Supervisory Control of Discrete Event Systems using Observers

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Abstract— **State-feedback Supervisory Controllers for Discrete Event Systems are synthesized. A state Observer providing the feedback information is integrated to the control scheme. Petri Nets models are used for modeling the system, the controller and the observer. The required behavior of the system is specified as Linear Constrains. The proposed technique allows the construction of controllers for systems with partial state observation. An application example is provided.**

Keywords— **Supervisory Control, Discrete-Even systems, Observability, Petri Nets**

I. Introduction

The design of controllers for Discrete Event Systems (DES) has been addressed in different ways. In Supervisory Control Theory (SCT) [4],[7], the problem of supervising a plant, modeled as a finite automata or as a vector-additive system, has been introduced. In case of a finite automata, the specifications are represented as a sublanguage of the automata modeling the plant. In vector-additive systems, specifications are represented as linear inequalities. However, these approaches do not consider a state observer in their control scheme reducing the usability of the technique.

In Petri Net (PN) control techniques [8], systems and their specifications are modeled as PN's. Nevertheless, these approaches assume that every state variable is measurable. Consequently, the cases where the system state is realistically partially observed are not considered.

In Control by Monitor Places (CM) [6], the system is also modeled as a PN. However, the specifications are represented as a linear inequality, similar to [4]. Unfortunately, as occurs in [4], the feedback information came exclusively from the system outputs.

In this paper, the problem of controlling a DES using a state-feedback supervisory controller is presented. The plant, the closed-loop system and the state observer are modeled as PN's. The technique is a generalization of that one presented in [6] but this approach considers upper and lower bounds for the state variables. Additionally, the cases when not all the system variables are measured is considered. An state observer is used for reconstruction the missing information. The proposed technique ensures the fulfillment of the specifications, even during the transitory period of the observer.

This paper is organized as follows. Section II reviews PN notation and concepts used in this article. Section III introduces the SCT techniques from the PN point of view. Section IV presents Observability concepts within the PN framework. Finally, section V provides a technique for controlling a plant where the controller uses the estimations produced by an observer as feedback information. An application example illustrates the techniques. Finally, section VI provides conclusions.

II. Petri Nets

The PN framework is suitable for the modeling and control of DES. An illustrative PN survey is presented in [2]. This work uses Interpreted Petri Nets (IPN) an extension to the Petri Net models [5].

Definition 1: An Interpreted Petri Net IPN is a 4-tuple $(N, \Sigma, \lambda, \varphi)$ where: $N = (Q, M_0)$ is a PN system, where Q is a PN structure and M_0 is the intial marking; $\Sigma = \{\alpha_1, \alpha_2, ..., \alpha_r\}$ is the input alphabet, where each α_i is an input symbol; $\lambda : T \rightarrow \Sigma \cup \{\varepsilon\}$ is a transition labeling function, where the symbol ε usually represents internal events in a DES; $\varphi : R(Q, M_0) \rightarrow \{\mathbb{Z}^+\}^{q}$ is an output function that associates each marking in $R(Q, M_0)$ to an output vector of dimension q, where q is the number of system outputs and $\mathbb{Z}^+$ represents the set of non-negative integers.

When a transition t_k fires in a marking M_k reaching M_{k+1}, denoted $M_k \xrightarrow{t_k} M_{k+1}$, is computed as:

$$M_{k+1} = M_k + C \cdot \vec{t}_k, \quad y_k = \varphi \cdot M_k \tag{1}$$

where C and $\vec{t}_k$ are the incidence matrix of (Q, M_0) and the Parikh vector of t_k, respectively. The vector $y_k \in (\mathbb{Z}^+)^q$ is the $k-th$ output vector of the IPN.

Accordingly to function λ, if $\lambda(t_i) \neq \varepsilon$, then t_i is said to be a *manipulated* transition; otherwise *non-manipulated*. Pictorially, non-manipulated transitions are gray filled. The submatrix formed by the columns in C corresponding to non-manipulated transitions is denoted by Q_{nm}.

