High Order Sliding Mode Block Control of Single Phase Induction Motor

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***Abstract*—A new sliding mode observer-based controller for single-phase induction motor is designed. The proposed control scheme is formulated using block control feedback linearization technique and high order sliding mode algorithms with measurements of the rotor speed and stator currents. The stability of the complete closed-loop system included the rotor flux second order sliding mode observer, is analyzed in the presence of model uncertainty, namely, rotor resistance variation and bounded time-varying load torque.**

***Index Terms*—Sliding Mode Control; Single-Phase Induction Motors; Robust Control; Nonlinear Systems.**

I. Nomenclature

$\lambda_{\alpha r}$, $\lambda_{\beta r}$	Rotor magnetic flux linkage components.
$i_{\alpha s}$, $i_{\beta s}$	Stator current components.
$v_{\alpha s}$, $v_{\beta s}$	Voltages of the main and auxiliary stator windings.
ω_r	Rotor speed.
T_L	Load torque.
$R_{\alpha s}$, $R_{\beta s}$	Stator resistances main and auxiliary winding.
R_r	Rotor resistance.
$L_{\alpha s}$, $L_{\beta s}$	Stator inductances main and auxiliary winding.
L_r	Rotor inductance.
L_m	Magnetization inductance.
J	Rotor moment inertia.
np	Numbers of pairs of poles.

II. Introduction

THE aim of this paper is to present an observer-based controller using High Order Sliding Mode (HOSM) algorithms for capacitor-run Single-Phase Induction Motor (SPIM). The SPIM is widely used in many household applications compressors, pumps, air conditioning systems, washer, refrigerators, and other equipment which require low power motors. These motors are basically powered directly from a commercial source phase, and usually operated in an open loop configuration, see [1] and references therein.

To improve the SPIM performance, a feedback control scheme can be designed. This problem consists of three subproblems: *a)* a feedback controller design mainly for speed profile tracking and flux magnitude regulation, *b)* an observer design to estimate the rotor flux, and *c)* stability analysis of the complete system closed by the designed observer-based feedback.

It can be noted that the SPIM dynamical model is similar to three phase motor (TPIM) one. Therefore, to solve the first subproblem, an application of several feedback controllers proposed for TPIM can be considered. For example, a classical vector control with field orientation control (FOC) technique, due to [2]; more recently, the application of back-stepping [3], passivity-based control [4], [5], input-output feedback linearization [6], [7], adaptive [8] and sliding mode (SM) [9]–[13], including neural networks [14], [15] and discrete time [16] controllers. However, the treatment of the SPIM control design problem is different from the TPIM controller, since the SPIM despite of symmetric TPIM has a basic control input which applies to the main winding, and the auxiliary winding is affected by the switched capacitor, it looks like a ”subactuated” system.

To apply, for instance, the vector control strategy for SPIM case [17], a transformation of the state variables was employed [18] eliminating the asymmetries and deriving the symmetrical model. However, this transformation as well as the field orientation control depends on the plant parameters that in practice are subject to variations as a result of a change in the system loading and/or in the system configuration. Moreover, this control scheme does not take into account practical constrain on the auxiliary control input that depends on switching parameter which can take just two values.

Concerning the second subproblem, the rotor flux estimation is usually obtained from machine model and the measurement of the speed, stator voltages and currents [19], [20]. Several flux observers have been proposed using adaptive [21], [22], neural network [14], [15] and sliding mode (SM) [10], [11], [23]–[25] approaches, including the cases of fault detection [26] and fault tolerant [27] schemes. The proposed observers strategies guarantee robustness in the presence of plant model uncertainty. However, they provide only asymptotic convergence of the flux observation error. A robust second order SM finite time observer for an induction motor has been proposed in [24]. This observer provides high efficiency, however, the proposed design procedure requires the knowledge oh the sliding function derivative, which is unknown. Moreover, the proposed sliding function depends on the motor parameters which in practice can vary. It can be noted that, for the SPIM, a flux observer which uses a linear compensator in the form of real differentiator, has been designed in [28] for the nominal plant only, therefore, this observer is sensitives to motor model uncertainty and to

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sensors noise due to using the real differentiator, moreover, stability analysis of the closed-loop system included the observer, was not carried on.

As far as stability analysis of the complete observer/control system is concerned, the separation principle proposed in [29], can be applied. However, this principle was developed for a certain class of nonlinear minimum phase systems that can be presented in observer canonical form, and then a high gain observer can be designed. The induction motor case covers a different observer/controller scenario, and the applicability of the observer/controller scheme described in [29] is questionable, and by far not trivial. A more precise substantiation of the induction motor observer/controller coupling, is necessary.

Revising the previous works it is possible to see that they have failed to design a complete robust SPIM controller which include both the flux observer and speed and capacitor control algorithms. For example, in [30]–[32], [28] proposed a decoupling control vector for SPIM and Flux-Oriented Control, where it is important to remove the pair-electromagnetic pulsating and torque control. But nevertheless there is no an auxiliary control for the switched capacitor, while in [33]–[35], it was considered the problem of controlling the capacitor only without any observer design. Moreover, the proposed separately in the various papers controllers and observers are sensitives the plant parameter variation and external disturbance, namely the load torque.

In this paper, in spite of the previous works, we propose a complete observer-based control scheme for the capacitor-run SPIM in the presence of uncertainty caused by plant parameter variations and external perturbations. The proposed control scheme is based on the motor dynamic model including the capacitor dynamics, described in a stationary reference frame $(\alpha\beta)$ fixed in the stator, that does not require any plant parameter dependent transformation. This scheme includes:

1) A second-order SM observer which is designed to estimate the rotor flux. It could be highlighted that the idea of applying a second order SM algorithm for electrical drives was successively used to design the flux observer in [25]. In spite of these results, the proposed observer is formulated such that the estimation error dynamics have a SM *super-twisting* algorithm structure [36], leading to a finite time and robust estimation. This idea was previous proposed for mechanical systems in [37] and, for the SPIM, was applied only in [38]. The designed observer in spite of the previously works (including the SPIM case [28]) ensures finite time estimation error convergence in presence of motor parameter variation.

2) A motor speed basic controller which is designed using the measured stator current and estimated rotor flux, and applying a combination of *Block Control feedback linearization* [39] and quasi-continuous SM [40] techniques, a sliding manifold on which the control tracking error is finite time zero, is formulated as shown in [41]. Then, again the super twisting SM algorithm for the basic control input and switching logic for the auxiliary input are proposed in order to ensure the designed sliding manifold be a finite time attractive. Comparing with [28] the proposed control scheme takes into account practical constraints, improves the produced motor torque by controlling the rotor flux, and exhibit robustness of the closed-loop system avoiding the estimation of the load torque, and allowing to overcome an uncertainty due to the rotor resistance variations, improving the tracking accuracy. As a result, the designed controller adjusts the equivalent capacitance incrementing the SPIM electromagnetic torque during start-up and improving its performance in steady state reducing pair-electromagnetic pulsating.

Comparing again with previous works, the stability analysis of the proposed complete closed-loop system is carried on. The system is represented in the control and estimation error state variables. Then, by using the SM finite time convergence property, first the reaching phase to the sliding manifold, and then the sliding motion stability is studied. That facilitates the stability analysis of the complete closed-loop system.

