A high order sliding mode observer for systems in triangular input observer form

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***Abstract*—This paper deals with the design of a high order sliding mode observer for a class of nonlinear systems that can be described in the so called *triangular input observer form*. The mathematical tools required to make the system transformation to such form are also presented. At last, we show the performance of the observers with several simulation examples, including the application of a CSTR process model.**

I. Introduction

Sliding mode approaches have been widely used for the problems of dynamic systems control and observation due to its characteristics of finite time convergence, robustness to uncertainties and insensitivity to external disturbances. In addition, the state observers have another important properties like the possibility of obtaining a step by step design and work with a reduced observation dynamics [1, Ch. 4], [2] . Often, sliding mode motion is obtained by means of a discontinuous term depending on the output error, into the controlling or observing system. Additionally, by using the sign of the error to drive the sliding mode observer, the observer trajectories become insensitive to many forms of noise. Hence, some sliding mode observers have attractive properties similar to the Kalman filter but with simpler implementation [3].

Several researchers have dealt with the issue of designing sliding-mode observers for different applications [4], [5], including the classical problem of non-linear state estimation [6]. In [7], a sliding-mode observer for non-linear system based over the equivalent control method is proposed. Some applications of the sliding mode techniques to control and robust differentiation are presented in [8] [9] [10]. Noting that the classical sliding mode techniques are a particular case of the high order sliding mode concept and can be considered as a first order sliding mode [11]. The high order sliding modes allows also take into account the sampling measurement delays. Some practical examples of the use high order sliding modes observers can be found in [12], [13]. All of these imply that high order sliding modes observers are very convenient for real implantations.

In this work, our purpose is to discuss about observer design for a system in a triangular input observer form. We discuss this form, which has been treated previously in [14], because for a such system it is possible to design a simple observer which does not use the input derivative. In fact, there are some applications, mainly in the electrical motor control systems, where the exact knowledge of the input derivative is very questionable [14]. Besides this, a system in a triangular input observer form does not face the problem of singular input. The another goal of this paper is to show that the proposed observer can also be applied to a chemical process system like the CSTR.

In the following, in section II some mathematical preliminaries are given. In section III a higher order sliding mode observer for systems in triangular input observer form is proposed. Three examples of the proposed observer, including two well-known systems [14], [1, Ch. 4] and a CSTR, are presented in section IV. Finally, in section V some conclusion are presented.

II. Mathematical Preliminaries

In this section, we present the mathematical concepts required to take a given SISO system to the triangular input observer form, and its relationship with a robust differentiator.

A. Triangular input observer form

Let us consider the following SISO system

$$\begin{aligned} \dot{\chi} &= f(\chi) + g(\chi, u) \\ y &= h(\chi) \end{aligned} \tag{1}$$

where $\chi \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}$ is the input, $y \in \mathbb{R}$ is the output and f, g, h are function vectors, with $g(x,0) = 0$ for all $x \in \mathbb{R}^n$. In addition for the system (1), lets suppose the following two conditions

Condition 1. *The codistribution*

$$\Omega^i = \operatorname{span}\left\{dh, \ldots, dL_f^i h\right\} \quad 0 \le i \le n-1 \tag{2}$$

is involutive.

and

Condition 2. *For any $u \in \mathbb{R}$, the vector field g fulfills*

$$dL_g L_f^i h \in \Omega^i \quad \forall i = \overline{1, n-1} \tag{3}$$

From (2) the coordinate change

$$\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} h \\ L_f(h) \\ \vdots \\ L_f^{n-1}(h) \end{pmatrix} \tag{4}$$

is a diffeomorphism $\underset{\chi\mapsto x}{T(\chi)\to x}$ which transforms the system (1) to