Definition 2: A firing transition sequence is a sequence $\sigma = t_i t_j ... t_k ... t_l$, such that $M_0 \xrightarrow{t_i} M_1 \xrightarrow{t_j} ... M_w \xrightarrow{t_k} M_k ... M_l \xrightarrow{t_l} M_r$. The length $|\sigma|$ of σ is the number of transition in σ. The prefix set of σ is denoted by $\bar{\sigma}$. The firing language $\pounds(Q, M_0)$ of (Q, M_0) is the set of firing transition sequences. Given a $\sigma = t_i t_j ... t_k ... t_l \in \pounds(Q, M_0)$, such that $M_0 \xrightarrow{t_i} M_1 \xrightarrow{t_j} ... M_w \xrightarrow{t_k} M_k ... M_l \xrightarrow{t_l} M_r$, the output word associated to σ is $\varphi(\sigma) = \varphi(M_0)\varphi(M_1)...\varphi(M_w)\varphi(M_k)...\varphi(M_l)\varphi(M_r)$.

Let $\sigma = t_i t_j ... t_k ... t_l \in \pounds(Q, M_0)$. The Parikh vector $\overrightarrow{\sigma} : T \longrightarrow (\mathbb{Z}^+)^m$ of σ maps every transition $t_r \in T$ to the number of its occurrences in σ, where $m = |T|$. Given a vector V, that can be a firing vector or a Marking vector, $[V]$ denotes the support of V, i.e., the set of elements in V different from 0.

Remark 1: This work considers safe nets (1-bounded nets) which markings are known as safe markings [2]. A safe marking which puts a token in every place in the set of places $\{p_i, ..., p_k\}$ will be expressed as $M_{\{p_i,...,p_k\}}$.

The following example presents an IPN model.

Example 1: The Fig. 1a) depicts an IPN model of a production system. It is composed by two machines coupled to three buffers. Machine 1 (K_1) performs two different processes over incoming inventory parts. These processes are represented by the transition loops from t_1^1 to t_1^6 and from t_1^7 to t_1^{12}, respectively. Machine K_1 through the first sequence (t_1^1 to t_1^6) process two different semifinished parts type A_5 and A_6, delivering them into Buffer 1 and 2, as shown. The first sequence processes two parts of type A_5, through A_1 and C_1, and one part of type A_6, through B_1. Similarly, the second sequence (t_1^7 to t_1^{12}) processes parts A_5 and A_6, but in this case, two parts of type A_6 and one part of type A_5. Then, Machine K_1 balances the buffers capacity depending on the selected process sequence. Machine 2 (I_2) delivers parts from Buffers 1 and 2 to Buffer 3 firing the transitions t_2^5 and t_2^{10}.

III. Supervisory Control of IPN

Linear constraints (LC) are specifications for restricting the system behavior used in the SCT framework [4],[6]. In an IPN, a linear constrain represents a bound in the number of tokens that the places hold:

$$lX \leq q \tag{2}$$

where every $a_i, b \in \mathbb{Z}^+$ for $1 \leq i \leq n$, $l := [a_1, ..., a_n]$ and $X := [p_1, ..., p_n]$. Every p_i is a token-bounded net place. A generalization for a linear constrain (GLC) results from extending a LC to a lower bound:

$$\vec{q}_{lwr} \leq LX \leq \vec{q}_{upp} := \bigwedge_{i=1}^{n} \left(q_{lwr}^i \leq l_i X \leq q_{upp}^i\right) \tag{3}$$

where $\vec{q}_{lwr}$ and $\vec{q}_{up}$ are vectors and L is a matrix.

Next example illustrates a GLC.

Example 2: Let (Q, M_0) be the IPN depicted in Fig. 1a). Let $G := (q_5^1 \leq p_5^1 \leq q_5^2) \wedge (q_6^1 \leq p_6^1 \leq q_6^2) \wedge (q_7^1 \leq p_7^1 \leq q_7^2)$ be a GLC. The hyper-volume depicted in Fig. 1b), representing the GLC G, denotes buffer constrains in the production line. Notice that only the places of interest are shown for an intuitive visualization. Every state inside the hyper-volume satisfies to G.