In the following, Section III provides the considered model of the SPIM. Sections IV and V describe the proposed observer and controllers, including a detailed stability and robustness analysis. Simulation results which demonstrate the main characteristics of the proposed controller, are presented in Section VII. Finally, in Section VIII the conclusions are given.

III. Mathematical Model for the *SPIM*

The dynamic model of the SPIM can be considered as the model of an unsymmetrical 2-phase (a, b) induction machine in the variables of circuit elements. After the transformation to a fixed frame $(\alpha\beta)$ [42], the single phase induction motor scheme with the stator current and the rotor flux as the state variables, is presented in Fig. 1.

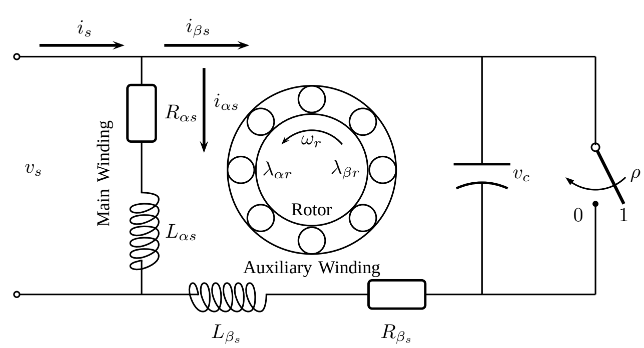

Fig. 1: Single phase induction motor.

and its dynamic equations are given by

$$\begin{aligned}
\frac{di_{\alpha s}}{dt} =& -c_1 a_{10} i_{\alpha s} + c_1 c_{40} \lambda_{\alpha r} - c_1 c_3 n_p \omega_r \lambda_{\beta r} \\
& + c_1 v_{\alpha s} + \Delta_{\alpha s} \\
\frac{di_{\beta s}}{dt} =& -c_2 a_{20} i_{\beta s} + c_2 c_{40} \lambda_{\beta_r} + c_2 c_3 n_p \omega_r \lambda_{\alpha r} \\
& + c_2 v_{\beta s} + \Delta_{\beta s} \\
\frac{d\lambda_{\alpha r}}{dt} =& -a_{30} \lambda_{\alpha r} + n_p \omega_r \lambda_{\beta r} + a_{40} i_{\alpha s} + \Delta_{\alpha r} \\
\frac{d\lambda_{\beta r}}{dt} =& -n_p \omega_r \lambda_{\alpha r} - a_{30} \lambda_{\beta r} + a_{40} i_{\beta s} + \Delta_{\beta r} \\
\frac{d\omega_r}{dt} =& d_1 d_2 \left(\lambda_{\beta r} i_{\alpha s} - \lambda_{\alpha r} i_{\beta s} \right) - d_2 T_L.
\end{aligned} \tag{1}$$

This model considers variations on rotor resistance of the form

$$R_r(t) = R_{r0} + \Delta R_r(t)$$

with $\Delta R_r(t)$ an unknown but bounded function of time, leading to a set of uncertain model parameters

$$\begin{aligned} &a_1(t) = a_{10} + \Delta a_1(t), \quad a_2(t) = a_{20} + \Delta a_2(t), \\ &a_3(t) = a_{30} + \Delta a_3(t), \quad a_4(t) = a_{40} + \Delta a_4(t) \text{ and,} \\ &c_4(t) = c_{40} + \Delta c_4(t) \end{aligned}$$

where $a_{10} = R_{\alpha s} + ({L_m^2}/{L_r^2})R_{r0}$, $a_{20} = R_{\beta s} + ({L_m^2}/{L_r^2})R_{r0}$, $a_{30} = ({1}/{L_r})R_{r0}$, $a_{40} = ({L_m}/{L_r})R_{r0}$ and, $c_{40} = ({L_m}/{L_r^2})R_{r0}$ are the parameter nominal values.

The parametric uncertainties are presented by $\Delta a_1(t) = \Delta a_2(t) = ({L_m^2}/{L_r^2})\Delta R_r(t)$, $\Delta a_3(t) = ({1}/{L_r})\Delta R_r(t)$, $\Delta a_4(t) = ({L_m}/{L_r})\Delta R_r(t)$, and $\Delta c_4(t) = ({L_m}/{L_r^2})\Delta R_r(t)$.

While the model parameters which do not depend on the resistance variations are given by $c_1 = {L_r}/{(L_{\alpha_s} L_r - L_m^2)}$, $c_2 = {L_r}/{(L_{\beta_s} L_r - L_m^2)}$, $c_3 = {L_m}/{L_r}$, $d_1 = ({L_m}/{L_r})n_p$ and $d_2 = {1}/{J}$.

Thus, the unknown terms in (1) are defined by

$$\begin{aligned} \Delta_{\alpha s} &= \Delta c_4(t) c_1 \lambda_{\alpha r} - \Delta a_1(t) c_1 i_{\alpha s}, \\ \Delta_{\beta s} &= \Delta c_4(t) c_2 \lambda_{\beta r} - \Delta a_2(t) c_2 i_{\beta s}, \\ \Delta_{\alpha r} &= -\Delta a_3(t) \lambda_{\alpha r} + \Delta a_4(t) i_{\alpha s} \text{ and,} \\ \Delta_{\beta r} &= -\Delta a_3(t) \lambda_{\beta r} + \Delta a_4(t) i_{\beta s}. \end{aligned}$$

The dynamics of the capacitor (see Fig.1) are given by

$$\frac{dv_c}{dt} = \omega_0 X_c i_{\beta s}$$

where X_c is the capacitor reactance and $\omega_0 = 2\pi f$, with f being the fundamental frequency.

Using the relation between the voltages $v_{\alpha s}$ and $v_{\beta s}$ in (1) of the form

$$\begin{aligned} v_{\alpha s} &= v_s \\ v_{\beta s} &= n^{-1} v_s - v_c \rho \end{aligned} \quad (2)$$

where the switching parameter $\rho \in \{0, 1\}$, the voltage $v_{\beta s}$ yields to

$$v_{\beta s} = \begin{cases} n^{-1} v_s - v_c & \text{if} \quad \rho = 1 \\ n^{-1} v_s & \text{if} \quad \rho = 0 \end{cases}$$

being $n^{-1} v_s$ as a referred voltage of the main winding to the auxiliary winding with $n = N_A/N_B$, where N_A is the number turns of main winding and N_B is the number turns of an auxiliary winding.

The switched capacitor circuit [1], which is shown in Fig. 1, consists of two parts: an ac capacitor and an electronic switch. The switch must allow bidirectional current flow as in a mechanical switch. The switch is short-circuited and opened for each cycle. It is closed the instant the voltage across the capacitor reaches zero. Thus, a zero voltage switching is performed. Before the switch is closed, the current flows through the capacitor which is placed in series with the auxiliary winding. When the switch is short-circuited, the current bypasses the capacitor, and the voltage across the capacitor is zero. As a result, the voltage across the capacitor may change only during the period of the switch being opened. By closing and opening the switch periodically, the effective value of the capacitor appears to be larger than the actual value [35]. When the short-circuited period of the switch is longer, the voltage across the capacitor is lower; however, the current flowing through the auxiliary winding is increased. Therefore, for a fixed frequency, it is obvious that the effective capacitor is increased when the shorting period of the switch is longer. So, it is possible to obtain a larger capacitor value to start the induction motor by using just a small capacitor (running capacitor) in parallel with a switch. The switch has a resistance which ensures the operation frequency of the capacitor and the relationship of change in the current to the switch, and it is not damaged.