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{pmatrix} = \begin{pmatrix} x_2 + \bar{g}_1(x,u) \\ x_3 + \bar{g}_2(x,u) \\ \vdots \\ x_n + \bar{g}_{n-1}(x,u) \\ \bar{f}_n(x) + \bar{g}_n(x,u) \end{pmatrix} \tag{5}$$

with $\bar{g}_i(\cdot,u) = 0$ for $u = 0$, $\forall i = \overline{1,n}$. Where $x = [x_1,\ldots,x_n]^T$. In addition, the equation (3) is equivalent to

$$d\bar{g}_i \in \text{span}\{d\chi_1,\ldots,d\chi_i\} \quad \forall i \in \{1,\ldots,n\} \tag{6}$$

this reduces the system (5) to the so-called *Triangular input observer form*, as follows

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{pmatrix} = \begin{pmatrix} x_2 + \bar{g}_1(x_1,u) \\ x_3 + \bar{g}_2(x_1,x_2,u) \\ \vdots \\ x_n + \bar{g}_{n-1}(x_1,\ldots,x_{n-1},u) \\ \bar{f}_n(x) + \bar{g}_n(x,u) \end{pmatrix} \tag{7}$$
$$y = x_1$$

with $\bar{g}_i(\cdot,u) = 0$ for $u = 0$, $\forall i = \overline{1,n}$

Remark 1. *The equation (2) follows from the observability rank condition*

$$\text{rank}\begin{pmatrix} dh \\ dL_f h \\ \vdots \\ dL_f^{n-1}h \end{pmatrix} = n \tag{8}$$

this gives an easy way to check the **Condition 1**, [15].

B. Arbitrary-order exact robust differentiator

Real-time differentiation is a well-known problem, several approaches have been proposed to obtain time derivatives for a given signal. Between these all solutions, sliding mode based methods have demonstrated high accuracy and robustness. For the calculation of higher order exact derivatives, successive implementation of a first order differentiator with finite time convergence is used in [8]. For the same objective, an arbitrary-order exact robust differentiator based in a recursive scheme and which provides the best possible asymptotic accuracy in presence of input noises and discrete sampling is proposed in [9]. Let $f(t) \in \mathcal{C}^{\bar{k}}[0,\infty)$ be a function to be differentiate and let $k \le \bar{k}$, then the k-th order differentiator is defined as follows:

$$\begin{aligned}
\dot{z}_0 &= v_0, \\
v_0 &= -\lambda_k L^{\frac{1}{k+1}} |z_0 - f(t)|^{\frac{k}{k+1}} \text{sign}(z_0 - f(t)) + z_1 \\
\dot{z}_1 &= v_1, \\
v_1 &= -\lambda_{k-1} L^{\frac{1}{k}} |z_1 - v_0|^{\frac{k-1}{k}} \text{sign}(z_1 - v_0) + z_2 \\
&\vdots \\
\dot{z}_{k-1} &= v_{k-1}, \\
v_{k-1} &= -\lambda_1 L^{\frac{1}{2}} |z_{k-1} - v_{k-2}|^{\frac{1}{2}} \text{sign}(z_{k-1} - v_{k-2}) + z_k \\
\dot{z}_k &= -\lambda_0 L \text{sign}(z_k - v_{k-1})
\end{aligned} \tag{9}$$

where z_i is the estimation of the true signal $f^{(i)}(t)$. The differentiator provides finite time exact estimation under ideal condition when neither noise nor sampling are present. The parameters $\lambda_0 = 1.1$, $\lambda_1 = 1.5$, $\lambda_2 = 2$, $\lambda_3 = 3$, $\lambda_4 = 5$, $\lambda_5 = 8$ are suggested for the construction of differentiators up to the 5-th order. The parameter L is selected such that be a upper bound for $|f^{(k+1)}|$. See [9] and [10] for further details on the estimation of time of convergence, the error bounds for the signal $f(t)$ and their derivatives in presence of noise or discrete sampling and other proprieties and constrains of the differentiator.

