for flux and load in induction motors. It is based on a block-wise representation of the motor model in  $\alpha\beta$  frame and second order sliding mode algorithms. The block structure provides a straightforward form to the application of uniform second order sliding mode algorithms, yielding to finite-time convergence with a predetermined settling time independent on initial conditions. The cases of single-phase, three-phase and linear induction motors are studied. Finally, numerical simulations show the efficiency and feasibility of the proposal.

*Keywords:* Induction Motors, Non-linear Estimation, Second-Order Sliding Mode Algorithms, Fixed Time Stability

# 1. INTRODUCTION

The aim of this paper is to present a Sliding Mode (SM) observer for the estimation of flux and mechanical load in Induction Motors (IM), due the difficulty of its direct measurement Kanellakopoulos et al. (1992). The SM methods are applied with the idea to drive the dynamics of a system to a sliding manifold that is an integral manifold with finite reaching time Drakunov and Utkin (1992), this approach exhibits very interesting and desirable features such as the work with reduced observation error dynamics, the possibility to decompose the design problem into two sub problems of the reduced order, the robustness of the closed - loop system in presence of parameter variations and external disturbances and, finite-time stability Utkin et al. (2009). Therefore, SM algorithms can be considered as an effective solution to the problem of observers design for nonlinear systems Walcott et al. (1987), specially when finite-time convergence of the observed states to the real ones is required. Most of the proposed SM observers use the equivalent control method Utkin (1972) to obtain information of the system by means of continuous equivalent values of the discontinuous observer inputs in SM motion Drakunov (1992). With this idea, several designs have been proposed as the cascade observers Krasnova et al. (2001) for nonlinear systems presented in so-called block form Loukianov (1998). Moreover, the continuous High Order SM (HOSM) algorithms Levant (1993) allow to obtain a better approximation of the equivalent control value without filtering, see for example the second order SM observer in Floquet and Barbot (2007) and an SM observer

in Bejarano and Fridman (2010) where the estimation of unknown inputs problem has been considered.

For electrical drives, specially induction motors, the SM algorithms have been successfully applied to design state observers Bartolini et al. (2003), Rubio et al. (2011) and fault tolerant schemes Djeghali et al. (2011). All the mentioned systems exhibit finite-time convergence to the equilibrium, however, the settling time of the observer convergence depends on initial conditions. Therefore, an interesting feature for a SM observer is related with the possibility to predefine a determined convergence time independently on initial conditions. This characteristic was introduced for observer design in Cruz-Zavala et al. (2010) by means of the concept of *uniform finite time stability* and, it has been applied to controller design in Polyakov (2012) under the concept of *fixed time stability*.

In this work, a SM observer design problem for an induction motors with the time independent to the initial conditions convergence, is considered. To solve this problem, uniform second order sliding mode algorithms Cruz-Zavala et al. (2010) are applied to estimate the rotor flux and mechanical load. The observer structure is based on the representation of induction motors including single-phase, three-phase and linear ones in the  $\alpha\beta$  stationary reference frame. In this case, it is possible to present in the block form which consists of two blocks: the first one is composed by the measured variables and, the second one is related with the variables to be observed. This block form allows a simple implementation of the SM algorithms.

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The effectiveness the proposed observer is demonstrated by means of numerical simulation, showing a good performance of this proposal.

This paper is organized as follows: Section 2 introduces the structured model of the IM in  $\alpha\beta$ . Section 3 describes the proposed observer, including the convergence analysis. The simulations are presented in Section 4. Finally, in Section 5 the conclusions are given.

# 2. MATHEMATICAL STRUCTURE OF THE INDUCTION MOTORS IN $\alpha\beta$ FRAME

In this Section, we describe the structure of a model that represents different types of induction motors as: three-phase, single-phase and three-phase with linear motion. This model can be described by equations for the stator current and rotor fluxes in stationary reference frame  $\alpha\beta$  as follows:

$$\frac{d\Theta}{dt} = d_1 \left( \lambda_{\alpha r} i_{\beta s} - \lambda_{\beta r} i_{\alpha s} \right) - d_2 \Gamma - d_3 \Theta$$

$$\frac{d\lambda_{\alpha r}}{dt} = -\eta_1 \lambda_{\alpha r} + \eta_2 \Theta \lambda_{\beta r} + \eta_3 i_{\alpha s}$$

