Integral High Order Sliding Mode Control of Single-Phase Induction Motor

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***Abstract*—An observer-based controller for the single-phase induction motor is proposed in this paper. The scheme presented is formulated using block control feedback linearization technique and high order sliding mode algorithms with measurements of the rotor speed and stator currents. A second order sliding mode observer is included into the controller design in order to obtain estimates of the rotor flux. The stability of the complete closed-loop system is analyzed in the presence of model uncertainty, namely, rotor resistance variation and bounded time-varying load torque.**

I. Introduction

This paper is aimed to present an observer-based controller using high order sliding mode (HOSM) algorithms for capacitor-run single-phase induction motor (SPIM). It is well known the SPIM is widely used in many household applications as compressors, pumps, air conditioning systems, washer, refrigerators, and other equipment which require low power motors [1]. Therefore, the design of control algorithms which improve the SPIM performance is a relevant task.

An important class of solutions for this problem is the observer-based controllers. For the SPIM case, it consists of: (i) a feedback controller for speed profile tracking and flux magnitude regulation, (ii) a state observer to estimate the rotor flux, and (iii) stability analysis of the whole system closed by the designed observer-based feedback.

For the feedback controller case, several approaches have been proposed for the induction motor control. For example, a classical vector control with field orientation technique, due to [2], the application of back-stepping [3], passivity-based control [4], [5], input-output feedback linearization [6], adaptive [7] and sliding mode (SM) [8]–[11] including neural networks [12] and discrete controllers [13]. However, most of the mentioned proposals are for the three phase motor (TPIM). The treatment of the SPIM control design problem is different from the TPIM controller, since the SPIM despite of symmetric TPIM has a basic control input which applies to the main winding, and the auxiliary winding is affected by the switched capacitor, it looks like a ”subactuated” system. Moreover, this control input that depends on switching parameter which can take just two values ”0” or ”1”.

In addition, most of the proposed methods assume the rotor flux to be known. Hence, it is necessary the development of a tool that allows the estimation of this variable. The estimate is usually obtained from machine model and the measurement of speed and stator voltage and current [14], [15]. Several flux observers have been proposed using adaptive [16], [17] and sliding mode (SM) [8], [18], [19]. The proposed observers strategies guaranty robustness in the presence of plant model uncertainty.

Usually, the stability analysis of the complete observer-control system is carried by using the separation principle proposed in [20]. However, this principle was developed for a class of nonlinear minimum phase systems that can be presented in the observer canonical form. The induction motor case covers a different scenario and the applicability of the observer-controller scheme described in [20] is questionable and, by far not trivial. Thus, a more precise stability proof for the whole scheme is necessary.

In this paper, a robust observer-based controller design for the capacitor-run SPIM in the presence of uncertainty is considered. The proposed control scheme is based on the motor dynamic model including the capacitor dynamics, described in a stationary reference frame $(\alpha\beta)$ fixed in the stator. First, a second-order SM observer based on *equivalent control* [21] and a generalization [22] of the *super-twisting* algorithm [23] is designed to estimate the rotor flux. With the measured stator current and estimated rotor flux, the controller is proposed by using a combination of *block control feedback linearization* [24] and quasi-continuous SM algorithms [25] in order to design a nested integral structure as [26], [27] but with exact disturbance rejection, similar to the techniques presented in [28], [29]. The super twisting algorithm for the basic control input and switching logic for the auxiliary input is proposed in order to ensure the design sliding manifold be a finite time attractive. The closed-loop system exhibits the properties of exponential tracking and robustness, allowing to overcome the uncertainty due to the parameter variations and external disturbances as the load torque.

In the following, Section II provides the considered model of the SPIM. Sections III and IV describe the proposed observer and controllers, including a detailed analysis of stability and robustness. Simulation results which demonstrate the main characteristics of the proposed controller, are presented in Section VI. Finally, in Section VII the conclusions are given.

II. Mathematical Model for the *SPIM*

The dynamic model of the SPIM can be considered as the model of an unsymmetrical 2-phase (a, b) induction machine in the variables of circuit elements. After the transformation to a fixed frame $(\alpha\beta)$ [30], the single phase induction motor scheme with the stator current and the rotor flux as the state variables, is presented in Fig. 1.

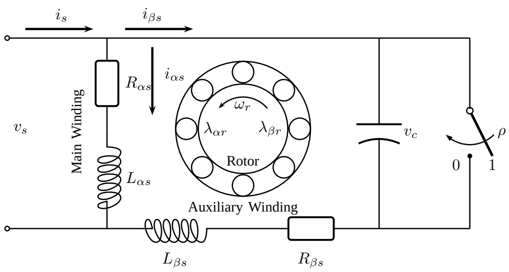

Fig. 1: Single phase induction motor.

and its dynamic equations are given by

$$\begin{aligned}\frac{di_{\alpha s}}{dt} =& - c_1 a_{10} i_{\alpha s} + c_1 c_{40} \lambda_{\alpha r} - c_1 c_3 n_p \omega_r \lambda_{\beta r} \\ &+ c_1 v_{\alpha s} + \Delta_{\alpha s} \\ \frac{di_{\beta s}}{dt} =& - c_2 a_{20} i_{\beta s} + c_2 c_{40} \lambda_{\beta_r} + c_2 c_3 n_p \omega_r \lambda_{\alpha r} \\ &+ c_2 v_{\beta s} + \Delta_{\beta s} \\ \frac{d\lambda_{\alpha r}}{dt} =& - a_{30} \lambda_{\alpha r} + n_p \omega_r \lambda_{\beta r} + a_{40} i_{\alpha s} + \Delta_{\alpha r} \\ \frac{d\lambda_{\beta r}}{dt} =& - n_p \omega_r \lambda_{\alpha r} - a_{30} \lambda_{\beta r} + a_{40} i_{\beta s} + \Delta_{\beta r} \\ \frac{d\omega_r}{dt} =& d_1 d_2 \left(\lambda_{\beta r} i_{\alpha s} - \lambda_{\alpha r} i_{\beta s}\right) - d_2 T_L\end{aligned} \tag{1}$$