A technique for enforcing LC's in the form of the equation (2) is found in [6]. It consists on the inhibition of the transition firings that otherwise would lead to a violation in the constrains. Such inhibition is achieved by means of "control places" linked to an IPN model [6],[4].

The inequality (2) is transformed into equality adding extra "slack-variables" [6], as shown in equation (4).

$$lX + X_c = \vec{q} \tag{4}$$

The term X_c represents the set of control places. The closed-loop system is another IPN $\left(D, \breve{M}_0\right)$ given by:

$$D = \begin{bmatrix} Q \\ D_c \end{bmatrix}, \quad \breve{M}_0 = \begin{bmatrix} M_0 \\ M_{0c} \end{bmatrix} \tag{5}$$

where D_c represents an "extra-structure" added to the system model. M_{0c} is a suitable marking for the control places [6].

If the restriction has the form of equation (3), the next theorem gives a solution.

Theorem 1: Let (Q, M_0) be an IPN where every transition is both *distinguishable* and controllable. Let $\vec{q}_{lwr} \leq LX \leq \vec{q}_{upp}$ be a GLC given by equation (3). If:

$$\vec{q}_{upp} - LM_0 \geq 0, \quad LM_0 - \vec{q}_{lwr} \geq 0 \tag{6}$$

holds, then the PN controller (D_c, M_{0c}) in equation (7):

$$D_c = \begin{bmatrix} -LQ \\ LQ \end{bmatrix}, \quad M_{0c} = \begin{bmatrix} \vec{q}_{upp} - LM_0 \\ LM_0 - \vec{q}_{lwr} \end{bmatrix} \tag{7}$$

enforces the GLC $\vec{q}_{lwr} \leq LX \leq \vec{q}_{upp}$ over (Q, M_0).

Proof: The IPN structure of the closed-loop system using equations (5) and (7) is:

$$D = \begin{bmatrix} Q \\ -LQ \\ LQ \end{bmatrix}, \quad \breve{M}_0 = \begin{bmatrix} M_0 \\ \vec{q}_{upp} - LM_0 \\ LM_0 - \vec{q}_{lwr} \end{bmatrix} \tag{8}$$

The GLC is divided into the single LC's:

$$LM' \leq \vec{q}_{upp} \tag{9}$$

and:

$$\vec{q}_{lwr} \leq LM' \tag{10}$$

To verify that inequality (9) is satisfied, consider the sub-system $\left(D_{c_1}, \breve{M}_{0c_1}\right)$ defined as:

$$D_{c_1} = \begin{bmatrix} Q \\ (-LQ) \end{bmatrix}, \quad \breve{M}_{0_{c_1}} = \begin{bmatrix} M_0 \\ \vec{q}_{upp} - LM_0 \end{bmatrix} \tag{11}$$

The equation (7) ensures that:

$$LM_0 + M_{0c_1} = \vec{q}_{upp} \tag{12}$$

where $M_{0c_1} = [\vec{q}_{upp} - LM_0]$.

Now, let $P_{L+c_1} := \begin{bmatrix} P_L & \vdots & P_{c_1} \end{bmatrix}$ be the row vector formed by those places in L and the control places in D_{c_1}. Multiplying $P_{L+c_1} D_{c_1}$ results in $P_L Q + (-LQ)$, since the control places in D_{c_1} are exactly those in $(-LQ)$.

Moreover, $P_L Q = LQ$. Therefore, $P_{L+c_1} D_{c_1} = P_L Q + (-LQ) = LQ + (-LQ) = 0$. As a consequence, every reachable marking M' satisfies the equation $LM' + M'_{c_1} = \vec{q}_{upp}$ and accordingly, $LM' \leq \vec{q}_{upp}$ as required.