III. Observer Design

In this section, based on the equation of the arbitrary-order exact robust differentiator (9), we propose an observer for a system represented in the triangular input observer form (7). The use of high order sliding mode terms allow us to avoid the use of classic low pass filters employed to obtain the equivalent controls [4]. Another feature of the differentiator (9) is the fact that the output does not depend directly on discontinuous functions but on an integrator output. So high frequency chattering, which can be very harmful for the systems, can also be avoided.

Based in the equation (9), we propose the following observer for (7):

$$\begin{aligned}
\dot{\hat{x}}_1 &= \zeta_1, \\
\zeta_1 &= \hat{x}_2 + \bar{g}_1(x_1,u) - \lambda_1 L^{\frac{1}{n+1}} |\hat{x}_1 - x_1|^{\frac{n}{n+1}} \text{sign}(\hat{x}_1 - x_1) \\
\dot{\hat{x}}_2 &= \zeta_2, \\
\zeta_2 &= \hat{x}_3 + \bar{g}_2(x_1,\hat{x}_2,u) - \lambda_2 L^{\frac{1}{n}} |\hat{x}_2 - \zeta_1|^{\frac{n-1}{n}} \text{sign}(\hat{x}_2 - \zeta_1) \\
&\vdots \\
\dot{\hat{x}}_n &= \zeta_n, \\
\zeta_n &= \bar{f}_n(\hat{x}) + \bar{g}_n(\hat{x},u) - \lambda_n L^{\frac{1}{2}} |\hat{x}_n - \zeta_{n-1}|^{\frac{1}{2}} \text{sign}(\hat{x}_n - \zeta_{n-1})
\end{aligned} \tag{10}$$

For this form, as it is stated in [9] and [10], we can obtain the convergence of the observation error to zero in finite time.

IV. Application Case

We highlight in this section the utility and the advantages of the high order sliding modes of the previous recalls in the resolution of the observation problem. At first, we use two systems previously proposed in the literature [11], [1, Ch. 4]. At last,we apply the same procedure to a *Continuous Stirred Tank Reactor* (CSTR) system [16].

A. Example 1

Using a system proposed in [11], let the Bounded Input Bounded State (BIBS) in finite time presented in the triangular input observer form

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 - x_2^3 + x_2 u \\ \dot{x}_3 &= -\left(x_2^2 + x_3^2\right) x_3^3 + u \\ y &= x_1 \end{aligned} \tag{11}$$

The proposed observer as in (10) for the system (11) is shown in equation (12). The parameter values are $L = 400$, $\lambda_1 = 1.1$, $\lambda_2 = 1.5$ and $\lambda_3 = 2$.

$$\begin{aligned} \dot{\hat{x}}_1 &= \zeta_1, \\ \zeta_1 &= \hat{x}_2 - \lambda_1 L^{\frac{1}{4}} \left|\hat{x}_1 - x_1\right|^{\frac{3}{4}} \operatorname{sign}\left(\hat{x}_1 - x_1\right) \\ \dot{\hat{x}}_2 &= \zeta_2, \\ \zeta_2 &= \hat{x}_3 - \hat{x}_2^3 + \hat{x}_2 u - \lambda_2 L^{\frac{1}{3}} \left|\hat{x}_2 - \zeta_1\right|^{\frac{2}{3}} \operatorname{sign}\left(\hat{x}_2 - \zeta_1\right) \\ \dot{\hat{x}}_3 &= -\left(\hat{x}_2^2 + \hat{x}_3^2\right) \hat{x}_3^3 + u - \lambda_3 L^{\frac{1}{2}} \left|\hat{x}_3 - \zeta_2\right|^{\frac{1}{2}} \operatorname{sign}\left(\hat{x}_3 - \zeta_2\right) \end{aligned} \tag{12}$$