$$\frac{d\lambda_{\beta r}}{dt} = -\eta_1 \lambda_{\beta r} - \eta_2 \Theta \lambda_{\alpha r} + \eta_3 i_{\beta s}$$

$$\frac{di_{\alpha s}}{dt} = -\eta_4 i_{\alpha s} + \eta_5 \lambda_{\alpha r} - \eta_6 \Theta \lambda_{\beta r} + \eta_7 v_{\alpha s}$$

$$\frac{di_{\beta s}}{dt} = -\eta_8 i_{\beta s} + \eta_9 \lambda_{\beta r} + \eta_{10} \Theta \lambda_{\alpha r} + \eta_{11} v_{\beta s}$$
(1)

where  $\lambda_{\alpha r}$  and  $\lambda_{\beta r}$  are the rotor magnetic-flux-linkage components, respectively;  $i_{\alpha s}$  and  $i_{\beta s}$  are the stator current components, respectively,  $v_{\alpha s}$  and  $v_{\beta s}$  are the voltage of  $\alpha$  and  $\beta$  axes in the stator, respectively.

#### 2.1 Single-phase

For the case of single-phase induction motor in  $\alpha\beta$  frame with permanent split capacitor, the voltages  $v_{\alpha s}$  and  $v_{\beta s}$ in (1) are of the form,

$$v_{\alpha s} = v_s \cos(\omega t) \tag{2}$$

$$v_{\beta s} = n^{-1} v_{\alpha s} - v_c \tag{3}$$

where the dynamics of the capacitor are governed by

$$lv_c/dt = \omega_0 X_c i_{\beta s} \tag{4}$$

with 
$$X_c$$
 being capacitor reactance and  $\omega_0 = 2\pi f$ .

Furthermore the parameters are: 
$$\eta_1 = \frac{R_r}{L_r}, \ \eta_2 = n_p,$$
  
 $\eta_3 = \frac{R_r L_m}{L_r}, \ \eta_4 = \left(\frac{R_{\alpha s}L_r^2 + R_r L_m^2}{L_r^2}\right) \left(\frac{L_r}{L_{\alpha s}L_r - L_m^2}\right), \ \eta_5 = \left(\frac{R_r L_m}{L_r^2}\right) \left(\frac{L_r}{L_{\alpha s}L_r - L_m^2}\right), \ \eta_6 = n_p \left(\frac{L_m}{L_r}\right) \left(\frac{L_r}{L_{\alpha s}L_r - L_m^2}\right), \ \eta_7 = \left(\frac{L_r}{L_{\alpha s}L_r - L_m^2}\right), \ \eta_8 = \left(\frac{R_{\beta s}L_r^2 + R_r L_m^2}{L_r^2}\right) \left(\frac{L_r}{L_{\beta s}L_r - L_m^2}\right), \ \eta_9 = \left(\frac{R_r L_m}{L_r^2}\right) \left(\frac{L_r}{L_{\beta s}L_r - L_m^2}\right), \ \eta_{10} = n_p \left(\frac{L_m}{L_r}\right) \left(\frac{L_r}{L_{\beta s}L_r - L_m^2}\right), \ \eta_{11} = \left(\frac{L_r}{L_{\beta s}L_r - L_m^2}\right), \ d_1 = \frac{n_p^2 L_m}{L_r J}, \ d_2 = \frac{n_p}{J}, \ d_3 = \frac{k_d}{J} \ \text{where } R_{\alpha s}, \ R_{\beta s}, \ L_{\alpha s} \ \text{and } L_{\beta s} \ \text{are the resistances and inductances of the main and auxiliary stator windings, respectively.  $k_d$  is the viscous friction,  $\Theta = \omega_r$  is the rotor speed,  $\Gamma = T_L$  is the load torque and  $J$  is the rotor moment of inertia.$ 