where $\lambda_{\alpha r}$ and $\lambda_{\beta r}$ are the rotor magnetic-flux-linkage, $i_{\alpha s}$ and $i_{\beta s}$ are the stator current, $v_{\alpha s}$ and $v_{\beta s}$ are the voltage of the main and auxiliary stator windings, respectively, ω_r is the rotor speed, n_p is the number of pole pairs, T_L is the load torque. This model considers variations on rotor resistance of the form$R_r(t) = R_{r0} + \Delta R_r(t)$ where $\Delta R_r(t)$ is an unknown but bounded function of time, leading to a set of uncertain model parameters $a_1(t) = a_{10} + \Delta a_1(t)$, $a_2(t) = a_{20} + \Delta a_2(t)$, $a_3(t) = a_{30} + \Delta a_3(t)$, $a_4(t) = a_{40} + \Delta a_4(t)$ and $c_4(t) = c_{40} + \Delta c_4(t)$ where $a_{10} = \left(R_{\alpha s} + R_{r0}\frac{L_m^2}{L_r^2}\right)$, $a_{20} = \left(R_{\beta s} + R_{r0}\frac{L_m^2}{L_r^2}\right)$, $a_{30} = \frac{R_{r0}}{L_r}$, $a_{40} = \frac{R_{r0}}{L_r}L_m$ and $c_{40} = \frac{R_{r0}}{L_r^2}L_m$ are the parameter nominal values. The parametric uncertainties are presented by $\Delta a_1(t) = \Delta a_2(t) = \frac{L_m^2}{L_r^2}\Delta R_r(t)$, $\Delta a_3(t) = \frac{1}{L_r}\Delta R_r(t)$, $\Delta a_4(t) = \frac{L_m}{L_r}\Delta R_r(t)$, and $\Delta c_4(t) = \frac{L_m}{L_r^2}\Delta R_r(t)$. While the model parameters which do not depend on the resistance variations are given by $c_1 = \frac{L_r}{L_{\alpha_s} L_r - L_m^2}$, $c_2 = \frac{L_r}{L_{\beta_s} L_r - L_m^2}$, $c_3 = \frac{L_m}{L_r}$, $d_1 = n_p \frac{L_m}{L_r}$ and $d_2 = \frac{1}{J}$.

Thus, the unknown terms in (1) are defined by $\Delta_{\alpha s} = \Delta c_4(t) c_1 \lambda_{\alpha r} - \Delta a_1(t) c_1 i_{\alpha s}$, $\Delta_{\beta s} = \Delta c_4(t) c_2 \lambda_{\beta r} - \Delta a_2(t) c_2 i_{\beta s}$, $\Delta_{\alpha r} = -\Delta a_3(t)\lambda_{\alpha r} + \Delta a_4(t) i_{\alpha s}$, and $\Delta_{\beta r} = -\Delta a_3(t)\lambda_{\beta r} + \Delta a_4(t) i_{\beta s}$.

The dynamics of the capacitor (see Fig.1) are given by

$$\frac{dv_c}{dt} = \omega_0 X_c i_{\beta s}$$

where X_c is the capacitor reactance and $\omega_0 = 2\pi f$, with f being the fundamental frequency.

Using the relation between the voltages $v_{\alpha s}$ and $v_{\beta s}$ in (1) of the form

$$\begin{aligned} v_{\alpha s} &= v_s \\ v_{\beta s} &= n^{-1} v_s - v_c \rho \end{aligned} \tag{2}$$

where the switching parameter $\rho \in \{0, 1\}$, the voltage $v_{\beta s}$ yields to

$$v_{\beta s} = \begin{cases} n^{-1} v_s - v_c & \text{if} \quad \rho = 1 \\ n^{-1} v_s & \text{if} \quad \rho = 0 \end{cases}$$

being $n^{-1} v_s$ as a referred voltage of the main winding to the auxiliary winding with $n = N_A / N_B$, where N_A is the number turns of main winding and N_B is the number turns of an auxiliary winding.

III. Second Order Sliding Mode Observer for Rotor Fluxes

Having the rotor speed ω_r and stator current $i_{\alpha s}$, $i_{\beta s}$ measurements only, in this section a second order SM observer is designed to estimate the rotor flux.

Considering the transformation

$$\lambda_{\alpha r}^* = i_{\alpha s} + c_1 c_3 \lambda_{\alpha r}, \quad \lambda_{\beta r}^* = i_{\beta s} + c_2 c_3 \lambda_{\beta r} \tag{3}$$

the flux and current dynamics (1) are represented in new variables of the form

$$\begin{aligned} \frac{di_{\alpha s}}{dt} &= -\vartheta_{11} i_{\alpha s} + \vartheta_{12} \lambda_{\alpha r}^* - \varphi_1 \omega_r \lambda_{\beta r}^* + c_1 v_{\alpha s} + \Delta_{\alpha s} \\ \frac{di_{\beta s}}{dt} &= -\vartheta_{21} i_{\beta s} + \vartheta_{22} \lambda_{\beta r}^* + \varphi_2 \omega_r \lambda_{\alpha r}^* + c_2 v_{\beta s} + \Delta_{\beta s} \\ \frac{d\lambda_{\alpha r}^*}{dt} &= \varsigma_{11} i_{\alpha s} + \varsigma_{12} \lambda_{\alpha r}^* + c_1 v_{\alpha s} + \Delta_{\alpha r} \\ \frac{d\lambda_{\beta r}^*}{dt} &= \varsigma_{21} i_{\beta s} + \varsigma_{22} \lambda_{\beta r}^* + c_2 v_{\beta s} + \Delta_{\beta r} \end{aligned} \tag{4}$$

