Leak Detection and Isolation Using an Observer based on Robust Sliding Mode Differentiators

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***Abstract*—A Leak Detection and Isolation algorithm with a State Observer is designed and tested in simulation to locate a water-leak on a pipeline. Most of the works presented in the literature make some linearization in the model limiting the performance of the approach. In order to avoid this situation, in this work, the State Estimation is designed with the nonlinear model using an observer based on robust sliding mode differentiators. The approach assumes only flow and pressure sensors at the ends of the duct. Simulation results with synthetic data obtained from a pipeline simulator are presented to assess the method efficiency. The proposed scheme ensures finite time convergence of the observer and the reduction of chattering effect.**

I. INTRODUCTION

Leak Detection and Isolation (LDI) is an important problem to solve. Due to this fact, the design of fast and robust methods to locate and isolate leaks is the subject of several research initiatives.

A successful technique to deal with the LDI problem is the Fault Model (FM) based approach. Many FM algorithms for LDI (FM-LDI) have been proposed in the literature, most of these procedures use a state space model of the pipeline in order to obtain a steady-state detection and isolation; some examples of FM-LDI applications are presented in [1], [2], [3] and [4]. All of these works have shown positive result, however, they use asymptotic observers in order to tackle the isolation task and, in this way, limiting the localization time.

On the other hand, sliding mode approach has been widely used for problems of dynamic systems control and observation due to their characteristics of finite time convergence, robustness to uncertainties and insensitivity to external bounded disturbances [5], [6]. In observers based on sliding mode, the sliding motion is obtained by means of a *Sliding Operator* depending on the output error [7]. Additionally, by using an *Sliding Operator* of the error (like the sign function) to drive the sliding mode observer, the observer trajectories become insensitive to many forms of noise. Hence, some sliding mode observers have attractive properties and simple implementation [8].

The purpose of this work is to design a FM-LDI scheme whit an observer based on robust exact sliding mode differentiators [9] (it should be noted that to the authors knowledge, there are not studies exploring the finite time convergence of the observer in a LDI problem). In order to obtain the estimate states, the system is transformed in a special case of *triangular form* based on the Lie derivatives of the system output [10] and, in this way, calculate the estimations in the original variables using an inverse transformation.

At first, we use the model shown in [4], where the states are flows, pressure heads, the leak position and a parameter related with the leak intensity. Then, the resulting continuous-time nonlinear model is employed together with robust sliding mode differentiator in order to design a state observer. The observer is used to isolate a leak by direct estimation of its location. Finally, the estimation of the pipeline state variables is done comparing the derivatives of the model with the sliding differentiator calculation. These high order sliding operators provide the observer with the properties of finite time convergence and reduction of chattering effect.

To assess the performance of the designed LDI system, it is tested with synthetic data obtained from a simulator based on the pipeline prototype described in [11].

The paper continues as follows: Section II states the considered model and Section III describes the proposed model-based detection approach. Section IV then presents some successful simulation results while Section V finally concludes the paper.

II. MODEL

This section presents the two Partial Differential Equations and the leak model which describe the pipeline dynamics. Also, a finite dimensional model is obtained from a space discretization.

A. Modelling equation

Assuming the fluid to be slightly compressible and the duct walls slightly deformable; the convective changes in velocity to be negligible; the cross section area of the pipe and the fluid density to be constant, then the dynamics of the pipeline fluid can be described by the following partial differential equations [12]:

Momentum Equation

$$\frac{\partial Q(z,t)}{\partial t} + gA\frac{\partial H(z,t)}{\partial z} + \mu Q(z,t)\left|Q(z,t)\right| = 0 \quad (1)$$

Continuity Equation

$$\frac{\partial H(z,t)}{\partial t} + \frac{b^2}{gA}\frac{\partial Q(z,t)}{\partial z} = 0 \quad (2)$$

where Q is the flow rate $[m^3/s]$, H is the pressure head $[m]$, z the length coordinate $[m]$, t the time coordinate $[s]$, g the gravity acceleration $[m/s^2]$, A the cross-section area $[m^2]$, b the speed of the pressure wave in the fluid $[m/s]$, $\mu = \frac{\tau}{2DA}$, D the diameter $[m]$ and τ the friction factor.