#### 2.2 Three-phase

For the three-phase induction motor in  $\alpha\beta$  frame, the voltages are presented as:

$$v_{\alpha s} = v_s \sqrt{2} \cos(\omega t) \tag{5}$$

$$v_{\beta s} = -v_s \sqrt{2} \sin(\omega t). \tag{6}$$

In the case the parameters are:  $\eta_1 = \frac{R_r}{(L_r + L_m)}, \ \eta_2 = n_p,$   $\eta_3 = \frac{R_r L_m}{(L_r + L_m)}, \ \eta_4 = \frac{R_s (L_r + L_m)^2 + R_r L_m^2}{(L_r + L_m)^2 (L_s + L_m) - (L_r + L_m) L_m^2}, \ \eta_5 = \frac{R_r L_m}{(L_r + L_m)^2 (L_s + L_m) - (L_r + L_m) L_m^2}, \ \eta_6 = \frac{L_m n_p}{(L_r + L_m) (L_s + L_m) - L_m^2},$   $\eta_7 = \frac{(L_r + L_m)}{(L_r + L_m) (L_s + L_m) - L_m^2}, \ \eta_8 = \eta_4, \ \eta_9 = \eta_5, \ \eta_{10} = \eta_6,$   $\eta_{11} = \eta_7, \ d_1 = \frac{3}{2} \frac{n_p L_m}{J (L_r - L_m)}, \ d_2 = \frac{1}{J}, \ d_3 = \frac{k_d}{J} \ \text{where} \ R_s$ and  $L_s$  are the resistance and inductance of the stator, respectively.  $k_d$  is the viscous friction,  $\Theta = \omega_r$  is the rotor speed and  $\Gamma = T_L$  is the load torque and J is the rotor moment of inertia.

#### 2.3 Three-phase Linear

For the three-phase linear induction motor in  $\alpha\beta$  frame, the voltages are presented of the form

$$v_{\alpha s} = v_s \sin(\omega t) \tag{7}$$

$$v_{\beta s} = -v_s \sin(\omega t). \tag{8}$$

Thus, in this case, the parameters are:  $\eta_1 = \frac{R_r}{L_r}, \eta_2 = n_p\left(\frac{\pi}{\tau}\right), \eta_3 = \frac{R_r L_m}{L_r}, \eta_4 = \frac{R_s}{\left(\frac{L_s^2 L_r - L_s Lm^2}{L_s L_r}\right)} + \frac{1 - \left(\frac{L_s L_r - Lm^2}{L_s L_r}\right)}{\left(\frac{L_s L_r - Lm^2}{L_s L_r}\right)}, \eta_5 = \frac{L_m R_r}{\left(\frac{L_s L_r - Lm^2}{L_s L_r}\right) L_s L_r^2}, \eta_6 = n_p\left(\frac{\pi}{\tau}\right) \frac{L_m}{\left(\frac{L_s L_r - Lm^2}{L_s L_r}\right) L_s L_r}, \eta_7 = \frac{1}{\left(\frac{L_s L_r - Lm^2}{L_s L_r}\right) L_s}, \eta_8 = \eta_4, \eta_9 = \eta_5, \eta_{10} = \eta_6, \eta_{11} = \eta_7, d_1 = \frac{3n_p \pi L_m}{2L_r \tau M}, d_2 = \frac{1}{M}, d_3 = \frac{D}{M}$  where  $R_s$  and  $L_s$  are the resistance and inductance of the stator, respectively.  $\tau$  is the pole pitch M is the total mass of the moving

the resistance and inductance of the stator, respectively.  $\tau$  is the pole pitch, M is the total mass of the moving element, D is viscous friction,  $\Theta = v$  is the linear velocity and  $\Gamma = F_L$  is the external force.

It can be noted that for all models in  $\alpha\beta$  frame, the meaning of some parameters are the same as:  $R_r$  and  $L_r$  are the rotor resistance and inductance, respectively,  $L_m$  is the magnetization inductance,  $n_p$  is the number of pair of poles.

#### 3. UNIFORM OBSERVER FOR INDUCTION MOTORS

#### 3.1 Observer Design

For the observer design, the availability of continuous measurements of motor speed and currents is assumed. In addition the mechanic load  $\Gamma$  is considered as an unknown and slowly-varying perturbation to be estimated, that is  $\dot{\Gamma} = 0$ . Thus, the system (1) can be written in the following block-wise form:

$$\dot{x}_1 = B_1(x_1)x_2 + f_1(x_1, u) 
\dot{x}_2 = B_2(x_1)x_2 + f_2(x_1) 
y = x_1$$
(9)