IV. Second Order Sliding Mode Observer for Rotor Fluxes

Having the rotor speed ω_r and stator current $i_{\alpha s}$ and $i_{\beta s}$ measurements only, in this section a second order SM observer is designed to estimate the rotor flux.

Consider the following transformation:

$$\begin{aligned} \lambda_{\alpha r}^* &= \lambda_{\alpha r} - l_1 i_{\alpha s} \\ \lambda_{\beta r}^* &= \lambda_{\beta r} - l_2 i_{\beta s} \end{aligned} \quad (3)$$

where l_1 and l_2 are the transformation gains to be chosen later. Using (3), the perturbed flux and current dynamics (1) are represented in new variables $\lambda_{\alpha r}^*$ and $\lambda_{\beta r}^*$ of the form

$$\begin{aligned} \frac{di_{\alpha s}}{dt} =& - p_{11} i_{\alpha s} - p_{12} n_p \omega_r i_{\beta s} - q_3 n_p \omega_r \lambda_{\beta r}^* + q_4 \lambda_{\alpha r}^* \\ &+ c_1 v_{\alpha s} + \Delta_{\alpha s} \\ \frac{di_{\beta s}}{dt} =& - p_{21} i_{\beta s} + p_{22} n_p \omega_r i_{\alpha s} + q_5 n_p \omega_r \lambda_{\alpha r}^* + q_6 \lambda_{\beta r}^* \\ &+ c_2 v_{\beta s} + \Delta_{\beta s} \\ \frac{d\lambda_{\alpha r}^*}{dt} =& - l_{11} \lambda_{\alpha r}^* + l_{12} n_p \omega_r \lambda_{\beta r}^* + \varsigma_{11} n_p \omega_r i_{\beta s} + \varsigma_{12} i_{\alpha s} \\ &- q_1 v_{\alpha s} - l_1 \Delta_{\alpha s} + \Delta_{\alpha r} \\ \frac{d\lambda_{\beta r}^*}{dt} =& - l_{21} \lambda_{\beta r}^* - l_{22} n_p \omega_r \lambda_{\alpha r}^* - \varsigma_{21} n_p \omega_r i_{\alpha s} + \varsigma_{22} i_{\beta s} \\ &- q_2 v_{\beta s} - l_2 \Delta_{\beta s} + \Delta_{\beta r} \end{aligned} \quad (4)$$

where $p_{11} = c_1 a_{10} - l_1 c_1 c_{40}$, $p_{12} = l_2 c_1 c_3$, $p_{21} = c_2 a_{20} - l_2 c_2 c_{40}$, $p_{22} = l_1 c_2 c_3$, $l_{11} = a_{30} + l_1 c_1 c_{40}$, $l_{12} = 1 + l_1 c_1 c_3$, $l_{21} = a_{30} + l_2 c_2 c_{40}$, $l_{22} = 1 + l_2 c_2 c_3$, $\varsigma_{11} = l_{12} l_2$, $\varsigma_{12} = a_{40} - l_{11} l_1 + l_1 c_1 a_{10}$, $\varsigma_{21} = l_{22} l_1$, $\varsigma_{22} = a_{40} - l_{21} l_2 + l_2 c_2 a_{20}$, $q_1 = l_1 c_1$, $q_2 = l_2 c_2$, $q_3 = c_1 c_3$, $q_4 = c_1 c_{40}$, $q_5 = c_2 c_3$, and $q_6 = c_2 c_{40}$.

Based on (4), a nonlinear observer is designed of the form which leads to a second order SM structure of the observer error dynamics (see Section VI). Thus, defining $\hat{\lambda}_{\alpha r}^*$, $\hat{\lambda}_{\beta r}^*$, $\hat{i}_{\alpha s}$, and $\hat{i}_{\beta s}$ as the estimates of $\lambda_{\alpha r}^*$, $\lambda_{\beta r}^*$, $i_{\alpha s}$, and $i_{\beta s}$, respectively, the observer is proposed as follows:

$$\begin{aligned}\frac{d\hat{i}_{\alpha s}}{dt} =& - p_{11}\hat{i}_{\alpha s} - p_{12}n_p\omega_r\hat{i}_{\beta s} - q_3 n_p\omega_r\hat{\lambda}^*_{\beta r} + q_4\hat{\lambda}^*_{\alpha r} \\ & + c_1 v_{\alpha s} + k_{1\alpha}\left|\tilde{i}_{\alpha s}\right|^{\frac{1}{2}}\text{sign}\left(\tilde{i}_{\alpha s}\right) + k_{3\alpha}\tilde{i}_{\alpha s} \\ \frac{d\hat{i}_{\beta s}}{dt} =& - p_{21}\hat{i}_{\beta s} + p_{22}n_p\omega_r\hat{i}_{\alpha s} + q_5 n_p\omega_r\hat{\lambda}^*_{\alpha r} + q_6\hat{\lambda}^*_{\beta r} \\ & + c_2 v_{\beta s} + k_{1\beta}\left|\tilde{i}_{\beta s}\right|^{\frac{1}{2}}\text{sign}\left(\tilde{i}_{\beta s}\right) + k_{3\beta}\tilde{i}_{\beta s} \\ \frac{d\hat{\lambda}^*_{\alpha r}}{dt} =& - l_{11}\hat{\lambda}^*_{\alpha r} + l_{12}n_p\omega_r\hat{\lambda}^*_{\beta r} + \varsigma_{11}n_p\omega_r i_{\beta s} + \varsigma_{12}i_{\alpha s} \\ & - q_1 v_{\alpha s} + \frac{k_{2\alpha}}{q_4}\text{sign}\left(\tilde{i}_{\alpha s}\right) \\ \frac{d\hat{\lambda}^*_{\beta r}}{dt} =& - l_{21}\hat{\lambda}^*_{\beta r} - l_{22}n_p\omega_r\hat{\lambda}^*_{\alpha r} - \varsigma_{21}n_p\omega_r i_{\alpha s} + \varsigma_{22}i_{\beta s} \\ & - q_2 v_{\beta s} + \frac{k_{2\beta}}{q_6}\text{sign}\left(\tilde{i}_{\beta s}\right)\end{aligned} \tag{5}$$

where $\tilde{i}_{\alpha s} = i_{\alpha s} - \hat{i}_{\alpha s}$, and $\tilde{i}_{\beta s} = i_{\beta s} - \hat{i}_{\beta s}$ are the observer errors and $k_{i\alpha}, k_{i\beta} > 0$ for $i = 1, 2, 3$.

As a result, from (3), the rotor fluxes estimates $\hat{\lambda}_{\alpha r}$ and $\hat{\lambda}_{\beta r}$ are obtained as $\hat{\lambda}_{\alpha r} = \hat{\lambda}^*_{\alpha r} + l_1\hat{i}_{\alpha s}$ and $\hat{\lambda}_{\beta r} = \hat{\lambda}^*_{\beta r} + l_2\hat{i}_{\beta s}$.