Leak model: One leak arbitrarily located at point z_1 (see Fig. 1) in a pipeline can be modeled as follows [12]:

$$Q_L = \lambda\sqrt{H_L} \quad (3)$$

where the constant λ is a function, among others of the orifice area and the discharge coefficient, Q_L is the flow through the leak and H_L is the head pressure at the leak point [12].

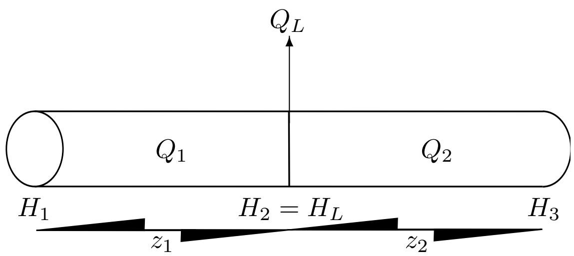

Fig. 1. Discretization of the pipeline with a leak Q_L

This leak produces a discontinuity in the system. Furthermore, due to the law of conservation of mass, Q_L must satisfy the next relation:

$$Q_b = Q_a + Q_L \quad (4)$$

where Q_b and Q_a are the flow before and after of the leak, respectively.

B. Spatial Discretization of the Modelling Equations

In order to obtain a state space representation of model (1) and (2), the partial differential equations are discretized with respect to the spatial variable z, as in [2], [13], by using the following relation:

$$\frac{\partial H}{\partial z} \approx \frac{H_{j+1} - H_j}{z_j} \quad (5)$$

$$\frac{\partial Q}{\partial z} \approx \frac{Q_j - Q_{j-1}}{z_j} \quad (6)$$

Assuming only two partitions in the pipeline, as shown in Fig. 1, then z_j ($j = \{1,2\}$) becomes the distance from the beginning of the pipe to point of the leak and from the point of the leak to the end of the pipe, respectively. Notice that $z_2 = L - z_1$ where L is the total length of the pipeline. Applying the approximation (5) and (6) to the equation (1) and (2) together with (3) and (4), and then incorporating as additional states z_1 and λ, we get:

$$\begin{bmatrix} \dot{Q}_1 \\ \dot{H}_2 \\ \dot{Q}_2 \\ \dot{z}_1 \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} -\frac{gA}{z_1}(H_2 - u_1) - \mu Q_1\left|Q_1\right| \\ -\frac{b^2}{gAz_1}(Q_2 - Q_1 + \lambda\sqrt{H_2}) \\ -\frac{gA}{L-z_1}(u_2 - H_2) - \mu Q_2\left|Q_2\right| \\ 0 \\ 0 \end{bmatrix} \quad (7)$$

Here, the input vector is $u = [H_1 \; H_3]^T = [u_1 \; u_2]^T$, since these signals are measured, and the output vector is $y = [Q_1 \; Q_2]^T$.

III. FAULT MODEL BASED OBSERVER SCHEME

A. Method Description

Let us consider the following system

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \quad (8)$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the input, $y \in \mathbb{R}^p$ is the output and f, g, h are sufficiently differentiable function vectors.

For the system (8), the vector of output derivatives is given by:

$$V(t) = \begin{pmatrix} y_1(t) \\ \dot{y}_1(t) \\ \vdots \\ y_1^{(k)}(t) \\ \vdots \\ y_p(t) \\ \dot{y}_p(t) \\ \vdots \\ y_p^{(k)}(t) \\ \vdots \end{pmatrix} \quad (9)$$

From (8), $V(t)$ can be expressed as a fuction of $x, u, \dot{u}, \ldots, u^{(k)}, \ldots$:

$$V(t) = H(x, u, \dot{u}, \ldots, u^{(k)}, \ldots) \quad (10)$$

Observability somehow means that this relationship is invertible, and that one can find elements among the components of H defining an invertible map with respect to x [14].

Let us denote by $\bar{H}(x,u,t)$ this map, and consider the vector $\bar{V}(t)$ of the n corresponding elements of $V(t)$. Then:

$$x = \bar{H}^{-1}\left(\bar{V}(t), u, t\right). \quad (11)$$

Now nothing that the system output time derivative being unknown, the problem reduces to the estimation $\hat{\bar{V}}(t)$ of $\bar{V}(t)$ so that an estimation of the state can be given by:

$$\hat{x} = \bar{H}^{-1}\left(\hat{\bar{V}}(t), u, t\right) \quad (12)$$

B. Robust sliding mode differentiators

The estimation of the output derivatives are made by means of a k-order exact robust differentiator $\mathcal{SD}_k(\cdot)$ [9]. Let $s(t) \in \mathcal{C}^{\bar{k}}[0,\infty)$ be a function to be differentiated and let $k \leq \bar{k}$, then the k-th order differentiator is defined as an application

$$\mathcal{SD}_k(s(t)) \mapsto [s(t), \xi_1, \ldots, \xi_k]^T \tag{13}$$

where

$$\begin{aligned}
\dot{\xi}_0 &= \zeta_0, \\
\zeta_0 &= -\varphi_k \Gamma^{\frac{1}{k+1}} |\xi_0 - s(t)|^{\frac{k}{k+1}} \operatorname{sign}(\xi_0 - s(t)) + \xi_1 \\
\dot{\xi}_1 &= \zeta_1, \\
\zeta_1 &= -\varphi_{k-1} \Gamma^{\frac{1}{k}} |\xi_1 - \zeta_0|^{\frac{k-1}{k}} \operatorname{sign}(\xi_1 - \zeta_0) + \xi_2 \\
&\vdots \\
\dot{\xi}_{k-1} &= \zeta_{k-1}, \\
\zeta_{k-1} &= -\varphi_1 \Gamma^{\frac{1}{2}} |\xi_{k-1} - \zeta_{k-2}|^{\frac{1}{2}} \operatorname{sign}(\xi_{k-1} - \zeta_{k-2}) + \xi_k \\
\dot{\xi}_k &= -\varphi_0 \Gamma \operatorname{sign}(\xi_k - \zeta_{k-1})
\end{aligned} \tag{14}$$

where ξ_i is the estimation of the true signal $s^{(i)}(t)$.

The differentiator provides finite time exact estimation under ideal condition when neither noise nor sampling are present. For the gain Γ case (φ_i, $i = \{0, \ldots, k\}$ can be computed proceeding as [9]), the following condition is provided:

Condition 1. *The parameter Γ is selected such that it is an upper bound for $|s^{(k+1)}|$.*

If **Condition 1** is fulfilled, then $[s(t), \xi_1, \ldots, \xi_{n-1}]^T = [s(t), \dot{s}(t), \ldots, s^{(n)}(t)]^T$ in finite time.

Applying the form (13) and (14) to the model (8), the vector of the system output derivatives $\hat{V}(t)$ could be defined as:

$$\hat{V}(t) = \begin{pmatrix} \mathcal{SD}_\infty(y_1(t)) \\ \mathcal{SD}_\infty(y_2(t)) \\ \vdots \\ \mathcal{SD}_\infty(y_p(t)) \end{pmatrix}. \tag{15}$$

It is possible to form the vector $\hat{\bar{V}}(t)$ selecting the n corresponding elements of $\hat{V}(t)$. Now, just rest to find the function $\bar{H}$ in (12) for a specific application.