To verify that inequality (10) is satisfied, consider the sub-system $\left(D_{c_2}, \breve{M}_{0c_2}\right)$ defined as:

$$D_{c_2} = \begin{bmatrix} (-LQ) \\ LQ \end{bmatrix}, \quad \breve{M}_{0_{c_2}} = \begin{bmatrix} \vec{q}_{upp} - LM_0 \\ LM_0 - \vec{q}_{lwr} \end{bmatrix} \tag{13}$$

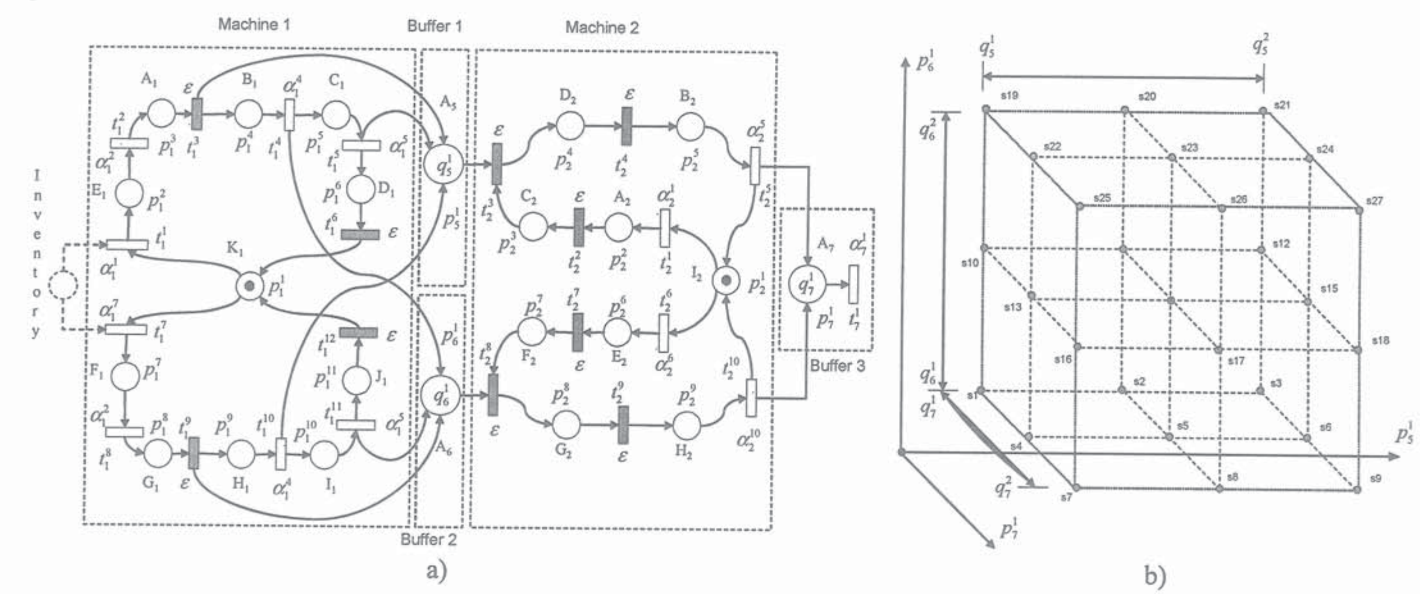

Fig. 1. a) An IPN model for a Production System; b) A convex volume produced by a GLC

The inequality (10) can be transformed as:

$$\vec{q}_{lwr} \leq \vec{q}_{upp} - M_{0c_1} \quad (14)$$

since by equation (12) it is hold that $LM_0 = \vec{q}_{upp} - M_{0c_1}$.

Then, equation (14) can be written as:

$$M_{0c_1} \leq \vec{q}_{upp} - \vec{q}_{lwr} \quad (15)$$

That is, the inequality (10) is satisfied restricting the tokens in the places of equation (11).

Again, equations (7) and (14) ensure that, for the subsystem given by equation (13), the following holds:

$$M_{c_1} + M_{c_2} = \vec{q}_{upp} - \vec{q}_{lwr} \quad (16)$$

where $M_{0c_2} = [LM_0 - \vec{q}_{lwr}]$.

From equation (13) it follows that the identity row vector, denoted as P_I, is a positive S-invariant that includes every place of D_{c_2}. Therefore, equations (16) and (10) are kept for any reached marking.