where 
$$x = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T$$
 and, the blocks are  $x_1 = \begin{bmatrix} \Theta & i_{\alpha s} & i_{\beta s} \end{bmatrix}^T$   
and,  $x_2 = \begin{bmatrix} \lambda_{\alpha r} & \lambda_{\beta r} & \Gamma \end{bmatrix}^T$ , with  $u = \begin{bmatrix} v_{\alpha s} & v_{\beta s} \end{bmatrix}^T$ . Here  
 $B_1(x_1) = \begin{bmatrix} d_{1i\beta s} & d_{1i\alpha s} & -d_2 \\ \eta_5 & -\eta_6 \Theta & 0 \\ \eta_{10} \Theta & \eta_9 & 0 \end{bmatrix}$ ,  $f_1(x_1, u) = \begin{bmatrix} -d_3 \Theta \\ \eta_7 v_{\alpha s} - \eta_4 i_{\alpha s} \\ \eta_{10} \Theta \end{bmatrix}$   
 $B_2(x_1) = \begin{bmatrix} -\eta_1 & \eta_2 \Theta & 0 \\ -\eta_2 \Theta - \eta_1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$  and,  $f_2(x_1) = \begin{bmatrix} \eta_{3i\alpha s} \\ \eta_{3i\beta s} \\ 0 \end{bmatrix}$ .

Based on the system (9), the following observer is proposed in order to provide a uniform finite estimation of the state x:

$$\dot{\hat{x}}_1 = B_1(x_1)\hat{x}_2 + f_1(x_1, u) + M_1\phi_1(\tilde{x}_1) \dot{\hat{x}}_2 = B_2(x_1)\hat{x}_2 + f_2(x_1) + B_1^{-1}(x_1)M_2\phi_2(\tilde{x}_1)$$
(10)

where  $\hat{x}_1$  and  $\hat{x}_2$  are the estimates of  $x_1$  and  $x_2$ , respectively and, the observer errors are given by  $\tilde{x}_1 = \hat{x}_1 - x_1$  and  $\tilde{x}_2 = \hat{x}_2 - x_2$ . The observer inputs  $\phi_1(\tilde{x}_1)$ , and  $\phi_2(\tilde{x}_1)$  are defined as

$$\phi_2(\tilde{x}_1) = \frac{\mu_1}{2} \operatorname{sign}(\tilde{x}_1) + 2\mu_2^2 \tilde{x}_1 + \frac{3}{2}\mu_1 \mu_2 |\tilde{x}_1|^2 \operatorname{sign}(\tilde{x}_1)$$

with  $\mu_1 \geq 0$ ,  $\mu_2 \geq 0$  scalars,  $M_1 = \text{diag}(m_{1,1}, m_{1,2}, m_{1,3})$ and  $M_2 = \text{diag}(m_{2,1}, m_{2,2}, m_{2,3})$  are two  $3 \times 3$  diagonal matrices with positive entries that are the gains of the observer. The function  $(\bullet)^{\frac{1}{2}}$  is extended to the form  $\xi^{\frac{1}{2}} = (\xi_1^{\frac{1}{2}}, \ldots, \xi_{n_i}^{\frac{1}{2}})$  for the expression  $|\tilde{x}_i|^{\frac{1}{2}}$ , a similar definition for  $(\bullet)^{\frac{3}{2}}$  must be underteed, and in the correspondence of

for  $(\bullet)^{\frac{1}{2}}$  must be understood, and in the expressions of the form  $|\tilde{x}_i|^p \operatorname{sign}(\tilde{x}_i)$  the product is element to element,  $B_1^{-1}(x_1)$  is the inverse of the matrix  $B_1(x_1)$  and is given by

$$B_1^{-1}(x_1) = \begin{bmatrix} 0 & \frac{\eta_3}{\eta_6\eta_{10}\Theta^2 + \eta_5\eta_9} \\ 0 & -\frac{\eta_{10}\Theta}{\eta_6\eta_{10}\Theta^2 + \eta_5\eta_9} \\ -\frac{1}{d_2} & \frac{d_1(\eta_9i_{\beta s} + \eta_{10}i_{\alpha s}\Theta)}{d_2(\eta_6\eta_{10}\Theta^2 + \eta_5\eta_9)} & -\frac{\frac{\eta_6\Theta}{\eta_6\eta_{10}\Theta^2 + \eta_5\eta_9}}{d_1(\eta_5i_{\alpha s} - \eta_6i_{\beta s}\Theta)} \\ -\frac{1}{d_2} & \frac{d_1(\eta_9i_{\beta s} + \eta_{10}i_{\alpha s}\Theta)}{d_2(\eta_6\eta_{10}\Theta^2 + \eta_5\eta_9)} & -\frac{d_1(\eta_5i_{\alpha s} - \eta_6i_{\beta s}\Theta)}{d_2(\eta_6\eta_{10}\Theta^2 + \eta_5\eta_9)} \end{bmatrix}$$

### 3.2 Convergence Analysis

To analyze the observer convergence, consider the dynamics of the errors  $\tilde{x}_1$  and  $\tilde{x}_2$ . From (9) and (10) it follows

$$\tilde{x}_1 = B_1(x_1)\tilde{x}_2 - M_1\phi_1(\tilde{x}_1) 
\dot{\tilde{x}}_2 = B_2(x_1)\tilde{x}_2 - B_1^{-1}(x_1)M_2\phi_2(\tilde{x}_1).$$
(12)

Defining  $q = B_1(x_1)\tilde{x}_2$ , the system (12) is transformed to

$$\dot{\hat{x}}_1 = q + M_1 \phi_1(\tilde{x}_1) 
\dot{q} = B(x_1)q + M_2 \phi_2(\tilde{x}_1)$$
(13)  
where  $B(x_1) = \left[\dot{B}_1(x_1) + B_1(x_1)B_2(x_1)\right] B_1^{-1}(x_1).$ 

The equation (13) is in the form of the so-called *generalized*  
super-twisting Cruz-Zavala et al. (2010). Hence, with a  
suitable choice of the matrices 
$$M_1$$
 and  $M_2$ , the system  
(12) with (11) is globally uniformly finite time stable.

#### 4. SIMULATION RESULTS

This section shows numerical simulations results of the proposed observer for each case of induction motor. For the simulation purpose for all cases, the initial conditions of the state variables of the observer are selected equal to one. The tracking signals are the flux and the load torque these is introduced as step form.

# '4.1 Single Phase Induction Motor

For Single-phase induction motor the parameter are presented as Krause P. C. (2002):

H.P.	Single-Phase 0.25	$V_s$	110 (V)
f	60 (Hz)	$n_p$	2
$n = \frac{N_A}{N_B}$	1.18	$R_{\alpha s}$	$2.02~(\Omega)$
$R_{\beta s}$	$5.13~(\Omega)$	$R_r$	$4.12~(\Omega)$
$L_{\alpha s}$	0.1846(H)	$L_{\beta s}$	0.1833 (H)
$L_r$	0.1828 (H)	$L_m$	0.1772 (H)
J	$0.0146 \ (Kgm^2)$	$k_d$	$0 \ (kgm^2/s)$
$I_{\max}$	15 (A)	$C_{run}$	$35 \ \mu f$
$\mu_1$	1	$\mu_2$	1
$m_{11}$	640	$m_{12}$	640
$m_{13}$	630	$m_{21}$	64000
$m_{22}$	64000	$m_{23}$	155.5
1	64000	$m_{23}$	155.5

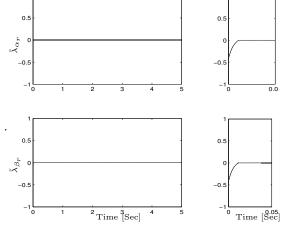


Fig. 1. Error of rotor flux  $\tilde{\lambda}_{\alpha_r}$  and  $\tilde{\lambda}_{\beta_r}$  of SPIM.

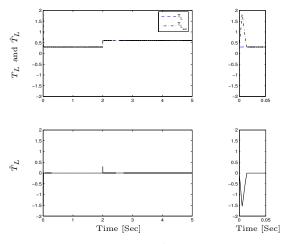


Fig. 2. Load torque estimated  $\hat{T}_L$  and error of load torque  $\tilde{T}_L$  of SPIM.