C. Observer Design

Equation (7) can be written in compact form (8) as:

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \tag{16}$$

with $x \doteq [Q_1\ H_2\ Q_2\ z_1\ \lambda]^T = [x_1\ x_2\ x_3\ x_4\ x_5]^T$. Moreover, if we consider unidirectional flow (i.e. $x_1 > 0$ and $x_3 > 0$), $f(x)$, $g(x)$ and $h(x)$ are as follows:

$$f(x) = \begin{bmatrix} -\frac{gA}{x_4}x_2 - \mu x_1^2 \\ -\frac{b^2}{gAx_4}(x_3 - x_1 + x_5\sqrt{x_2}) \\ \frac{gA}{L-x_4}x_2 - \mu x_3^2 \\ 0 \\ 0 \end{bmatrix}$$

$$g(x) = \begin{bmatrix} \frac{gA}{x_4} & 0 \\ 0 & 0 \\ 0 & -\frac{gA}{L-x_4} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$h(x) = \begin{bmatrix} h_1(x) \\ h_2(x) \end{bmatrix} = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix}$$

Here it is easy to check that the elements of $\bar{H}$, in (12), correspond to output time derivatives is as follows:

$$\hat{\bar{V}}(t) = \bar{H}(x,u,\dot{u}) = \begin{bmatrix} y_1(t) \\ \dot{y}_1(t) \\ \ddot{y}_1(t) \\ y_2(t) \\ \dot{y}_2(t) \end{bmatrix} = \begin{bmatrix} \bar{H}_1(x,u,\dot{u}) \\ \bar{H}_2(x,u,\dot{u}) \\ \bar{H}_3(x,u,\dot{u}) \\ \bar{H}_4(x,u,\dot{u}) \\ \bar{H}_5(x,u,\dot{u}) \end{bmatrix} \tag{17}$$

where $\bar{H}_1(x) = x_1$, $\bar{H}_2(x) = -\frac{Ag}{x_4}(x_2 - u_1) - \mu x_1^2$, $\bar{H}_3(x) = 2\mu^2 x_1^3 + \frac{b^2}{x_4^2}\left(x_3 - x_1 + x_5\sqrt{x_2}\right) + \frac{Ag}{x_4}(\dot{u}_1 + 2\mu x_1 x_2 - 2\mu x_1 u_1)$, $\bar{H}_4(x) = x_3$ and $\bar{H}_5(x) = \frac{Ag}{L-x_4}(u_2 - x_2) - \mu x_3^2$. Therefore, the state estimate in terms of the output derivatives (11), is written as:

$$\begin{aligned}
x_1 &= y_1 \\
x_2 &= \frac{\dot{y}_2 + \mu y_2^2}{Ag}\left[\frac{L\left(\dot{y}_1 + \mu y_1^2\right) - Ag\left(\dot{y}_2 + \mu y_2^2\right)}{\left(\dot{y}_1 + \mu y_1^2\right) - \left(\dot{y}_2 + \mu y_2^2\right)}\right] + u_2 \\
x_3 &= y_2 \\
x_4 &= \frac{L\left(\dot{y}_2 + \mu y_2^2\right) - Ag\left(u_1 - u_2\right)}{\left(\dot{y}_2 + \mu y_2^2\right) - \left(\dot{y}_1 + \mu y_1^2\right)} \\
x_5 &= \frac{x_4^2}{b^2\sqrt{x_2}}\left(\ddot{y}_1 + 2\mu y_1 \dot{y}_1 - \frac{Ag}{x_4}\dot{u}_1\right) + \frac{1}{\sqrt{x_2}}(y_1 - y_2).
\end{aligned} \tag{18}$$

Finally, from (12), the estimated values of the state variables in (18), $\hat{x}_i$, $i = \{1, \ldots, 5\}$, are obtained replacing the output, the inputs and their derivatives by the robust sliding mode differentiators. To do that, the form (15) is applied to the vector of the system output derivatives $\hat{\bar{V}}(t)$ defined as follows:

$$\hat{\bar{V}}(t) = \begin{pmatrix} \mathcal{SD}_3(y_1(t)) \\ \mathcal{SD}_2(y_2(t)) \end{pmatrix}. \tag{19}$$

where $y_1(t) = x_1(t)$, $y_2(t) = x_3(t)$, $\mathcal{SD}_3(y_1(t))$ provides an estimation of $y_1(t)$ and its first and second derivatives, and $\mathcal{SD}_2(y_2(t))$ provides an estimation of $y_2(t)$ and its first derivative.