It follows that the extended row vectors $P_{upp} := \begin{bmatrix} P_{L+c_1} & \vdots & 0 \end{bmatrix}$ and $P_{lwr} := \begin{bmatrix} 0 & \vdots & P_I \end{bmatrix}$ are place invariants for the closed-loop system described by equation (8).

Finally, the result follows from the fact that equations (12) and (16) are valid for every marking in the closed-loop system of equation (8). As consequence, inequalities (10) and (9) are satisfied for every reachable marking and the result holds. ■

By Theorem (1):

$$LM + M_{c_1} = \vec{q}_{upp} \quad (17)$$

for every marking M in the closed-loop system. That is, the marking in the control places for the LC $LX \leq \vec{q}_{upp}$ is always determined by $\vec{q}_{upp} - P_L$. From this fact, the next theorem follows.

Theorem 2: Let (Q, M_0) be an IPN and let $\vec{q}_{lwr} \leq LX \leq \vec{q}_{upp}$ be a GLC. Then L can be enforced on (Q, M_0) by means of a supervisor if there exist linear transformations R_1, R_2, R_1' and R_2' such that:

$$\begin{gathered} (R_1 + R_2 L) \cdot Q_{nm} \leq 0 \\ (R_1 + R_2 L) \cdot M_0 \leq R_2 \cdot \left(\vec{q}_{upp} + \vec{1}\right) - \vec{1} \end{gathered} \quad (18)$$

and

$$\begin{gathered} (R_1' + R_2'(-L)) \cdot Q_{nm} \leq 0 \\ (R_1' + R_2'(-L)) \cdot M_{0c_1} \leq R_2' \cdot \left(\vec{q}_{upp} - \vec{q}_{lwr} + \vec{1}\right) - \vec{1} \end{gathered} \quad (19)$$

Proof: If linear transformations satisfying equation (18) exist, then Theorem 1 guarantees that the LC $LX \leq \vec{q}_{upp}$ can be enforced.

If equation (19) is satisfied, then $(R_1' + R_2'(M_{c_1} - \vec{q}_{upp})) \cdot Q_{nm} \leq 0$ is also satisfied, since by equation (17), it is hold that $-L = M_{c_1} - \vec{q}_{upp}$. It means that $(R_1' + R_2' M_{c_1} - R_2' q_{upp}) \cdot Q_{nm} \leq 0$, or equivalently, $(R_3' + R_2' M_{c_1}) \cdot Q_{nm} \leq 0$, for $R_3' = R_1' - R_2' \cdot \vec{q}_{upp}$, as required. ■

For the computation of the linear transformations R_1, R_2, R_1' and R_2', the next extended version of the Linear Optimization Problem (LOP) in [6] is used.

Algorithm 1: Computation of Linear Transformations

Inputs: An IPN (Q, M_0) and a GLC $\vec{q}_{lwr} \leq LX \leq \vec{q}_{upp}$ in the form of equation (3).

Outputs: Linear transformations R_1, R_2, R_1' and R_2'.

Let $R := \begin{bmatrix} R_1 & R_2^* & R_3 \end{bmatrix}$, where $R_2^* := R_2 - \vec{1}$. The

LOP for the LC $LX \leq \vec{q}_{upp}$ is constructed as:

$$\min_{R}\left(R\begin{bmatrix} M_0 \\ LM_0-\vec{q}-\vec{1} \\ \vec{0}\end{bmatrix}\right)$$
$$s.t.\left\{\begin{array}{c} R\begin{bmatrix} Q_{nm} \\ LQ_{nm} \\ I\end{bmatrix} = -LQ_{nm} \\ R\geq 0, R\neq LX \end{array}\right\} \tag{20}$$

Let $R' := \begin{bmatrix} R'_1 & R^{*\prime}_2 & R'_3 \end{bmatrix}$, where $R^{*\prime}_2 := R'_2-\vec{1}$. The LOP for the LC $\vec{q}_{lwr} \leq LX$ is constructed as:

$$\min_{R'}\left(R'\begin{bmatrix} M_0 \\ -LM_0-\vec{q}-\vec{1} \\ \vec{0}\end{bmatrix}\right)$$
$$s.t.\left\{\begin{array}{c} R'\begin{bmatrix} Q_{nm} \\ -LQ_{nm} \\ I\end{bmatrix} = LQ_{nm} \\ R'\geq 0, R'\neq LX \end{array}\right\} \tag{21}$$

Once the matrices R_1, R_2, R'_1 and R'_2 are computed, the controller structure is obtained as follows.