## 4.2 Three Phase Induction Motor

For three-phase induction motor the parameter are presented as Krause P. C. (2002):

	Three-phase		
H.P.	3	$V_s$	220 (V)
f	60 (Hz)	$n_p$	2
$R_s$	$0.435~(\Omega)$	$R_r$	$0.816~(\Omega)$
$L_s$	$0.0020({\rm H})$	$L_r$	$0.0020~({\rm H})$
$L_m$	0.0693 (H)	M	2.78~(kg)
J	$0.0089~((Kgm^2)$	$k_d$	$0~(kgm^2/s)$
$I_{\rm max}$	18 (A)		
$\mu_1$	1	$\mu_2$	1
$m_{11}$	640	$m_{12}$	640
$m_{13}$	357.25	$m_{21}$	64000
$m_{22}$	64000	$m_{23}$	1.5

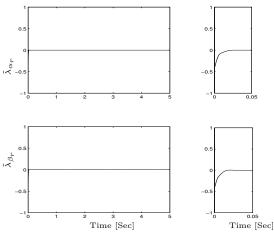


Fig. 3. Error of rotor flux  $\tilde{\lambda}_{\alpha_r}$  and  $\tilde{\lambda}_{\beta_r}$  of TIM.

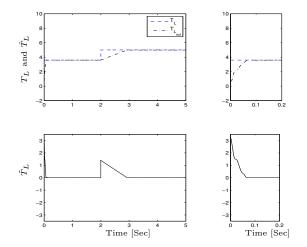


Fig. 4. Load torque estimated  $\hat{T}_L$  and error of load torque  $\tilde{T}_L$  of TIM.

# 4.3 Three-Phase Linear Induction Motor

For three-phase linear induction motor the parameter are presented as Boldea and Nasar (1997):

	Three-phase linear		
H.P.	4	$V_s$	180 (V)
f	60 (Hz)	$n_p$	2
$R_s$	$5.3685~(\Omega)$	$R_r$	$3.5315~(\Omega)$
$L_s$	0.02846(H)	$L_r$	$0.02846~({\rm H})$
$L_m$	0.02419 (H)	M	2.78~(kg)
D	36.0455~(Kg/s)	au	0.027~(m)
$I_{\rm max}$	14.2 (A)		
$\mu_1$	1	$\mu_2$	1
$m_{11}$	640	$m_{12}$	640
$m_{13}$	45	$m_{21}$	64000
$m_{22}$	64000	$m_{23}$	20

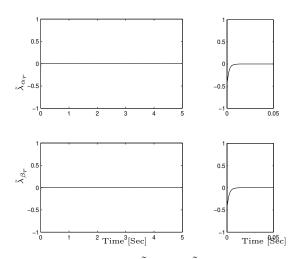


Fig. 5. Error of rotor flux  $\tilde{\lambda}_{\alpha_r}$  and  $\tilde{\lambda}_{\beta_r}$  of TLIM.

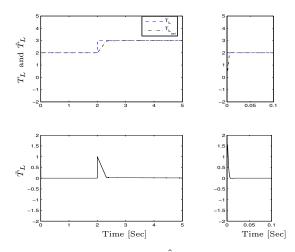


Fig. 6. Load torque estimated  $\hat{T}_L$  and error of load torque  $\tilde{T}_L$  of TLIM.

In Fig. 1, 3 and 5 the time evolutions of the rotor flux  $\tilde{\lambda}_{\alpha_r}$ and  $\tilde{\lambda}_{\beta_r}$  errors of induction motors are shown, while Fig. 2, 4 and 6 present the time evolution of the estimated load torque  $\hat{T}_L$  and the load estimation error  $\tilde{T}_L$  of induction motors cases.

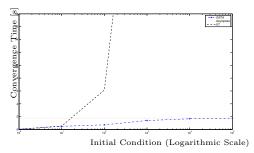


Fig. 7. Convergence time of both observers by growing initial condition norm.

The Figure 7 presents a comparison of the proposed observer with another which uses the so-called *super-twisting* Levant (1993) algorithm, that means fixing  $\mu_1 = 1$  and  $\mu_2 = 0$  in (11). Here is highlighted that the convergence time for the super-twisting grows unboundedly with the norm of the initial condition, while the convergence time of the uniform observer is asymptotically bounded by a constant for growing initial condition's norm.

### 5. CONCLUSIONS

In this work a uniform observer scheme based on the model on the stationary frame  $\alpha\beta$  for induction motors are proposed. The flux and load torque were estimated, all of them are shown to give appreciable results in order of convergence time to estimate the rotor flux and the load torque.

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