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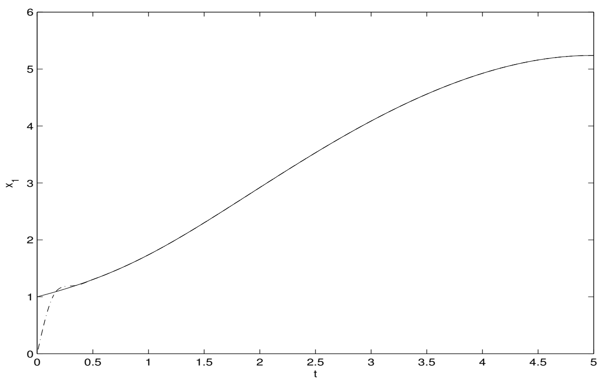

Fig. 1. Real x_1 and observed $\hat{x}_1$(- -) states for system (11)

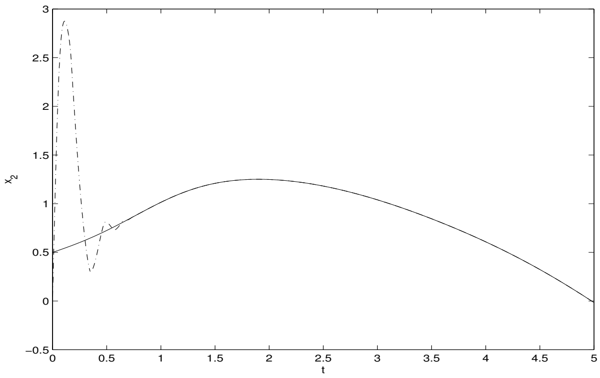

Fig. 2. Real x_2 and observed $\hat{x}_2$(- -) states for system (11)

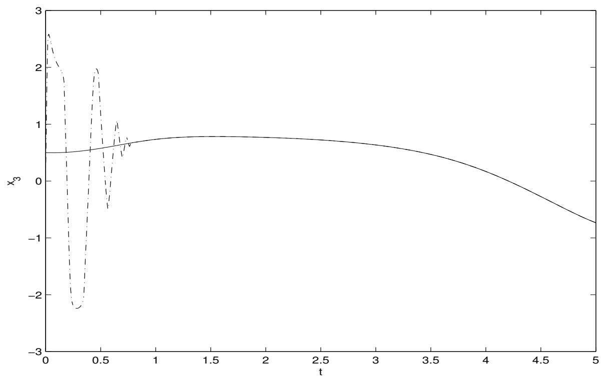

Fig. 3. Real x_3 and observed $\hat{x}_3$(- -) states for system (11)

Figures 1, 2 and 3 shows the simulation results for the system (11). The initial conditions for this system were $x(0) = [1\,0.5\,0.5]^T$ and $\hat{x}(0) = [0\,0\,0]^T$. The state $\hat{x}_1$ converges to x_1 in finite time of 0.25s. Then $\hat{x}_2$ reaches to x_2 in finite time 0.75s. Note that $\hat{x}_2$ only reaches x_2 at a time of 0.75s, and after $\hat{x}_1$ converges to its state. Finally at 1s, $\hat{x}_3$ converges to x_3.

B. Example 2

Let us consider the following system in the triangular input observer form [1, Ch. 4]:

$$\begin{aligned} \dot{x}_1 &= x_2 - x_1^3 u \\ \dot{x}_2 &= x_3 - x_1 x_2 u \\ \dot{x}_3 &= -3x_3 - 3x_2 - x_1 - x_3^3 - u \\ y &= x_1 \end{aligned} \tag{13}$$

For the system (13), the observer of equation (10) takes the form shown in equation (14), with the parameter values $L = 400$, $\lambda_1 = 1.1$, $\lambda_2 = 1.5$ and $\lambda_3 = 2$.

$$
\begin{aligned}
\dot{\hat{x}}_1 &= \zeta_1, \\
\zeta_1 &= \hat{x}_2 - x_1^3 u - \lambda_1 L^{\frac{1}{4}} \left|\hat{x}_1 - x_1\right|^{\frac{3}{4}} \operatorname{sign}\left(\hat{x}_1 - x_1\right) \\
\dot{\hat{x}}_2 &= \zeta_2, \\
\zeta_2 &= \hat{x}_3 - x_1 \hat{x}_2 u + \lambda_2 L^{\frac{1}{3}} \left|\hat{x}_2 - \zeta_1\right|^{\frac{2}{3}} \operatorname{sign}\left(\hat{x}_2 - \zeta_1\right) \\
\dot{\hat{x}}_3 &= -3\hat{x}_3 - 3\hat{x}_2 - x_1 - \hat{x}_3^3 - u - \lambda_3 L^{\frac{1}{2}} \left|\hat{x}_3 - \zeta_2\right|^{\frac{1}{2}} \operatorname{sign}\left(\hat{x}_3 - \zeta_2\right)
\end{aligned}
\tag{14}
$$

This approach has been tested by simulation with initial conditions $x(0) = [1\ 0.5\ 0.5]^T$ and $\hat{x}(0) = [0\ 0\ 0]^T$. In Figure 4, we see that $\hat{x}_1$ reaches x_1 in finite time around 0.5s. In Figure 5, we see that $\hat{x}_2$ also reaches x_2 in finite time of about 0.75s. But $\hat{x}_2$ will only reach x_2 after $\hat{x}_1$ has been able to get to the value of x_1. In Figure 6, we see that $\hat{x}_3$ reaches x_3 in finite time of 1.2s.

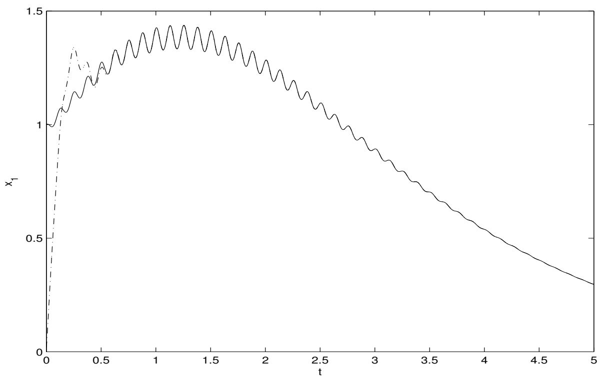

Fig. 4. Real x_1 and observed $\hat{x}_1$(- -) states for system (13)

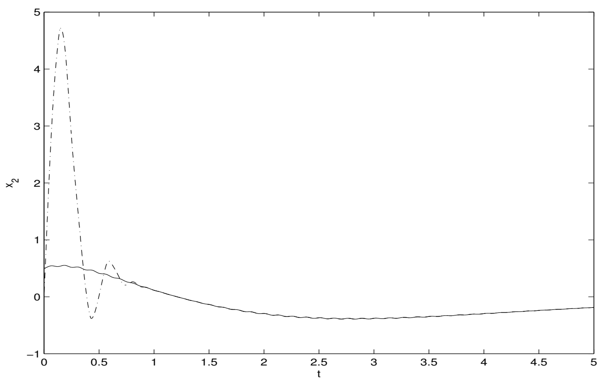

Fig. 5. Real x_2 and observed $\hat{x}_2$(- -) states for system (13)

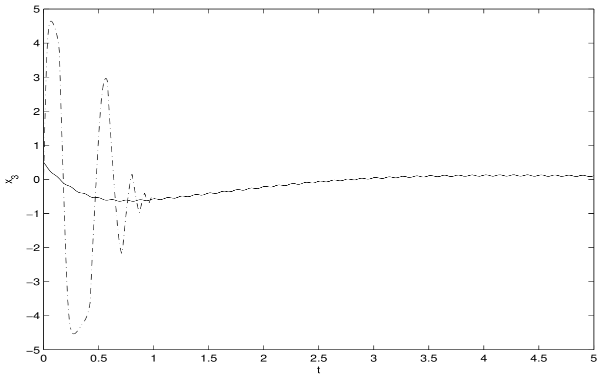

Fig. 6. Real x_3 and observed $\hat{x}_3$(- -) states for system (13)