Theorem 3: Suppose that the system has only distinguishable transitions. Given the linear transformations R_1, R_2, R'_1 and R'_2, the controller computed as:

$$D_c = \begin{bmatrix} -(R_1+R_2L)\cdot Q \\ R'_1+R'_2(-L)\cdot Q\end{bmatrix}$$
$$M_{0c} = \begin{bmatrix} \vec{q}_{upp}-(R_1+R_2L)\cdot M_0 \\ -(R'_1+R'_2(-L))\cdot M_0-\vec{q}_{lwr}\end{bmatrix} \tag{22}$$

enforces the GLC $\vec{q}_{lwr} \leq LX \leq \vec{q}_{upp}$ over (Q, M_0).

The proof of the previous theorem is straightforward from Theorem 2 and is omitted due to space requirements.

The following example illustrates the theorems.

Example 3: The uncontrollable matrix Q_{nm} of the IPN model in Fig. 1 is formed by the transitions $\{t_1^3, t_1^6, t_1^9, t_1^{12}, t_2^2, t_2^3, t_2^4, t_2^7, t_2^8, t_2^9\}$. The LC's are $l_1 = (q_5^1 \leq p_5^1 \leq q_5^2)$, $l_2 = (q_6^1 \leq p_6^1 \leq q_6^2)$ and $l_3 = (q_7^1 \leq p_7^1 \leq q_7^2)$. The GLC is $G := l_1 \wedge l_2 \wedge l_3$.

The LC's l_1 and l_2 hold that $l_1 \cdot Q_{nm} \nleq 0$ and $l_2 \cdot Q_{nm} \nleq 0$. Thus, the linear transformations $R_1^{l_1} = p_1^3$, $R_2^{l_1} = 1$ and $R_1^{l_2} = p_1^8$, $R_2^{l_2} = 1$ are new GLC computed by Algorithm 1, for l_1 and l_2, respectively.

The GLC l_3 directly satisfies that $l_3 \cdot Q_{nm} \leq 0$ and $-l_3 \cdot Q_{nm} \leq 0$. Thus, it does not require the computation of new linear restrictions.

These linear transformations and equations (20) and (21) are used for constructing the closed-loop system that can be depicted as in Fig. 2. Suitable markings have been selected for buffers p_5^1, p_6^1 and p_7^1 and control places c_5^2, c_6^2 and c_7^2.

The system in Fig. 2 guarantees the fulfilment of the GLC. However, the controller requires the exact knowledge about every transition firing in the net. If sensors in the system fail, the synthesized controller is no longer valid and the bounds for the buffers may be violated.

Next section is devoted to the analysis of the observability property within the IPN framework for reconstructing the missing information.

IV. Observability for IPN Models

There exist several approaches dealing with the observability problem in the IPN framework [9],[3],[1]. This section follows the concepts and approach presented in [1].

Definition 3: An IPN (Q, M_0) represented by the equation (1), where M_0 is probably unknown, is Observable if there exists an integer $k < \infty$ such that for any transition sequence $\sigma = t_a t_b ... t_r$ where $M_0 \xrightarrow{t_a} M_1 ... \xrightarrow{t_r} M_q$ and $|\sigma| \geq k$, the output word $\varphi(\sigma) = \varphi(M_0)\varphi(M_1)...\varphi(M_q)$ and the mathematical structure of (Q, M_0) suffice for determinning the initial marking M_0 and the current marking M_q.

In general, deciding the observability property of an IPN is a computational complex problem [9],[3],[1]. Fortunately, the notions of Firing-Vector-Detectability and Marking-Detectability lead to efficient solutions [1]

Definition 4: An *IPN* (Q, M_0) described by the state equation (1), where M_0 is probably unknown, is Firing-Vector-Detectable if there exists an integer $k_P < \infty$ such that the Parikh vector of any transition sequence $\sigma \in \pounds(Q, M_0)$, fulfilling $|\sigma| \geq k_P$, can be uniquely determined using the output word $\varphi(\sigma)$ and the mathematical structure of (Q, M_0).