IV. SIMULATION EXAMPLE

In this section we present simulation results in order to evaluate the performance of the designed LDI scheme. The simulator has the same structure as the model system (equation

TABLE I
PIPELINE PARAMETERS

Parameter	Symbol	Value	Units
Length between sensors	L	86.49	$[m]$
Internal diameter	D	6.54×10^{-2}	$[m]$
Pressure wave speed	b	3.75×10^{2}	$[m/s]$
Friction factor coefficient	τ	1.72×10^{-2}	[-]
Gravity	g	9.81	$[m/s^2]$

(7)). The parameters of pipeline simulated are given in Table I. The initial pressure head at the upstream and the downstream point of the pipe (the simulator and observer inputs) were fixed in 14.15 $[m]$ and 7.15 $[m]$, respectively. The simulator was initialized in steady state condition as follows: $Q_1(0) = Q_2(0) = 7.75 \times 10^{-3}$ $[m^3/s]$ since, in a free-leak regime, the inflow is equal to the outflow; $z_1(0) = 0.5L$ $[m]$, i.e. the discretized point is located in the middle of the pipe length; $\lambda(0) = 0$ $[m^{\frac{5}{2}}/s]$ since, at the beginning of the simulation, the pipe is not leaking; finally, $H_2(0)$ was computed with the well-known Darcy-Weisbach equation at the distance $z_1(0)$. The parameters of the observer, following [9], are fixed as follows: $\varphi_0 = 1.1$, $\varphi_1 = 1.5$, $\varphi_2 = 2$, $\varphi_3 = 3$, $\varphi_4 = 5$, $\varphi_5 = 8$ are suggested for the construction of differentiators up to the 5-th order; finally, the Γ parameter was fixed as $\Gamma = 10000$.

To test the previous scheme, a leak at $z_L = 72$ m was suddenly induced in the simulator at time 500 s whit $\lambda = 2.7e-5$.

The initial conditions for the observer are equal to zero:

$$\begin{bmatrix} \hat{Q}_1^0 \\ \hat{H}_2^0 \\ \hat{Q}_2^0 \\ \hat{z}_1^0 \\ \hat{\lambda}^0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Fig. 2 presents the pressure heads at inlet ($H_{in} = u_1$) and outlet ($H_{out} = u_2$) of the pipe (i.e. the observer inputs). Fig. 3 shows the evolution of the inflow and outflow, Q_1 and Q_2, respectively. On the other hand, the pressure head, H_2, at the leak point and its estimation $\hat{H}_2$ are presented in Fig. 4.

In Fig. 5, the λ parameter and its estimation valued $\hat{\lambda}$ are shown. Using (3) with λ and $\hat{H}_2$ we can compute a leak intensity of 3.36×10^{-5} m^3/s which is equivalent proximately to 0.4% of the nominal flow.

Finally, the leak position is well estimated as seen in Fig. 6.

V. CONCLUSIONS

A Leak Detection and Isolation algorithm based on Robust Sliding Mode Differentiators has been designed and tested with synthetic data to locate a water-leak on a pipeline.

Flow estimations have been well estimated in the presence of a leak, which means that the Sliding Observer correctly follows the dynamics of the model with a leak. In the same way, the observer has estimated the leak position and its intensity in a very acceptable way.

The proposed scheme ensures finite time convergence of the observer and the reduction of chattering effect.

As a future work, this algorithm will be tested with real data obtained from a pipeline prototype.

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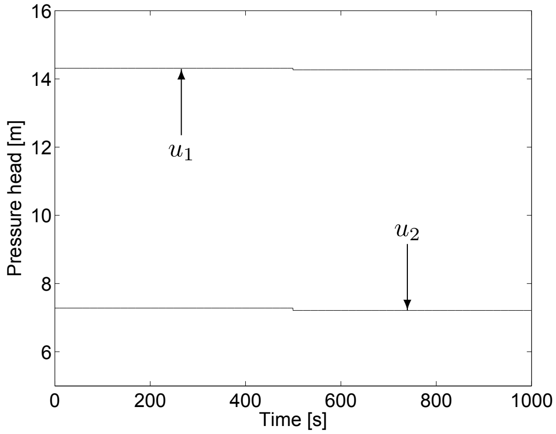

Fig. 2. Pressure head at inlet and outlet of the pipeline (observer inputs).

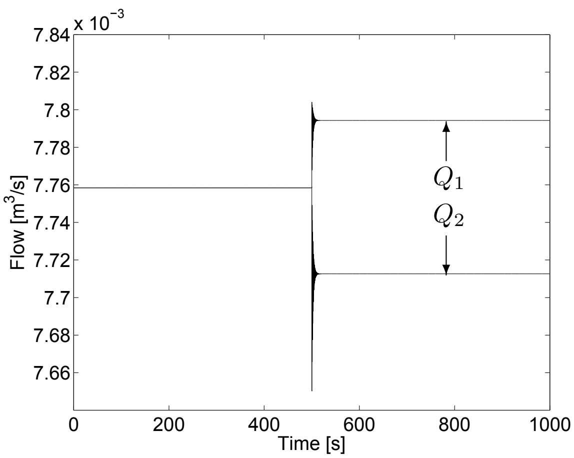

Fig. 3. Flow rate at inlet and outlet of the pipeline.

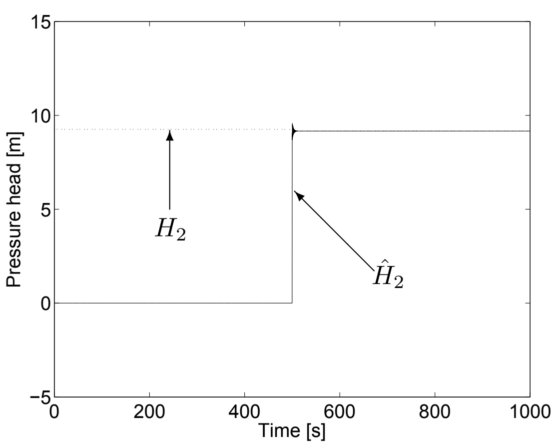

Fig. 4. Pressure head at leak point and its estimation $\hat{H}_2$.

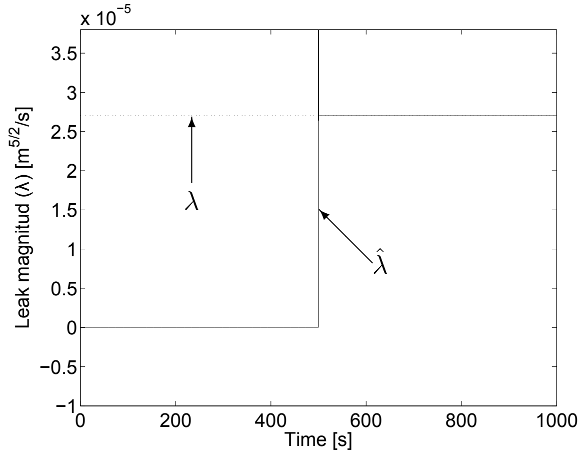

Fig. 5. λ and its estimation $\hat{\lambda}$.

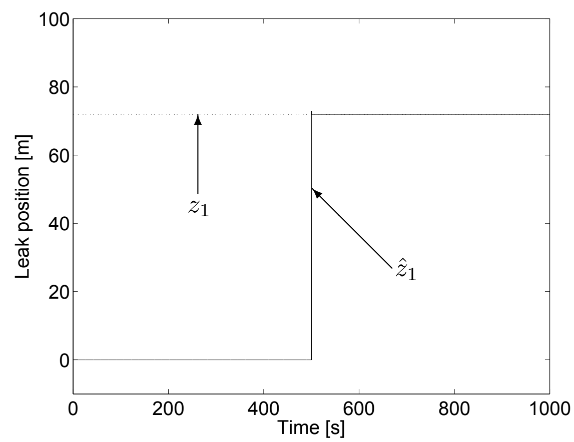

Fig. 6. Leak position and its estimation $\hat{z}_1$.