V. Sliding Mode Controller Design

Provided that the currents and speed are continuously measured and the rotor fluxes are estimated, the objective here is to design a SM controller which can effectively track the desired speed ω_{ref} and the module to the square of the rotor flux ϕ_{ref} reference signals by means of the continuous basic control v_s and auxiliary control ρ as a discontinuous function.

A. Sliding Manifold Design

As first step, the state variables x_1 and x_2 are defined as

$$x_1 = \begin{bmatrix} \omega_r & \phi \end{bmatrix}^T \quad \text{and} \quad x_2 = \begin{bmatrix} i_{\alpha s} & i_{\beta s} \end{bmatrix}^T. \tag{6}$$

where $\phi = |\psi|^2 = \lambda^2_{\alpha r} + \lambda^2_{\beta r}$. Then, using (6) the system (1) can be represented in the *Nonlinear Block Controllable form* with disturbance [39], which consists of two blocks

$$\begin{aligned}\frac{dx_1}{dt} =& f_1(\phi) + B_1(\lambda_r)x_2 + D_1 T_L + \Delta_r \\ \frac{dx_2}{dt} =& f_2(\omega_r, \lambda_r, i_s) + B_2 u + \Delta_s\end{aligned} \tag{7}$$

where $\lambda_r = \begin{bmatrix} \lambda_{\alpha r} & \lambda_{\beta r} \end{bmatrix}^T$, $u = \begin{bmatrix} v_{\alpha s} & v_{\beta s} \end{bmatrix}^T$, $f_1(\phi) = \begin{bmatrix} f_{11} & f_{12} \end{bmatrix}^T = \begin{bmatrix} 0 & -2a_{30}\phi \end{bmatrix}^T$, $D_1 = \begin{bmatrix} -d_2 & 0 \end{bmatrix}^T$, $f_2 = \begin{bmatrix} f_{21} & f_{22} \end{bmatrix}^T$, $\Delta_r = \begin{bmatrix} 0 & 2\Delta_{\alpha r}\lambda_{\alpha r} + 2\Delta_{\beta r}\lambda_{\beta r} \end{bmatrix}^T$, $\Delta_s = \begin{bmatrix} \Delta_{\alpha s} & \Delta_{\beta s} \end{bmatrix}^T$, $B_1(\lambda_r) = \begin{bmatrix} d_1 d_2 \lambda_{\beta r} & -d_1 d_2 \lambda_{\alpha r} \\ 2a_{40}\lambda_{\alpha r} & 2a_{40}\lambda_{\beta r} \end{bmatrix}$ and, $B_2 = \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix}$, with $f_{21} = -a_{10}c_1 i_{\alpha s} + c_1 c_{40}\lambda_{\alpha r} - c_1 c_3 \omega_r \lambda_{\beta r}$ and $f_{22} = -a_{20}c_2 i_{\beta s} + c_2 c_3 \omega_r \lambda_{\alpha r} + c_2 c_{40}\lambda_{\beta r}$.

Taking into account that only the estimates of the rotor fluxes are available for the control design, we define the following estimate variables $\hat{\phi} = \hat{\lambda}^2_{\alpha r} + \hat{\lambda}^2_{\beta r}$, $\hat{\lambda}_r = (\hat{\lambda}_{\alpha r}, \hat{\lambda}_{\beta r})$ and its errors $\tilde{\phi} = \phi - \hat{\phi}$, $\tilde{\lambda}_r = \lambda_r - \hat{\lambda}_r$, respectively.

Setting the *controller-used* error and *real* tracking errors, respectively

$$\hat{z}_1 = \begin{bmatrix} z_{11} & \hat{z}_{12} \end{bmatrix}^T, \; z_1 = \begin{bmatrix} z_{11} & z_{12} \end{bmatrix}^T$$

with $z_{11} = \omega_r - \omega_{ref}(t)$, $\hat{z}_{12} = \hat{\phi} - \phi_{ref}(t)$ and, $z_{12} = \phi - \phi_{ref}(t) = \hat{\phi} + \tilde{\phi} - \phi_{ref}(t) = \hat{z}_{12} + \tilde{\phi}$, the dynamics of the first transformed block (7) become

$$\frac{dz_1}{dt} = f_1\left(\hat{\phi}\right) + B_1\left(\hat{\lambda}_r\right)x_2 + \tilde{\Phi} + \bar{\Delta}_1 \tag{8}$$

where $\tilde{\Phi} = \begin{bmatrix} 0 & \frac{d\tilde{\phi}}{dt} \end{bmatrix}^T$ and $\bar{\Delta}_1 = D_1 T_L + \Delta_r + \begin{bmatrix} \frac{d\omega_{ref}(t)}{dt} & \frac{d\phi_{ref}(t)}{dt} \end{bmatrix}^T$.

To impose a desired dynamics for z_1, the Block Control technique [39] is applied. The desired value $x_{2des} = \begin{bmatrix} i^{des}_{\alpha s} & i^{des}_{\beta s} \end{bmatrix}^T$ for the virtual control x_2 in (8) is proposed of the form

$$\begin{aligned}\frac{dz_0}{dt} &= \hat{z}_1 \\ \frac{d\nu_2}{dt} &= -k_{a2}\frac{\frac{d\hat{z}_{12}}{dt} + |\hat{z}_{12}|^{\frac{1}{2}}\text{sign}\left(\hat{z}_{12}\right)}{\left|\frac{d\hat{z}_{12}}{dt}\right| + |\hat{z}_{12}|^{\frac{1}{2}}} \\ \frac{d\nu_1}{dt} &= -k_{a1}\frac{\frac{dz_{11}}{dt} + |z_{11}|^{\frac{1}{2}}\text{sign}\left(z_{11}\right)}{\left|\frac{dz_{11}}{dt}\right| + |z_{11}|^{\frac{1}{2}}} \\ x_{2des} &= B_1^{-1}(\hat{\lambda}_r)\left(-f_1(\hat{\phi}) - K_0 z_0 - K_1\hat{z}_1 + \nu\right)\end{aligned} \tag{9}$$

where $z_0 = \begin{bmatrix} z_{01} & z_{02} \end{bmatrix}^T$, $\nu = \begin{bmatrix} \nu_1 & \nu_2 \end{bmatrix}^T$, $K_0 = \text{diag}\,(k_{11}, k_{12})$, $K_1 = \text{diag}\,(k_1, k_2)$ with k_{a1}, k_{a2}, k_{11}, k_{12} , k_1 and, k_2 being positive constants. Here, the derivatives $\frac{dz_{11}}{dt}$ and $\frac{d\hat{z}_{12}}{dt}$ are obtained using a SM differentiator [43].