where $\vartheta_{11} = c_1 a_1 + \frac{c_1}{c_4}$, $\vartheta_{12} = \vartheta_{22} = \frac{c_4}{c_3}$, $\vartheta_{21} = c_2 a_2 + \frac{c_4}{c_3}$, $\varphi_1 = c_1 c_3 n_p$, $\varphi_2 = c_2 c_3 n_p$, $\varsigma_{11} = c_1 c_3 a_4 - c_1 a_1 - \varsigma_{12}$, $\varsigma_{21} = c_2 c_3 a_4 - c_2 a_1 - \varsigma_{22}$, $\varsigma_{12} = \left(\frac{c_1 c_4 - c_1 c_3 a_3}{c_1 c_3}\right)$ and, $\varsigma_{22} = \left(\frac{c_2 c_4 - c_2 c_3 a_3}{c_2 c_3}\right)$. Here, the disturbances $\Delta_{\alpha s}$ and $\Delta_{\beta s}$ are considered to be slow-varying, that is $\frac{d\Delta_{\alpha s}}{dt} = \frac{d\Delta_{\beta s}}{dt} = 0$.

Based on (4), and defining $\hat{\lambda}_{\alpha r}^*$, $\hat{\lambda}_{\beta r}^*$, $\hat{i}_{\alpha s}$, and $\hat{i}_{\beta s}$ as the estimates of $\lambda_{\alpha r}^*$, $\lambda_{\beta r}^*$, $i_{\alpha s}$, and $i_{\beta s}$, respectively, an observer based on the equivalent control method [21] is designed as

$$\begin{aligned} \frac{d\hat{i}_{\alpha s}}{dt} =& -\vartheta_{11} i_{\alpha s} + \vartheta_{12} \hat{\lambda}_{\alpha r}^* - \varphi_1 \omega_r \hat{\lambda}_{\beta r}^* + c_1 v_{\alpha s} + \hat{\Delta}_{\alpha s} \\ &+ l_{11} \rho_1 \left(\tilde{i}_{\alpha s}\right) + V_1 \\ \frac{d\hat{i}_{\beta s}}{dt} =& -\vartheta_{21} i_{\beta s} + \vartheta_{22} \hat{\lambda}_{\beta r}^* + \varphi_2 \omega_r \hat{\lambda}_{\alpha r}^* + c_2 v_{\beta s} + \hat{\Delta}_{\beta s} \\ &+ l_{21} \rho_1 \left(\tilde{i}_{\beta s}\right) + V_2 \\ \frac{d\hat{\lambda}_{\alpha r}^*}{dt} =& \varsigma_{11} i_{\alpha s} + \varsigma_{12} \hat{\lambda}_{\alpha r}^* + c_1 v_{\alpha s} + l_3 V_1 \\ \frac{d\hat{\lambda}_{\beta r}^*}{dt} =& \varsigma_{21} i_{\beta s} + \varsigma_{22} \hat{\lambda}_{\beta r}^* + c_2 v_{\beta s} + l_4 V_2 \\ \frac{d\hat{\Delta}_{\alpha s}}{dt} =& l_5 V_1, \quad \frac{dV_1}{dt} = l_{12} \rho_2 \left(\tilde{i}_{\alpha s}\right) \\ \frac{d\hat{\Delta}_{\beta s}}{dt} =& l_6 V_2, \quad \frac{dV_2}{dt} = l_{22} \rho_2 \left(\tilde{i}_{\beta s}\right) \end{aligned} \tag{5}$$

where $\tilde{i}_{\alpha s} = i_{\alpha s} - \hat{i}_{\alpha s}$, and $\tilde{i}_{\beta s} = i_{\beta s} - \hat{i}_{\beta s}$ are the estimation errors of $i_{\alpha s}$, and $i_{\beta s}$, respectively. With $\rho_1(\cdot) = \mu_1|\cdot|^{\frac{1}{2}}\text{sign}(\cdot) + \mu_2(\cdot) + \mu_3|\cdot|^{\frac{3}{2}}\text{sign}(\cdot)$, $\rho_2(\cdot) = \frac{1}{2}\mu_1^2\text{sign}(\cdot) + \frac{3}{2}\mu_1\mu_2|\cdot|^{\frac{1}{2}}\text{sign}(\cdot) + (\mu_2^2 + 2\mu_1\mu_3)(\cdot) + \frac{5}{2}\mu_2\mu_3|\cdot|^{\frac{3}{2}}\text{sign}(\cdot) + \frac{3}{2}\mu_3^2|\cdot|^2\text{sign}(\cdot)$, $l_j > 0$ for $j = 1, \ldots, 6$ and, $\mu_i > 0$ for $i = 1, 2, 3$.

As a result, the rotor flux estimates $\hat{\lambda}_{\alpha r}$ and $\hat{\lambda}_{\beta r}$ are obtained as $\hat{\lambda}_{\alpha r} = \frac{\hat{\lambda}^*_{\alpha r} - \hat{i}_{\alpha s}}{c_1 c_3}$ and $\hat{\lambda}_{\beta r} = \frac{\hat{\lambda}^*_{\beta r} - \hat{i}_{\beta s}}{c_2 c_3}$.

IV. Sliding Mode Controller Design

Provided that the currents and speed are continuously measured and the rotor fluxes are estimated, the objective here is to design a SM controller which can effectively track the desired speed ω_{ref} and the module to the square of the rotor flux ϕ_{ref} reference signals by means of the continuous basic control v_s and auxiliary control ρ as a discontinuous function.

A. Sliding Manifold Design

As first step, the state variables x_1 and x_2 are defined as $x_1 = \begin{bmatrix} \omega_r & \phi \end{bmatrix}^T$ and $x_2 = \begin{bmatrix} i_{\alpha s} & i_{\beta s} \end{bmatrix}^T$, where $\phi = |\psi|^2 = \lambda_{\alpha r}^2 + \lambda_{\beta r}^2$. Thus, the system (1) can be represented in the *nonlinear block controllable form* with disturbance [24]

$$\begin{aligned} \frac{dx_1}{dt} &= f_1(\phi) + B_1(\lambda_r)x_2 + D_1 T_L + \Delta_r \\ \frac{dx_2}{dt} &= f_2(\omega_r, \lambda_r, i_s) + B_2 u + \Delta_s \end{aligned} \tag{6}$$

where $\lambda_r = \begin{bmatrix} \lambda_{\alpha r} & \lambda_{\beta r} \end{bmatrix}^T$, $u = \begin{bmatrix} v_{\alpha s} & v_{\beta s} \end{bmatrix}^T$, $f_1(\phi) = \begin{bmatrix} f_{11} & f_{12} \end{bmatrix}^T = \begin{bmatrix} 0 & -2a_{30}\phi \end{bmatrix}^T$, $D_1 = \begin{bmatrix} -d_2 & 0 \end{bmatrix}^T$, $f_2 = \begin{bmatrix} f_{21} & f_{22} \end{bmatrix}^T$, $\Delta_r = \begin{bmatrix} 0 & 2\Delta_{\alpha r}\lambda_{\alpha r} + 2\Delta_{\beta r}\lambda_{\beta r} \end{bmatrix}^T$, $\Delta_s = \begin{bmatrix} \Delta_{\alpha s} & \Delta_{\beta s} \end{bmatrix}^T$, $B_1(\lambda_r) = \begin{bmatrix} d_1 d_2 \lambda_{\beta r} & -d_1 d_2 \lambda_{\alpha r} \\ 2a_{40}\lambda_{\alpha r} & 2a_{40}\lambda_{\beta r} \end{bmatrix}$ and, $B_2 = \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix}$, with $f_{21} = -a_{10}c_1 i_{\alpha s} + c_1 c_{40}\lambda_{\alpha r} - c_1 c_3 \omega_r \lambda_{\beta r}$ and $f_{22} = -a_{20}c_2 i_{\beta s} + c_2 c_3 \omega_r \lambda_{\alpha r} + c_2 c_{40}\lambda_{\beta r}$.

Only the estimates of the rotor fluxes are available for the control design. Hence, the estimated variables $\hat{\phi} = \hat{\lambda}_{\alpha r}^2 + \hat{\lambda}_{\beta r}^2$, $\hat{\lambda}_r = (\hat{\lambda}_{\alpha r}, \hat{\lambda}_{\beta r})$ and its errors $\tilde{\phi} = \phi - \hat{\phi}$, $\tilde{\lambda}_r = \lambda_r - \hat{\lambda}_r$, are defined.

Setting the *controller-used* error $\hat{z}_1 = \begin{bmatrix} z_{11} & \hat{z}_{12} \end{bmatrix}^T$ and *real* tracking errors $z_1 = \begin{bmatrix} z_{11} & z_{12} \end{bmatrix}^T$, with $z_{11} = \omega_r - \omega_{ref}(t)$, $\hat{z}_{12} = \hat{\phi} - \phi_{ref}(t)$ and, $z_{12} = \phi - \phi_{ref}(t) = \hat{\phi} + \tilde{\phi} - \phi_{ref}(t) = \hat{z}_{12} + \tilde{\phi}$, the dynamics of the first transformed block (6) become

$$\frac{dz_1}{dt} = f_1(\hat{\phi}) + B_1(\hat{\lambda}_r)x_2 + \tilde{\Phi} + \bar{\Delta}_1 \tag{7}$$

where $\tilde{\Phi} = \begin{bmatrix} 0 & \frac{d\tilde{\phi}}{dt} \end{bmatrix}^T$ and $\bar{\Delta}_1 = D_1 T_L + \Delta_r + \begin{bmatrix} \frac{d\omega_{ref}(t)}{dt} & \frac{d\phi_{ref}(t)}{dt} \end{bmatrix}^T$.

To stabilize the dynamics for z_1, x_2 can be selected as a stabilizing term in form of a virtual controller. Therefore, the desired value for x_2 is defined as $x_{2\text{des}} = \begin{bmatrix} i_{\alpha s}^{\text{des}} & i_{\beta s}^{\text{des}} \end{bmatrix}^T$, and it is proposed of the form

$$x_{2\text{des}} = x_{2\text{des}}^0 + x_{2\text{des}}^1 \tag{8}$$

where $x_{2\text{des}}^1$ will be designed to reject the disturbance $\bar{\Delta}_1$ in finite time by using the integral sliding mode technique [31]. The term $x_{2\text{des}}^0$ is such that z_1 converges exponentially to zero.

To establish the control $x_{2\text{des}}$ in (7), the error variable $z_2 = \begin{bmatrix} z_{21} & z_{22} \end{bmatrix}^T$ is defined as

$$z_2 = x_2 - x_{2\text{des}}. \tag{9}$$

and (7) is rewritten as

$$\begin{aligned} \frac{dz_1}{dt} &= f_1(\hat{\phi}) + B_1(\hat{\lambda}_r)x_{2\text{des}}^0 + B_1(\hat{\lambda}_r)x_{2\text{des}}^1 \\ &\quad + B_1(\hat{\lambda}_r)z_2 + \tilde{\Phi} + \bar{\Delta}_1. \end{aligned} \tag{10}$$

In order to calculate $x_{2\text{des}}^1$, the variable $\sigma = \begin{bmatrix} \sigma_1 & \sigma_2 \end{bmatrix}^T$ is proposed as

$$\sigma = \hat{z}_1 + \xi \tag{11}$$

where $\xi = \begin{bmatrix} \xi_1 & \xi_2 \end{bmatrix}^T$ is an integral variable to be defined below.

From (11), the dynamics of σ are given by

$$\begin{aligned} \frac{d\sigma}{dt} &= f_1(\hat{\phi}) + B_1(\hat{\lambda}_r)x_{2\text{des}}^0 + B_1(\hat{\lambda}_r)x_{2\text{des}}^1 \\ &\quad + B_1(\hat{\lambda}_r)z_2 + \tilde{\Phi} + \bar{\Delta}_1 + \frac{d\xi}{dt}. \end{aligned} \tag{12}$$

With the selection of $\frac{d\xi}{dt}$ as

$$\frac{d\xi}{dt} = -f_1(\hat{\phi}) - B_1(\hat{\lambda}_r)x_{2\text{des}}^0 \tag{13}$$

where $\xi(0) = z_1(0)$, the system (12) reduces to

$$\frac{d\sigma}{dt} = B_1(\hat{\lambda}_r)x_{2\text{des}}^1 + B_1(\hat{\lambda}_r)z_2 + \tilde{\Phi} + \bar{\Delta}_1. \tag{14}$$

To enforce sliding motion on the manifold $\sigma = 0$ despite of the disturbance $\bar{\Delta}_1$, the term $x_{2\text{des}}^1$ in (14) is chosen as $x_{2\text{des}}^1 = B_1^{-1}(\hat{\lambda}_r)\nu$, with $\nu = \begin{bmatrix} \nu_1 & \nu_2 \end{bmatrix}^T$ defined as the solution to

$$\begin{aligned} \frac{d\nu_1}{dt} &= -k_{\sigma_1}\frac{\frac{d\sigma_1}{dt} + k_{\delta_1}|\sigma_1|^{\frac{1}{2}}\text{sign}(\sigma_1)}{\left|\frac{d\sigma_1}{dt}\right| + k_{\delta_1}|\sigma_1|^{\frac{1}{2}}} \\ \frac{d\nu_2}{dt} &= -k_{\sigma_2}\frac{\frac{d\sigma_2}{dt} + k_{\delta_2}|\sigma_2|^{\frac{1}{2}}\text{sign}(\sigma_2)}{\left|\frac{d\sigma_2}{dt}\right| + k_{\delta_2}|\sigma_2|^{\frac{1}{2}}}. \end{aligned} \tag{15}$$

Here, the derivatives $\frac{d\sigma_1}{dt}$ and $\frac{d\sigma_2}{dt}$ are obtained using a SM differentiator [32].

When the motion on the manifold $\sigma = 0$ is reached, the solution to $\frac{d\sigma}{dt} = 0$ in (14)

$$\{B_1(\hat{\lambda}_r)x_{2\text{des}}^1\}_{\text{eq}} = B_1(\hat{\lambda}_r)z_2 + \tilde{\Phi} + \bar{\Delta}_1 \tag{16}$$

shows that the disturbance $\tilde{\Phi} + \bar{\Delta}_1$ is rejected by the equivalent control $\{B_1(\hat{\lambda}_r)x_{2\text{des}}^1\}_{\text{eq}}$ [15]. Therefore, the dynamics on $\sigma = 0$ are given by

$$\frac{dz_1}{dt} = f_1(\hat{\phi}) + B_1(\hat{\lambda}_r)x_{2\text{des}}^0. \tag{17}$$

Thus, the desired dynamics $-K_1\hat{z}_1$ for $\frac{dz_1}{dt}$ in (17) are introduced by means of

$$x^0_{2\text{des}} = B_1^{-1}(\hat{\lambda}_r)\left[-f_1(\hat{\phi}) - K_1\hat{z}_1\right] \tag{18}$$

where $K_1 = \text{diag}(k_1, k_2)$ with $k_1 > 0$, $k_2 > 0$. Hence, with (18) in (13), $\frac{d\xi}{dt}$ reduces to

$$\frac{d\xi}{dt} = K_1\hat{z}_1. \tag{19}$$

B. Inducing Sliding Modes

From (9) , it follows that

$$\frac{dz_2}{dt} = f_2(\omega_r, \hat{\lambda}_s, i_s) + B_2 u + \Delta_2 \tag{20}$$

where the term $\Delta_2 = \Delta_r - \frac{dx_{2\text{des}}}{dt}$ is a bounded disturbance.

To induce a SM motion on the manifold on $z_{21} = 0$ or $i_{\alpha s} = i^{\text{des}}_{\alpha s}$ in the current loop, taking into account (2), the basic control v_s is formulated as [23]

$$\begin{aligned} v_s =& -\alpha_1 \left|z_{21}\right|^{1/2} \text{sign}\left(z_{21}\right) - \alpha_3 z_{21} + u_1 \\ \frac{du_1}{dt} =& -\alpha_2 \text{sign}\left(z_{21}\right) \end{aligned} \tag{21}$$

with $\alpha_1 > 0$, $\alpha_2 > 0$, and $\alpha_3 > 0$. And to induce a quasi-sliding mode motion on the manifold $z_{22} = 0$ or $i_{\beta s} = i^{\text{des}}_{\beta s}$, the auxiliary control ρ for the capacitor is designed by means of the following switching logic:

$$\rho = \frac{1}{2}\text{sign}\left(z_{22}v_c\right) + \frac{1}{2}. \tag{22}$$

V. Stability Analysis of the Observer-Based Controller

The extended closed loop system is presented as

$$\left\{ \frac{dz_1}{dt} = -K_1\hat{z}_1 + \nu + B_1(\hat{\lambda}_r)z_2 + \tilde{\Phi} + \Delta_1 \right. \tag{23}$$

$$\begin{cases} \frac{d\sigma}{dt} = \nu + B_1(\hat{\lambda}_r)z_2 + \tilde{\Phi} + \bar{\Delta}_1 \\ \frac{d\nu_1}{dt} = -k_{\sigma_1}\dfrac{\frac{d\sigma_1}{dt} + k_{\delta_1}|\sigma_1|^{\frac{1}{2}}\text{sign}(\sigma_1)}{\left|\frac{d\sigma_1}{dt}\right| + k_{\delta_1}|\sigma_1|^{\frac{1}{2}}} \\ \frac{d\nu_2}{dt} = -k_{\sigma_2}\dfrac{\frac{d\sigma_2}{dt} + k_{\delta_2}|\sigma_2|^{\frac{1}{2}}\text{sign}(\sigma_2)}{\left|\frac{d\sigma_2}{dt}\right| + k_{\delta_2}|\sigma_2|^{\frac{1}{2}}} \end{cases} \tag{24}$$

$$\left\{ \frac{dz_2}{dt} = f_2\left(\omega_r, \hat{\lambda}_s, i_s\right) + B_2 u + \Delta_2 \right. \tag{25}$$

$$\begin{cases} \frac{d\tilde{i}_{\alpha s}}{dt} = \vartheta_{12}\tilde{\lambda}^*_{\alpha r} - \varphi_1\omega_r\tilde{\lambda}^*_{\beta r} + \tilde{\Delta}_{\alpha s} \\ \qquad -l_{11}\rho_1\left(\tilde{i}_{\alpha s}\right) - V_1 \\ \frac{d\tilde{i}_{\beta s}}{dt} = \vartheta_{22}\tilde{\lambda}^*_{\beta r} + \varphi_2\omega_r\tilde{\lambda}^*_{\alpha r} + \tilde{\Delta}_{\beta s} \\ \qquad -l_{21}\rho_1\left(\tilde{i}_{\beta s}\right) - V_2 \end{cases} \tag{26}$$

$$\begin{cases} \frac{d\tilde{\lambda}^*_{\alpha r}}{dt} = \varsigma_{12}\tilde{\lambda}^*_{\alpha r} + \Delta_{\alpha r} - l_3 V_1 \\ \frac{d\tilde{\lambda}^*_{\beta r}}{dt} = \varsigma_{22}\tilde{\lambda}^*_{\beta r} + \Delta_{\beta r} - l_4 V_2 \\ \frac{d\tilde{\Delta}_{\alpha s}}{dt} = -l_5 V_1 \\ \frac{d\hat{\Delta}_{\beta s}}{dt} = -l_6 V_2. \end{cases} \tag{27}$$

It is possible to demonstrate for the block (24) that there is a SM on the manifold $\sigma = 0$ in finite time by using the results exposed in [25]. Similarly, it can be shown the finite time convergence of the system (25) to the manifold $z_2 = 0$ [11]. Finally, the uniform finite time convergence to zero of the estimation errors $\tilde{i}_{\alpha s}$ and $\tilde{i}_{\beta s}$ in the system (26) can be proved with the results presented by [22].

The SM motion is given by the systems (23) and (27) constrained to the set $\begin{bmatrix} \sigma_1 & \sigma_2 & z_{21} & z_{22} & \tilde{i}_{\alpha s} & \tilde{i}_{\beta s} \end{bmatrix}^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$ as follows:

$$\begin{aligned} \frac{dz_1}{dt} &= -K_1(z_1 + \tilde{z}_1) \\ \frac{d\tilde{\lambda}^*_{\alpha r}}{dt} &= (\varsigma_{12} - l_3\vartheta_{12})\tilde{\lambda}^*_{\alpha r} - l_3\tilde{\Delta}_{\alpha s} + \Delta_{\alpha r} \\ &\quad + l_3\varphi_1\omega_r\tilde{\lambda}^*_{\beta r} \\ \frac{d\tilde{\lambda}^*_{\beta r}}{dt} &= (\varsigma_{22} - l_4\vartheta_{22})\tilde{\lambda}^*_{\beta r} - l_4\tilde{\Delta}_{\beta s} + \Delta_{\beta r} \\ &\quad - l_4\varphi_2\omega_r\tilde{\lambda}^*_{\alpha r} \\ \frac{d\tilde{\Delta}_{\alpha s}}{dt} &= -l_5\vartheta_{12}\tilde{\lambda}^*_{\alpha r} - l_5\tilde{\Delta}_{\alpha s} + l_5\varphi_1\omega_r\tilde{\lambda}^*_{\beta r} \\ \frac{d\hat{\Delta}_{\beta s}}{dt} &= -l_6\vartheta_{22}\tilde{\lambda}^*_{\beta r} - l_6\tilde{\Delta}_{\beta s} - l_6\varphi_2\omega_r\tilde{\lambda}^*_{\alpha r}. \end{aligned} \tag{28}$$

To analyze the stability of the system (28), it can be written as a linear system with non-vanishing disturbance of the form

$$\dot{\mathbf{e}} = \mathbf{M}\mathbf{e} + \Delta \tag{29}$$

where $\mathbf{e} = \begin{bmatrix} z_1^T & \tilde{\lambda}^*_{\alpha r} & \tilde{\lambda}^*_{\beta r} & \tilde{\Delta}_{\alpha s} & \tilde{\Delta}_{\beta s} \end{bmatrix}^T$, M is the block matrix

$$\mathbf{M} = \begin{bmatrix} -K_1 & 0 & 0 & 0 & 0 \\ 0 & \varsigma_{12} - l_3\vartheta_{12} & 0 & -l_3 & 0 \\ 0 & 0 & \varsigma_{22} - l_4\vartheta_{22} & 0 & -l_4 \\ 0 & -l_5\vartheta_{12} & 0 & -l_5 & 0 \\ 0 & 0 & -l_6\vartheta_{22} & 0 & -l_6 \end{bmatrix}$$

and $\Delta = \begin{bmatrix} \tilde{z}_1^T & \Delta_{\alpha r} + l_3\varphi_1\omega_r\tilde{\lambda}^*_{\beta r} & \Delta_{\beta r} - l_4\varphi_2\omega_r\tilde{\lambda}^*_{\alpha r} \\ l_5\varphi_1\omega_r\tilde{\lambda}^*_{\beta r} & l_6\varphi_2\omega_r\tilde{\lambda}^*_{\alpha r} \end{bmatrix}^T$.

For (29), the Lyapunov candidate function $V = \frac{1}{2}\mathbf{e}^T\mathbf{P}\mathbf{e}$ is proposed, with $\mathbf{P} > 0$. With the adequate choice of l_i when $i = 3, \ldots, 6$ and K_1, the matrix $\mathbf{M}$ is Hurwitz. Hence, there exists one unique solution $\mathbf{P}$ to the Lyapunov equation $\mathbf{M}^T\mathbf{P} + \mathbf{P}\mathbf{M} = -\mathbf{Q}$, where $\mathbf{Q} = \mathbf{Q}^T$ and $\mathbf{Q} > 0$.

For the system (29), it is satisfied

$$\begin{aligned} \lambda_{\text{min}}(\mathbf{P})\left\|\mathbf{e}\right\|_2^2 \leq \mathbf{e}^T\mathbf{P}\mathbf{Q} \leq \lambda_{\text{max}}(\mathbf{P})\left\|\mathbf{e}\right\|_2^2 \\ \frac{\partial V}{\partial \mathbf{e}}\mathbf{M}\mathbf{e} = -\mathbf{e}^T\mathbf{Q}\mathbf{e} \leq -\lambda_{\text{min}}(\mathbf{Q})\left\|\mathbf{e}\right\|_2^2 \end{aligned} \tag{30}$$

and the perturbation term is considered to be bounded by $\|\Delta\| \leq \alpha_2\|\mathbf{e}\|_2 + \beta_2$, with $\alpha_2 > 0$ and $\beta_2 > 0$.

The derivative of V, yields to

$$\dot{V} = -\mathbf{e}^T\mathbf{Q}\mathbf{e} - 2\mathbf{e}^T\mathbf{P}\Delta \tag{31}$$

and, substituting the bounds (30) in (31), results

$$\dot{V} \leq \left(-\lambda_{\min}(\mathbf{Q}) + 2\alpha_2\lambda_{\max}(\mathbf{P})\right)\|\mathbf{e}\|_2^2 + 2\beta_2\lambda_{\max}(\mathbf{P})\|\mathbf{e}\|_2$$
$$= -\alpha(1-\theta)\|\mathbf{e}\|_2^2 - \alpha\theta\|\mathbf{e}\|_2^2 + \beta\|\mathbf{e}\|_2$$

where $\alpha = \lambda_{\min}(\mathbf{Q}) - 2\alpha_2\lambda_{\max}(\mathbf{P})$, $\beta = 2\beta_2\lambda_{\max}(\mathbf{P})$ and $0 < \theta < 1$. Finally, $\dot{V} \leq -\alpha(1-\theta)\|\mathbf{e}\|_2^2$, $\forall \|\mathbf{e}\|_2 > \delta$, with $\delta = \frac{\beta}{\alpha\theta}$.

Thus, the nominal system $\dot{\mathbf{e}} = \mathbf{M}\mathbf{e}$ has an exponentially stable equilibrium point $\mathbf{e} = 0$ and, the solution $\mathbf{e}(t)$ of (29) is ultimately bounded. The ultimate bound is given by $\|\mathbf{e}\|_2 \leq \delta\frac{\sqrt{\lambda_{\max}(\mathbf{P})}}{\sqrt{\lambda_{\min}(\mathbf{P})}}$.

VI. Numerical Simulation Results

To verify the effectiveness and efficiency of the proposed observer-based controller, numerical simulations are conducted using the Euler integration method with a time step $t_s = 1 \times 10^{-4}$.

Parameters and data of the SPIM are in the Table 1. [30]:

	Single-Phase		
H.P.	0.25	V_s	110 (V)
f	60 (Hz)	n_p	2
$n = \frac{N_A}{N_B}$	1.18	$R_{\alpha s}$	2.02 (Ω)
$R_{\beta s}$	5.13 (Ω)	R_r	4.12 (Ω)
$L_{\alpha s}$	0.1846(H)	$L_{\beta s}$	0.1833 (H)
L_r	0.1828 (H)	L_m	0.1772 (H)
J	0.0146 (Kgm^2)	k_d	0 (kgm^2/s)
$I_{\max}$	15 (A)	C_{run}	35 μf

TABLE I: Parameters of SPIM

The controller gains are adjusted to $k_1 = 500$, $k_2 = 750$, $k_{\sigma_1} = 30$, $k_{\sigma_2} = -10$, $k_{\delta_1} = 1$, $k_{\delta_2} = 0.0015$, $\alpha_1 = 36$, and $\alpha_3 = 1$. And, the gains for the observer are $l_{11} = 15000$, $l_{21} = 17000$, $l_{12} = 0.01$, $l_{22} = 0.01$, $l_5 = 50$ and, $l_6 = 50$.

For the simulation purposes, the initial conditions of the state variables are selected to zero. Tracking performance is verified for the two plant outputs: driving the square of rotor flux ϕ to a constant reference $\phi_{ref} = 0.15$, and a speed profile ω_{ref} for ω_r, proposed as follows:

1) The SPIM starts on repose with the reference speed on 100 rad/sec.
2) At the first second, a change of the speed reference from 100 rad/sec to 120 rad/sec, is presented.
3) Finally, at 3 seconds, a change of the speed reference from 120 rad/sec to 140 rad/sec, is presented.

In addition, the system is subject to disturbances which are introduced as follows:

1) The SPIM starts on repose with a load torque of 0.5 N-m, then at 1 sec. a change of load torque from 0.5 N-m to 0.8 N-m. After that at 3 sec. another change of load torque from 0.8 N-m to 1 N-m. And finally at 4 sec. one more change of load torque from 1 N-m to 0.5 N-m.
2) At 2 seconds, a 30% increase in the value of the rotor resistance is presented.
3) The SPIM starts whit a increase in the value of inductances at 15%.

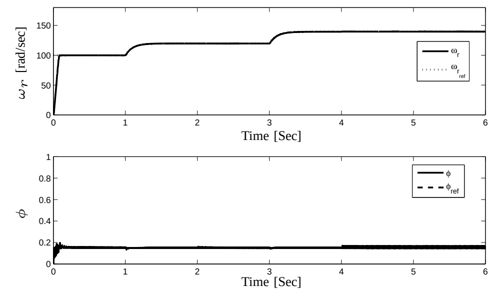

Fig. 2: Rotor speed ω_r and module to the square of rotor flux ϕ.

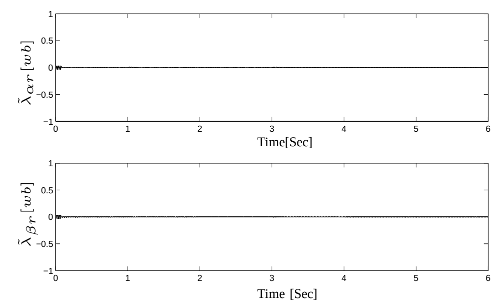

Fig. 3: Error of rotor flux in axis frame α β.

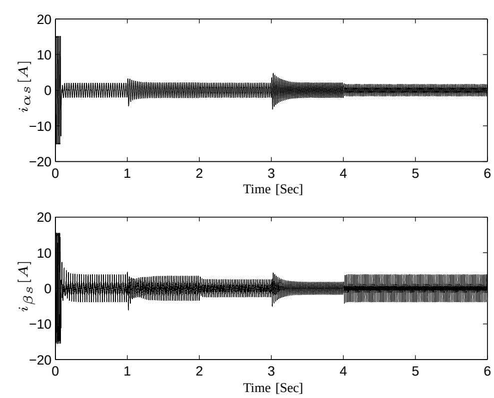

Fig. 4: Stator currents in axis frame α β.

The rotor speed tracking response is depicted in Fig. 2 which shows a satisfactory performance under the change of the speed reference at $t = 1,04$ sec. and $t = 3.04$ sec., where the speed tracking effect is achieved almost totally after 0.082 sec. Fig. 2 shows the module to the square of the rotor flux ϕ response too; it is possible to see that the module is maintained over the given reference. The errors responses of rotor fluxes are shown in Fig. 3.

On the other hand, the stator currents (see Fig. 4) are in the appropriate range during the start $(0 < t < 0.2)$ that corresponds to the proposed control algorithm. Finally, in Fig. 5, the responses of the voltages are presented, where $v_{\alpha s}$ is the *super-twisting* SM control and, $v_{\beta s}$ is the discontinuous SM control.

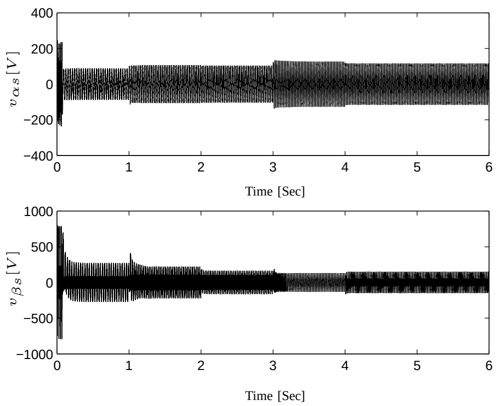

Fig. 5: Stator control voltages of axis α β.

VII. Conclusions

An observer-based control scheme was proposed to track the rotor angular speed ω_r and module to the square of rotor flux ϕ. It is based on SM algorithms allowing a robust design. The stability conditions of the closed-loop system was derived. The simulation results have shown a robust performance of the designed controller with respect to the perturbations caused by the load torque, fulfilling the stator current constraints.

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