C. CSTR Application

The CSTR process is a recognized benchmark frequently used for controller proofs [16] and represents many processes typically employed in industry. The model of the CSTR is described by the set of equations (15). See [16] for a more in-depth description and modelling of the process and its parameters. The simulation values for the system were also extracted from the same reference, and they are summarized in Table I.

$$
\begin{aligned}
\frac{dT}{dt} &= \frac{F}{V}\left(T_{in} - T\right) - \frac{\Delta H}{\rho C_p} k_0 C_A e^{-\frac{E}{RT}} + \frac{UA}{\rho C_p V}\left(T_j - T\right) \\
\frac{dC_A}{dt} &= \frac{F}{V}\left(C_{in} - C_A\right) - k_0 C_A e^{-\frac{E}{RT}}.
\end{aligned}
\tag{15}
$$

Assuming as the state variables the temperature T of the reactive mass and the concentration C_A of the reactant respectively, the model in space-state representation is shown in the set of equations (16). We suppose that T can be measured and acts as the model output y. The goal is the estimation of C_A (denoted by χ_2 from T (denoted by χ_1).

$$
\begin{aligned}
\dot{\chi}_1 &= \frac{F}{V}\left(T_{in} - \chi_1\right) - \frac{\Delta H}{\rho C_p} k_0 \chi_2 e^{-\frac{E}{R\chi_1}} + \frac{UA}{\rho C_p V}\left(T_j - \chi_1\right) \\
\dot{\chi}_2 &= \frac{F}{V}\left(C_{in} - \chi_2\right) - k_0 \chi_2 e^{-\frac{E}{R\chi_1}} \\
y &= h(\chi) = \chi_1
\end{aligned}
\tag{16}
$$

In order to find an state observer for the system, we have to describe the process in the triangular input observer form. From (4),

$$
\begin{aligned}
x_1 &= \chi_1 \\
x_2 &= \frac{F}{V}\left(T_{in} - \chi_1\right) - \frac{\Delta H}{\rho C_p} k_0 \chi_2 e^{-\frac{E}{R\chi_1}} + \frac{UA}{\rho C_p V}\left(T_j - \chi_1\right)
\end{aligned}
\tag{17}
$$

,

and the resultant system is:

TABLE I
NOMINAL PARAMETERS OF CSTR

Parameter	Value	Unit
F	0.1605	$m^3 \cdot min^{-1}$
V	2.4069	m^3
C_{in}	2114.5	$gmol \cdot m^{-3}$
k_0	2.8267·10^{11}	min^{-1}
E	75361.14	$J \cdot gmol^{-1}$
R	8.3174	$J \cdot gmol^{-1} K^{-1}$
T_{in}	295.22	K
ΔH	-9.0712·10^4	$J \cdot gmol^{-1}$
ρ	1000	$kg \cdot m^{-3}$
C_p	3571.3	$J \cdot kg^{-1}$
U	2.5552·10^4	$J \cdot (s \cdot m^2 \cdot K)^{-1}$
A	8,1755	m^{-2}
T_j	279	K

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \left(k_0 e^{-\frac{E}{Rx_1}} + \frac{F}{V} - \frac{Ex_2}{Rx_1^2}\right) \times \\
&\left(\frac{F}{V}\left(T_{in} - x_1\right) + \frac{UA}{\rho C_p V}\left(T_j - x_1\right) - x_2\right) - \\
&\left(\frac{UA}{V\rho C_p} + \frac{F}{V}\right) x_2 - \frac{C_{in} F \Delta H}{\rho C_p V} k_0 e^{-\frac{E}{Rx_1}}
\end{aligned} \tag{18}$$

. The observer obtained for the CSTR, with parameter values $L = 250$, $\lambda_1 = 2.5$ and $\lambda_2 = 1$ is presented in eq. (19).

$$\begin{aligned}
\dot{\hat{x}}_1 &= \zeta_1, \\
\zeta_1 &= \hat{x}_2 - \lambda_1 L^{\frac{1}{3}} \left|x_1 - \hat{x}_1\right|^{\frac{2}{3}} \operatorname{sign}\left(\hat{x}_1 - x_1\right) \\
\dot{\hat{x}}_2 &= \left(k_0 e^{-\frac{E}{Rx_1}} + \frac{F}{V} - \frac{E\hat{x}_2}{Rx_1^2}\right) \times \\
&\left(\frac{F}{V}\left(T_{in} - x_1\right) + \frac{UA}{\rho C_p V}\left(T_j - x_1\right) - \hat{x}_2\right) - \\
&\left(\frac{UA}{V\rho C_p} + \frac{F}{V}\right) \hat{x}_2 - \frac{C_{in} F \Delta H}{\rho C_p V} k_0 e^{-\frac{E}{Rx_1}} - \\
&\lambda_2 L^{\frac{1}{2}} \left|\hat{x}_2 - \zeta_1\right|^{\frac{1}{2}} \operatorname{sign}\left(\hat{x}_2 - \zeta_1\right)
\end{aligned} \tag{19}$$

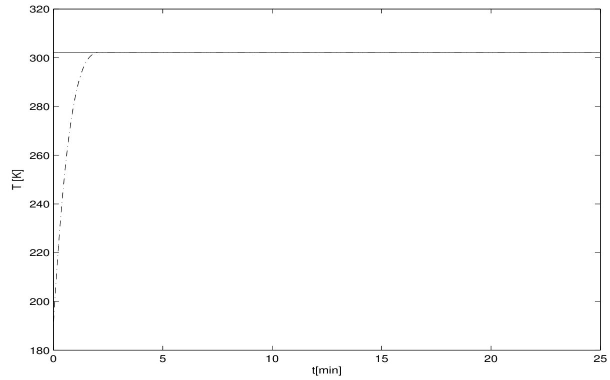

Fig. 7. Temperature x_1 and its observation $\hat{x}_1$(- -) for CSTR

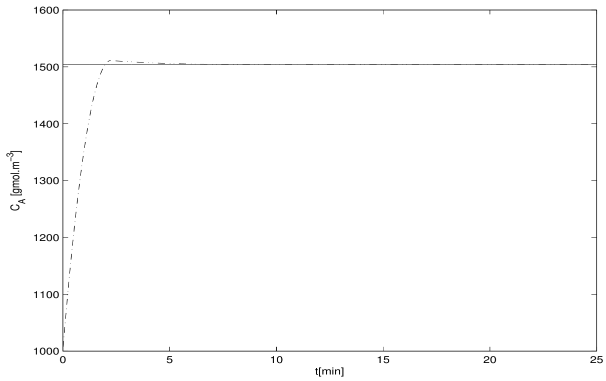

Fig. 8. Concentration x_2 and its observation $\hat{x}_2$(- -) for CSTR

This approach has been tested by simulation with the the initial conditions $x(0) = [302.24\; 1504.37]^T$ and $\hat{x}(0) = [190\; 1000]^T$. In Figure 7, we can see $\hat{x}_1$ reaching the real temperature value in less than 5 minutes. In Figure 8, we see that $\hat{x}_2$ also converges to the real value of the concentration in finite time of about 12 minutes.

V. CONCLUSION

We show in this paper that we can design a high order sliding mode observer for single output systems which can be transformed into triangular observer form. From this form, many observer designs work well, and in this case, advantages of the sliding mode observer were principally the design simplicity and the finite time convergence. Moreover, we also showed the application of the proposed schemes to a real process model like the CSTR. This model is a well documented benchmark that includes many dynamical processes; in this regard, the results of this work can be expanded towards a plethora of different application cases.

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