Now, the error variable z_2 is defined as follows:

$$z_2 = x_2 - x_{2des}. \tag{10}$$

Using the transformation (9)-(10), the system (7)-(8) is represented in the new coordinates z_1, $z_2 = \begin{bmatrix} z_{21} & z_{22} \end{bmatrix}^T$, $z_{21} = i_{\alpha s} - i^{des}_{\alpha s}$, $z_{22} = i_{\beta s} - i^{des}_{\beta s}$ of the form

$$\begin{cases} \frac{dz_0}{dt} &= z_1 - \tilde{z}_1 \\ \frac{d\nu_1}{dt} &= -k_{a1}\frac{\frac{dz_{11}}{dt} + |z_{11}|^{\frac{1}{2}}\text{sign}(z_{11})}{\left|\frac{dz_{11}}{dt}\right| + |z_{11}|^{\frac{1}{2}}} \\ \frac{d\nu_2}{dt} &= -k_{a2}\frac{\frac{d\hat{z}_{12}}{dt} + |\hat{z}_{12}|^{\frac{1}{2}}\text{sign}(\hat{z}_{12})}{\left|\frac{d\hat{z}_{12}}{dt}\right| + |\hat{z}_{12}|^{\frac{1}{2}}} \\ \frac{dz_1}{dt} &= -K_0 z_0 - K_1 z_1 + \nu + B_1(\hat{\lambda}_r)z_2 + \tilde{\Phi} + \Delta_1 \end{cases} \tag{11}$$

$$\left\{ \frac{dz_2}{dt} = f_2\left(\omega_r, \hat{\lambda}_s, i_s\right) + B_2 u + \Delta_2 \right. \tag{12}$$

where $\tilde{z}_1 = \begin{bmatrix} 0 & \tilde{\phi} \end{bmatrix}^T$, $\Delta_1 = \bar{\Delta}_1 + K_1\tilde{z}_1$, $\Delta_2 = \Delta_r - \frac{dx_{2des}}{dt}$.

B. Inducing Sliding Modes

To induce a SM motion on the manifold on $z_{21} = 0$ or $i_{\alpha s} = i^{des}_{\alpha s}$ in the current loop (12), taking into account (2), the basic control v_s is formulated as [36]

$$\begin{aligned} v_s &= -\alpha_1 |z_{21}|^{1/2}\text{sign}(z_{21}) - \alpha_3 z_{21} + u_1 \\ \frac{du_1}{dt} &= -\alpha_2\text{sign}(z_{21})\end{aligned} \tag{13}$$

with $\alpha_1 > 0$, $\alpha_2 > 0$, and $\alpha_3 > 0$. And to induce a quasi-sliding mode motion on the manifold $z_{22} = 0$ or $i_{\beta s} = i_{\beta s}^{des}$, the auxiliary control ρ for the capacitor is designed by means of the following switching logic:

$$\rho = \frac{1}{2}\text{sign}\left(z_{22}v_c\right) + \frac{1}{2}. \tag{14}$$

VI. Stability Analysis of the Observer-Based Controller

Substituting the control law (13) in (12) and using (4)-(5), the closed-loop system in the control z_{11}, z_{12}, z_{21}, z_{22} and observer $\tilde{i}_{\alpha s}$, $\tilde{i}_{\beta s}$, $\tilde{\lambda}^*_{\alpha r}$, $\tilde{\lambda}^*_{\beta r}$ errors variables become

$$\begin{cases} \frac{dz_{01}}{dt} & = z_{11} \\ \frac{dz_{11}}{dt} & = -k_1 z_{11} - k_{11}z_{01} + \nu_1 \\ & \quad +\Delta_{11} + d_1 d_2(\hat{\lambda}_{\beta r} z_{21} - \hat{\lambda}_{\alpha r} z_{22}) \\ \frac{d\nu_1}{dt} & = -k_{a1}\frac{\frac{dz_{11}}{dt} + |z_{11}|^{\frac{1}{2}}\text{sign}(z_{11})}{\left|\frac{dz_{11}}{dt}\right| + |z_{11}|^{\frac{1}{2}}}, \\ \frac{dz_{02}}{dt} & = z_{12} - \tilde{\phi} \\ \frac{dz_{12}}{dt} & = -k_2 z_{12} - k_{12}z_{02} + \nu_2 \\ & \quad +\Delta_{12} + 2a_{40}(\hat{\lambda}_{\alpha r} z_{21} + \hat{\lambda}_{\beta r} z_{22}) \\ \frac{d\nu_2}{dt} & = -k_{a2}\frac{\frac{d\hat{z}_{12}}{dt} + |\hat{z}_{12}|^{\frac{1}{2}}\text{sign}(\hat{z}_{12})}{\left|\frac{d\hat{z}_{12}}{dt}\right| + |\hat{z}_{12}|^{\frac{1}{2}}} \end{cases} \tag{15}$$

$$\begin{cases} \frac{dz_{21}}{dt} & = f_s(\hat{\lambda}_r) - c_1\alpha_1 |z_{21}|^{\frac{1}{2}}\text{sign}\left(z_{21}\right) \\ & \quad -c_1\alpha_3 z_{21} + c_1 u_1 \\ \frac{du_1}{dt} & = -\alpha_2 \text{sign}\left(z_{21}\right) \end{cases} \tag{16}$$

$$\left\{ \frac{dz_{22}}{dt} = -a_{22}z_{22} + \bar{f}_{22}\left(\bar{z}\right) - c_2 v_c \rho \right. \tag{17}$$

$$\begin{cases} \frac{d\tilde{i}_{\alpha s}}{dt} & = -k_{1\alpha}\left|\tilde{i}_{\alpha s}\right|^{\frac{1}{2}}\text{sign}\left(\tilde{i}_{\alpha s}\right) \\ & \quad -\left(k_{3\alpha} + p_{11}\right)\tilde{i}_{\alpha s} + \Delta_{1\alpha} \\ \frac{d\tilde{\lambda}^*_{\alpha r}}{dt} & = -\frac{k_{2\alpha}}{q_4}\text{sign}\left(\tilde{i}_{\alpha s}\right) + \Delta_{2\alpha} \\ \frac{d\tilde{i}_{\beta s}}{dt} & = -k_{1\beta}\left|\tilde{i}_{\beta s}\right|^{\frac{1}{2}}\text{sign}\left(\tilde{i}_{\beta s}\right) \\ & \quad -\left(k_{3\beta} + p_{21}\right)\tilde{i}_{\beta s} + \Delta_{1\beta} \\ \frac{d\tilde{\lambda}^*_{\beta r}}{dt} & = -\frac{k_{2\beta}}{q_6}\text{sign}\left(\tilde{i}_{\beta s}\right) + \Delta_{2\beta} \end{cases} \tag{18}$$

where

$$\tilde{\lambda}^*_{\alpha r} = \lambda^*_{\alpha r} - \hat{\lambda}^*_{\alpha r} \text{ and } \tilde{\lambda}^*_{\beta r} = \lambda^*_{\beta r} - \hat{\lambda}^*_{\beta r}$$

are the fluxes estimation errors, with the following perturbations: $\Delta_{11} = -d_2 T_L - \frac{d\omega_{ref}(t)}{dt}$, $\Delta_{12} = 2\Delta_{\alpha s}\hat{\lambda}_{\alpha r} + 2\Delta_{\beta s}\hat{\lambda}_{\beta r} - \frac{d\phi_{ref}(t)}{dt} + k_2\tilde{\phi}$, $\Delta_{1\alpha} = -p_{12}n_p\omega_r\tilde{i}_{\beta s} - q_3 n_p\omega_r\tilde{\lambda}^*_{\beta r} + q_4\tilde{\lambda}^*_{\alpha r} + \Delta_{\alpha r}$, $\Delta_{2\alpha} = -l_{11}\tilde{\lambda}^*_{\alpha r} + l_{12}n_p\omega_r\tilde{\lambda}^*_{\beta r} - l_1\Delta_{\alpha r} + \Delta_{\alpha s}$, $\Delta_{1\beta} = p_{22}n_p\omega_r\tilde{i}_{\alpha} + q_5 n_p\omega_r\tilde{\lambda}^*_{\alpha r} + q_6\tilde{\lambda}^*_{\beta r} + \Delta_{\beta r}$, $\Delta_{2\beta} = -l_{21}\tilde{\lambda}^*_{\beta r} - l_{22}n_p\omega_r\tilde{\lambda}^*_{\alpha r} + l_2\Delta_{\beta r} + \Delta_{\beta s}$ and $f_s(\hat{\lambda}_r) = f_{21}(\hat{\lambda}_r) - a_{10}c_1 i_{\alpha s}^{des} - \frac{di_{\alpha s}^{des}}{dt} + \Delta_{\alpha r}$.

First, the finite time reaching phase stability will be analyzed, and then a SM equation stability will be studied.

A. Finite Time Reaching Phase Stage Stability

To analyze the stability of the reaching phase stage for the errors variables $\tilde{i}_{\alpha s}$, $\tilde{i}_{\beta s}$, $\tilde{\lambda}^*_{\alpha r}$, $\tilde{\lambda}^*_{\beta r}$, z_{21} and, z_{22} in (16)-(18), the following bounds for the disturbance terms are considered

$$\begin{aligned} |\Delta_{1\alpha}|,\ \left|\frac{d\Delta_{1\alpha}}{dt}\right| &< \delta_{1\alpha}, \\ |\Delta_{2\alpha}| &< \delta_{2\alpha}, \\ |\Delta_{1\beta}|,\ \left|\frac{d\Delta_{1\beta}}{dt}\right| &< \delta_{1\beta}, \\ |\Delta_{2\beta}| &< \delta_{2\beta} \text{ and,} \\ \left|f_s(\hat{\lambda}_r)\right| &\leq \delta_1 |z_{21}| + \delta_2. \end{aligned} \tag{19}$$

with positive constants δ_1, δ_2, $\delta_{1\alpha}$, $\delta_{2\alpha}$, $\delta_{1\beta}$ and, $\delta_{2\beta}$.

Taking into account the conditions (19) and following the Lyapunov approach proposed in [44], if the gains for the observer (5) and controller (13) are selected under the conditions

$$\begin{aligned} & k_{1\alpha} > 0,\ k_{1\beta} > 0 \\ & k_{2\alpha} > \frac{k_{1\alpha}\delta_{2\alpha} + \frac{1}{9}\delta_{2\alpha}^2}{2\left(\frac{1}{8}k_{1\alpha} - \delta_{2\alpha}\right)}k_{1\alpha},\ k_{2\beta} > \frac{k_{1\beta}\delta_{2\beta} + \frac{1}{9}\delta_{2\beta}^2}{2\left(\frac{1}{8}k_{1\beta} - \delta_{2\beta}\right)}k_{1\beta} \\ & k_{3\alpha} > \frac{17}{8}\delta_{1\alpha} - p_{11},\ k_{3\beta} > \frac{17}{8}\delta_{1\beta} - p_{21} \\ & \alpha_1 > 0,\ \alpha_2 > c_1\alpha_1\frac{\left(\delta_2 c_1\alpha_1 + \frac{1}{9}\delta_2^2\right)}{2\left(\frac{1}{8}c_1\alpha_1 - \delta_2\right)},\ \alpha_3 > \frac{17}{8}\delta_1, \end{aligned}$$

then the systems (15) and (18) state will reach the manifold $(\tilde{i}_{\alpha s}, \tilde{i}_{\beta s}, \tilde{\lambda}^*_{\alpha r}, \tilde{\lambda}^*_{\beta r}, z_{21}) = (0, 0, 0, 0, 0)$ in finite time.

In SM motion on this manifold, the equivalent value $v_{s,eq}$ [19] of the control v_s is calculated as a continuous solution to $\frac{dz_{21}}{dt} = f_s(\lambda_r) - c_1 v_s = 0$ (16), of the form

$$v_{s,eq} = c_1^{-1} f_s(\lambda_r). \tag{20}$$

Substituting (14) and (20) in the equation (17) yields

$$\frac{dz_{22}}{dt} = -a_{22}z_{22} + \bar{f}_{22}\left(\bar{z}\right) - c_2 v_c\left[\frac{1}{2}\text{sign}\left(z_{22}v_c\right) + \frac{1}{2}\right] \tag{21}$$

where $\bar{z} = \begin{bmatrix} z_{11} & z_{12} & z_{22} \end{bmatrix}^T$, $a_{22} = a_2 c_2$ and $\bar{f}_{22}\left(\bar{z}\right) = f_{22} + c_2\left(nc_1\right)^{-1} f_s(\lambda_r) - a_{22}i_{\beta s}^{des} - \frac{di_{\beta s}^{des}}{dt} + \Delta_{\beta r}$.

Here, $\bar{f}_{22}\left(\bar{z}\right)$ is considered as a disturbance bounded by

$$\left\|\bar{f}_{22}\left(\bar{z}\right)\right\| \leq \gamma_1 |z_{22}| + \gamma_2, \quad \gamma_1 > 0,\ \gamma_2 > 0. \tag{22}$$

Thus, to analyze the stability of (21) the quadratic Lyapunov function candidate $V = \frac{1}{2}z_{22}^2$ is proposed. Then, its time derivative along the trajectories of (21) is calculated as

$$\frac{dV}{dt} = -a_{22}z_{22}^2 + \bar{f}_{22}\left(\bar{z}\right)z_{22} - \frac{1}{2}c_2\left(|z_{22}v_c| + z_{22}v_c\right)$$

and, using (22) results in

$$\frac{dV}{dt} \leq -(a_{22} - \gamma_1)|z_{22}|^2 + \gamma_2|z_{22}| - \frac{1}{2}c_2\left(|z_{22}v_c| + z_{22}v_c\right).$$

Assume now that $a_{22} > \gamma_1$. For the case $z_{22}v_c < 0$ we have

$$\frac{dV}{dt} \leq -(a_{22} - \gamma_1)|z_{22}|^2 + \gamma_2|z_{22}|.$$

Adding and subtracting the term $(a_{22} - \gamma_1)\beta_1|z_{22}|^2$ with $0 < \beta_1 < 1$ yields

$$\begin{aligned} \frac{dV}{dt} &\leq -(a_{22} - \gamma_1)(1 - \beta_1)|z_{22}|^2 \\ &\quad - \left[(a_{22} - \gamma_1)\beta_1|z_{22}| - \gamma_2\right]|z_{22}| \\ &< -\left[(a_{22} - \gamma_1)\beta_1|z_{22}| - \gamma_2\right]|z_{22}|. \end{aligned}$$

If $|z_{22}| > \varepsilon_1$ with $\varepsilon_1 = \frac{\gamma_2}{(a_{22}-\gamma_1)\beta_1}$, then $\frac{dV}{dt} < 0$. Thus, there exists $T_1 > 0$ such that the solution $z_{22}(t)$ satisfies [45]

$$|z_{22}(t)| < \varepsilon_1, \forall t \geq t_0 + T_1.$$

On the other hand, for the case $z_{22}v_c > 0$, we have

$$\frac{dV}{dt} \leq -(a_{22}-\gamma_1)|z_{22}|^2 - (c_2\,|v_c| - \gamma_2)|z_{22}|.$$

If $|v_c| > \frac{\gamma_2}{c_2}$, then

$$\frac{dV}{dt} \leq -(c_2 v^- - \gamma_2)|z_{22}|$$

with $v^- = \frac{\gamma_2}{c_2} + \epsilon$ and $\epsilon > 0$. Hence, $z_{22}(t)$ converges to zero in finite time.

Now, if $|v_c| \leq \frac{\gamma_2}{c_2}$ then adding and subtracting $(a_{22}-\gamma_1)\beta_2|z_{22}|^2$ with $0 < \beta_2 < 1$, we have

$$\frac{dV}{dt} < -\left[(a_{22}-\gamma_1)\beta_2|z_{22}| - (\gamma_2 - c_2 v^+)\right]|z_{22}|$$

with $v^+ \in (-\frac{\gamma_2}{c_2}, \frac{\gamma_2}{c_2})$. Thus, the solution $z_{22}(t)$ satisfies

$$|z_{22}(t)| < \varepsilon_2, \forall t \geq t_0 + T_2$$

for some $T_2 > 0$, where $\varepsilon_2 = \frac{\gamma_2 - c_2 v^+}{(a_{22}-\gamma_1)\beta_2}$. Therefore, a quasi-sliding motion is induced in the vicinity defined by $|z_{22}| < \varepsilon_0$, where $\varepsilon_0 = \max\{\varepsilon_1, \varepsilon_2\}$.

B. Sliding Mode Equation Stability

The SM motion on the manifold $(\tilde{i}_{\alpha s}, \tilde{i}_{\beta s}, \tilde{\lambda}^*_{\alpha r}, \tilde{\lambda}^*_{\beta r}, z_{21}) = (0,0,0,0,0)$ with the constraint $|z_{22}| \leq \varepsilon_0$ is described by the reduced order system (15)

$$\begin{aligned}
\frac{d\xi_1}{dt} &= \xi_2,\ \frac{d\xi_2}{dt} = \xi_3,\\
\frac{d\xi_3}{dt} &= -k_1\xi_3 - k_{11}\xi_2 - k_{a1}\frac{\xi_3 + |\xi_2|^{\frac{1}{2}}\operatorname{sign}(\xi_2)}{|\xi_2| + |\xi_2|^{\frac{1}{2}}} + \bar{\Delta}_{11},\\
\frac{d\xi_4}{dt} &= \xi_5,\ \frac{d\xi_5}{dt} = \xi_6,\\
\frac{d\xi_6}{dt} &= -k_2\xi_6 - k_{12}\xi_5 - k_{a2}\frac{\xi_6 + |\xi_5|^{\frac{1}{2}}\operatorname{sign}(\xi_5)}{|\xi_6| + |\xi_5|^{\frac{1}{2}}} + \bar{\Delta}_{12},
\end{aligned}$$

where $\xi_1 = z_{01}$, $\xi_2 = \frac{dz_{01}}{dt} = z_{11}$, $\xi_3 = \frac{dz_{11}}{dt}$, $\xi_4 = z_{02}$, $\xi_5 = \frac{dz_{02}}{dt} = z_{12}$, $\xi_6 = \frac{dz_{12}}{dt}$, $\bar{\Delta}_{11} = \frac{d}{dt}(\Delta_{11} - d_1 d_2 \lambda_{\alpha r}\varepsilon)$ and $\bar{\Delta}_{12} = \frac{d}{dt}(\Delta_{12} + 2a_{40}\lambda_{\beta r}\varepsilon)$, with $|\varepsilon| < \varepsilon_0$ and noticing that $\hat{z}_{12} = z_{12}$.

Assume that $|\bar{\Delta}_{11}| < \delta_{11}$ and $|\bar{\Delta}_{12}| < \delta_{12}$ with $\delta_{11},\ \delta_{12} > 0$. If $k_{a1} > \delta_{11}$ and $k_{a2} > \delta_{12}$ then the manifold $(\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6) = (z^{ss}_{01}, 0, 0, z^{ss}_{02}, 0, 0)$ will be reached in finite time [40], where z^{ss}_{01} and z^{ss}_{02} are steady state values for z_{01} and z_{02}, respectively. So, a finite time SM motion on the control tracking error manifold $(z_{11}, z_{12}) = (0,0)$ is established despite of the disturbances Δ_{11} and, Δ_{12} which are rejected in (15) by $(k_{11}z^{ss}_{01} - \nu_{1,eq})$ and $(k_{12}z^{ss}_{02} - \nu_{2,eq})$, respectively. Here $\nu_{1,eq}$ and $\nu_{2,eq}$ are the equivalent control values [19] calculated from (15) by putting $\frac{dz_{11}}{dt} = 0$ and $\frac{dz_{12}}{dt} = 0$, respectively.

Finally, to limit the stator currents we propose the following logic for the sliding variables z_{21} and z_{22}:

$$z_{21} = \begin{cases} i_{\alpha s} - \hat{i}^{des}_{\alpha s} & \text{for} \quad |i_{\alpha s}| \leq I_{\max} \\ i_{\alpha s} & \text{for} \quad |i_{\alpha s}| > I_{\max} \end{cases}$$

$$z_{22} = \begin{cases} i_{\beta s} - \hat{i}^{des}_{\beta s} & \text{for} \quad |i_{\beta s}| \leq I_{\max} \\ i_{\beta s} & \text{for} \quad |i_{\beta s}| > I_{\max} \end{cases}$$

where $I_{\max}$ is a maximum admissible current value, $I_{\max} \approx 3I_{\text{nom}}$, and I_{nom} is the nominal value of the current module.

This current limit provides maximum electrical torque produced by the motor during the closed-loop transient process.

VII. Numerical Simulation Results

To verify the effectiveness and efficiency of the proposed observer-based controller, numerical simulations are conducted using the Euler integration method with a time step $t_s = 1 \times 10^{-3}$.

Parameters and data of the SPIM are in the Table 1. [42]:

	Single-Phase		
H.P.	0.25	V_s	110 (V)
f	60 (Hz)	n_p	2
$n = \frac{N_A}{N_B}$	1.18	$R_{\alpha s}$	2.02 (Ω)
$R_{\beta s}$	5.13 (Ω)	R_r	4.12 (Ω)
$L_{\alpha s}$	0.1846(H)	$L_{\beta s}$	0.1833 (H)
L_r	0.1828 (H)	L_m	0.1772 (H)
J	0.0146 $(Kg \cdot m^2)$	k_d	0 $(Kg \cdot m^2/s)$
$I_{\max}$	15 (A)	C_{run}	35 μf

TABLE I: Parameters of SPIM

The controller gains are adjusted to $k_1 = k_2 = 500$, $k_{01} = k_{02} = 30$, $k_{a_1} = k_{a_2} = 5$, $\alpha_1 = 36$, and $\alpha_3 = 1$. And, the gains for the observer are $k_{1_\alpha} = 195$, $k_{1_\beta} = 140$, $k_{3_\alpha} = k_{3_\beta} = 7000$, $k_{2_\alpha} = k_{2_\beta} = 0.02$ and, $l_1 = l_2 = 0.01$.

For the simulation purposes, the initial conditions of the state variables are selected to zero. Tracking performance is verified for the two plant outputs: driving the square of rotor flux ϕ to a constant reference $\phi_{ref} = 0.15$, and a speed profile ω_{ref} for ω_r, proposed as follows:

1) The SPIM starts on repose with the reference speed on 100 rad/sec.
2) At the first second, a change of the speed reference – in ramp form – from 100 rad/sec to 120 rad/sec, is presented.
3) Finally, at 4 seconds, a change of the speed reference – in negative ramp form – from 120 rad/sec to 100 rad/sec, is presented.

In addition, the system is subject to disturbances which are introduced as follows:

1) The SPIM starts on repose with a load torque of $\frac{1}{2} + 0.1\sin(2.5t)$ N-m.
2) At 2 seconds, a 30% increase in the value of the rotor resistance is presented.

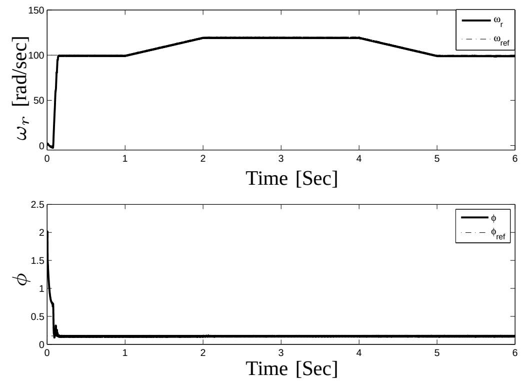

Fig. 2: Rotor speed ω_r and module to the square of rotor flux ϕ.

The rotor speed tracking response is depicted in Fig. 2 which shows a satisfactory performance under the change of the speed reference at $t = 1, 4$ sec., where the speed tracking effect is achieved almost totally after 0.087 sec. Fig. 2 shows the module to the square of the rotor flux ϕ response too; it is possible to see that the module is maintained over the given reference.

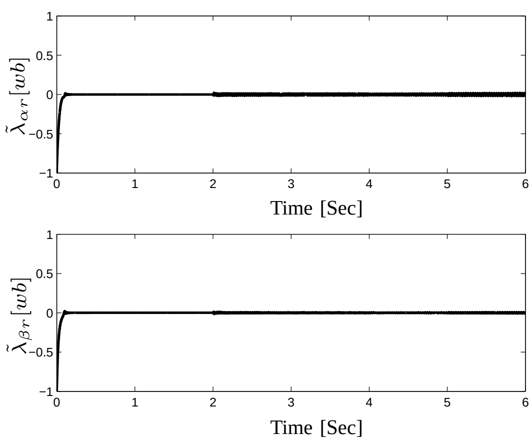

Fig. 3: Error of rotor flux in axis frame α β.

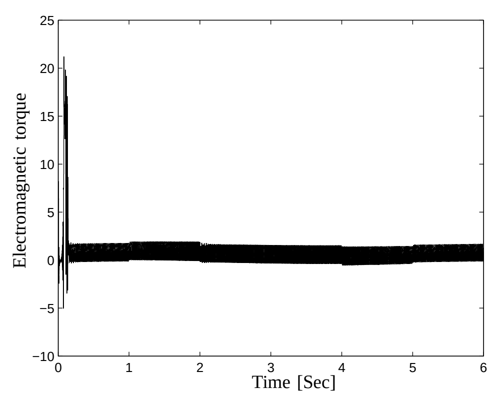

Fig. 4: Electromagnetic torque T_e.

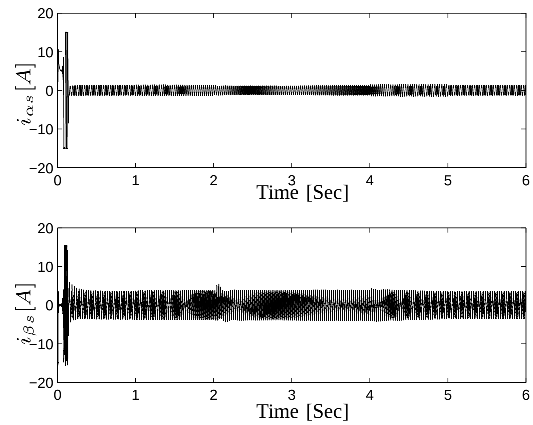

Fig. 5: Stator currents in axis frame α β.

The errors responses of rotor fluxes are shown in Fig. 3. The electromagnetic torque response is depicted in Fig. 4, showing that the torque has a high value 20 N-m during the interval $[0, 0.088]$ sec. This high value ensures a fast response of the speed (see Fig.2).

On the other hand, the stator currents (see Fig. 5) are in the appropriate range during the start $(0 < t < 0.2)$ that corresponds to the proposed control algorithm.

Finally, in Fig. 6, the responses of the voltages are presented, where $v_{\alpha s}$ is the *super-twisting* SM control and, $v_{\beta s}$ is the discontinuous SM control.

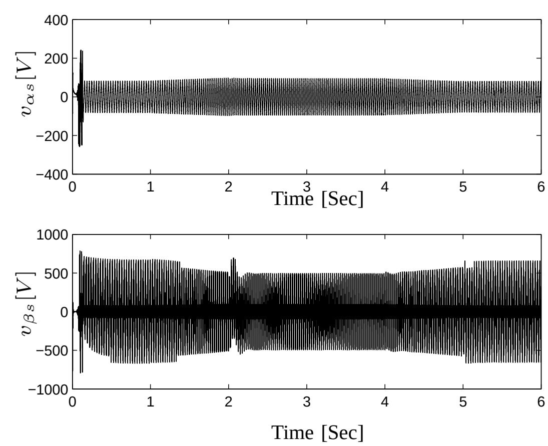

Fig. 6: Stator control voltages of axis α β.

VIII. CONCLUSIONS

A control scheme based on the Block Control technique, quasi-continuous SM manifold design, and the second order SM super-twisting algorithms, is proposed to track the rotor angular speed ω_r and the square module of rotor flux ϕ of the SPIM subject to perturbations. A nonlinear observer based on second order SM algorithms is designed to estimate the rotor flux in presence of plant parameter variations. The stability conditions of the complete closed-loop system with the proposed observer-based control, are derived.

The simulation results show a robust performance of the designed controller with respect to the disturbances caused by the load torque and the presence the rotor resistance variations. Moreover, the proposed controller ensures the constraints on the stator current.

In the near future, the proposed control scheme is planned to be implemented and tested in a real time prototype. This design will consist of two coupled motors: a single-phase to be controlled motor and DC one. The aim of the last motor is to emulate a desired load torque profile. The benchmark will include a DSpace1104 acquisition card, a pulse-width-modulation (PWM) unit for the power stage and a PC which will serve as user interface.

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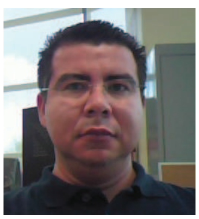

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