The Marking-Detectability is defined as follows.

Definition 5: An *IPN* (Q, M_0) described by the state equation (1), where M_0 is probably unknown, is Marking-Detectable if there exists an integer $k_M < \infty$ such that the current marking reached by the firing of any transition sequence $\sigma \in \pounds(Q, M_0)$, fulfilling $|\sigma| \geq k_M$, can be uniquely determined using the output word $\varphi(\sigma)$ and the mathematical structure of (Q, M_0).

Next theorem states that the previous detectability properties are sufficient conditions for Observability.

Theorem 4: A *IPN* (Q, M_0) which is Firing-Vector-Detectable and Marking-Detectable is Observable.

Proof: See [1]. ■

In the case of a safe SM, the Firing-Vector-Detectability implies the Marking-Detectability [1].

Proposition 1: If a strongly connected safe SM is Firing-Vector-Detectable then it is Observable.

Proof: See [1]. ■

The Sequence-Detectability is a stronger property used for testing the Firing-Vector-Detectability [1] in a safe SM.

Definition 6: An IPN (Q, M_0) described by the state equation (1), where M_0 is probably unknown, is Sequence-Detectable if there exists an integer $k_S < \infty$ such that any transition sequence $\sigma \in \pounds(Q, M_0)$, fulfilling $|\sigma| \geq k_S$, can be uniquely determined using the output word $\varphi(\sigma)$, and the mathematical structure of (Q, M_0).

Next theorem gives a sufficient condition for Sequence-Detectability in a strongly connected and safe SM.

Theorem 5: Let (Q, M_0) be a strongly connected safe SM described by the state equation (1). Let bT be a basis for

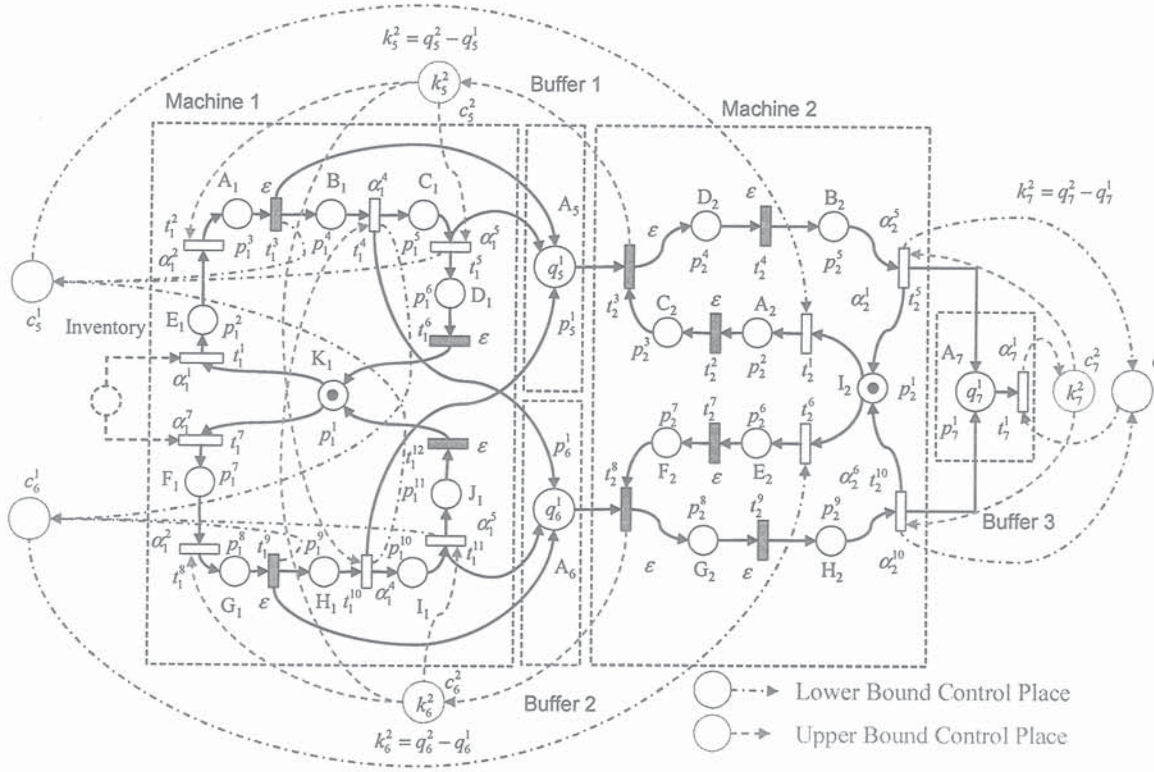

Fig. 2. Closed-loop System

the T-invariants in (Q, M_0). The net is sequence-detectable if:

$$\left(\ker\left(\varphi \cdot C^{-}\right) \cap \ker\left(\varphi \cdot C^{+}\right)\right) \cap span\,(bT) = \vec{0} \qquad (23)$$

Proof: See [1]. ■

The equation (23) is completely algebraic and can be easily tested in polynomial time.

Once a strongly connected and safe SM has verified this equation, a Sequences-Observer is used for producing estimations of the net marking [1].

Algorithm 2: Sequence Observer

Inputs: An IPN (Q, M_0) and the net's output vector $V_O = \varphi \cdot M_k$.

Outputs: Set of possible current markings $\{M_k\}$ and possible fired transitions $\{t_k\}$.

Let $N = (P, T, I, O)$ be the underlying net structure of (Q, M_0) and let C be its incidence matrix.

(Initialization) Initialize the Sequence Observer as follows:

- Let $\{M_k\} := \left\{M_{\{p_k\}} : \varphi \cdot M_{\{p_k\}} = V_O\right\}$;
- Let $\{t_k\} := \{\varepsilon\}$;
- Let $S := \{\varepsilon\}$;

(Estimations) Once a chance ΔV_O occurs, do:

1. Let σ be the top of S;
2. If $\sigma = \varepsilon$, let $U := \left\{t_u \in T : \varphi \cdot C \cdot \vec{t}_u = \Delta V_O;\right\}$
3. Otherwise $U := \left\{t_u \in T : \varphi \cdot C \cdot \vec{t}_u = \Delta V_O \wedge \sigma\bullet \supseteq \bullet t_u\right\}$
4. Put the set of transitions U in the top of S;
5. Update the stack S in the following way:

(a) Get the element w_{top} in the top of S;

(b) Get the next element w_{next} of S and update $w_{next} := w_{next} \cap \bullet\left(\bullet w_{top}\right)$;

(c) While the next element of S is different from ε, interchange $w_{top} := w_{next}$ and go to point (b);

6. Let w_{top} be the top element in S. Update the set $\{M_k\}$ and $\{t_k\}$ as follows:

(a) $\{M_k\} = \left\{M_{\{p_j\}} : p_j \in t_j\bullet, t_j \in w_{top}\right\}$;

(b) $\{t_k\} = w_{top}$;

7. Return $\{M_k\}$ and $\{t_k\}$.

The algorithm provides both, a set of possible current net markings and a set of possible fired transitions. These elements allow the utilization of controllers requiring state- and event-feedback, as well.

A simple further analysis of equation (23) allows for determining the largest transition sequences required by the observer in the Algorithm 2 to completely reconstruct the net marking and the fired transition. This number is known as the convergence constant of the observer [1].

V. Supervisory Control using Observers

Fig. 3 depicts the proposed control scheme. The controller uses the estimations of the observer for computing a control pattern. However, these estimations may carry an error during a "transitory" period, which could lead the system into a state that violates the specifications given by GLC.

Next theorem states a sufficient condition that allows to a controller for supervising a GLC using the scheme depicted in Fig. 3.

Theorem 6: Let (Q, M_0) be an IPN model representing a plant. Let $\vec{q}_{lwr} \leq LX \leq \vec{q}_{upp}$ be a GLC.

Suppose that the following holds: