

An Algebraic Observer for Leak Detection and Isolation in Plastic Pipelines

A. Navarro, Juan Diego Sánchez-Torres, O. Begovich, G. Besançon and Julián Alberto Patiño-Murillo

Abstract—In the continuation of authors' studies on leak diagnosis in pipelines, a new model-based Leak Detection and Isolation (LDI) algorithm is designed. This system only uses measures of flow and pressure coming from sensors placed at the ends of a pipeline. The present approach is based on a finite nonlinear pipeline model, and extended with variables related to the leak. On this basis, the purpose here is to investigate the use of a so-called algebraic observer to estimate the leak position and its magnitude. The corresponding observer design is thus presented, and its performances are illustrated both with simulation results, and experimental ones, with data taken from a real pipeline prototype.

I. INTRODUCTION

Several researchers have dealt with the issue of designing Leak Detection and Isolation (LDI) methods, and the main proposed approaches can be divided in *External Methods* [1] and *Model based algorithms*. For the case of model based methods several works have been developed [2]-[7]. In all these publications, a nonlinear asymptotic observer is the core of the LDI algorithm. The aim of the present paper is to investigate the use of an algebraic observer for the LDI problem, which has been introduced in the literature as a non-asymptotic observer [8]. This observer will be tested in simulation and real-time.

Algebraic approaches have been recently used for systems control and observation with emphasis on their characteristics of non-asymptotic convergence, robustness to uncertainties and capacity to deal with measured noise without any assumption on its statistical properties [9]. Some examples for controller design are given in [10], [11]. For the case of observers based on algebraic methods, the estimation is obtained by means of algebraic calculation of the output derivatives and, for observable systems, can be locally mapped to the state space [12], [13]. Those approaches have been related to the theory of linear observers in [14], but they still offer an alternative to classical implementations which can interestingly be inspected, in particular for nonlinear system [15], or in application for abrupt-change detection [16].

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In the present work, our purpose is to design an observer based on algebraic methods for direct application to the LDI problem. This means extending to a class of MIMO observable systems, a method previously presented for the SISO class [8], [17]. For the LDI application, we use the model derived in [7], where the states are: flows, pressure heads, the leak position and a parameter related with the leak intensity. Then, the resulting continuous-time nonlinear model is employed to design an algebraic state observer. To assess the performance of the designed LDI system, it is tested with synthetic data obtained from a simulator tuned with the parameters of the pipeline prototype described in [18]. After that, the method is applied to the same prototype but in real-time. It will be seen that, in both cases, the results are very satisfactory, specially in the real-time case where noisy signals are present.

In the following, Section II presents the considered model. Section III describes the proposed algebraic observer for MIMO systems in the LDI problem. The successful simulation and real-time results are shown in section IV. Finally the conclusions are given in Section V.

II. MODEL

This section presents the two Partial Differential Equations modelling the water dynamics in a pipeline. Also, the finite dimensional model to design the LDI system is described.

A. Modeling equation

Assuming the fluid to be slightly compressible and the duct walls slightly deformable; the convective changes in velocity to be negligible; the cross section area of the pipe and the fluid density to be constant, then the dynamics of the pipeline fluid can be described by the following partial differential equations [19]:

Momentum Equation

$$\frac{\partial Q(z, t)}{\partial t} + Ag \frac{\partial H(z, t)}{\partial z} + \mu Q(z, t) |Q(z, t)| = 0 \quad (1)$$

Continuity Equation

$$\frac{\partial H(z, t)}{\partial t} + \frac{b^2}{Ag} \frac{\partial Q(z, t)}{\partial z} = 0 \quad (2)$$

where Q is the flow rate [m/s^2], H the pressure head [m], z the length coordinate [m], t the time coordinate [s], g the gravity acceleration [m/s^2], A the cross-section area [m^2], b the speed of the pressure wave in the fluid [m/s], $\mu = \frac{\tau}{2DA}$, D the diameter [m] and τ the friction factor.

Leak model: on the other hand, one leak arbitrarily located at point z_1 (see Fig. 1) in a pipeline can be modeled as follows [19]:

$$Q_L = \lambda \sqrt{H_L} \quad (3)$$

where the constant λ is a function, among others of the orifice area and the discharge coefficient, Q_L is the flow through the leak and H_L is the head pressure at the leak point [19].

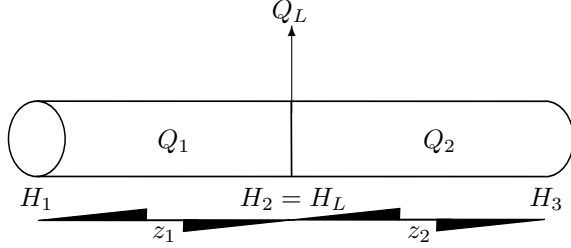


Fig. 1. Discretization of the pipeline with a leak Q_L

This leak produces a discontinuity in the system. Furthermore, due to the law of mass conservation, Q_L must satisfy the next relation:

$$Q_b = Q_a + Q_L \quad (4)$$

where Q_b and Q_a are the flows in a infinitesimal length before and after of the leak, respectively.

B. Spatial Discretization of the Modeling Equations

In order to obtain a finite dimensional model from (1) and (2), the partial differential equations are discretized with respect to the spatial variable z , as in [3], [5], by using the following relationships:

$$\frac{\partial H}{\partial z} \approx \frac{H_{j+1} - H_j}{z_j} \quad (5)$$

$$\frac{\partial Q}{\partial z} \approx \frac{Q_j - Q_{j-1}}{z_{j-1}} \quad (6)$$

Assuming only two partitions in the pipeline as shown in Fig. 1, z_j ($j = 1, 2$) becomes the distance from the beginning of the pipe to point of the leak and from the point of the leak to the end of the pipe, respectively. Notice that $z_2 = L - z_1$ where L is the total length of the pipeline and the leak position is assumed to be different from 0 and L in this description. Applying approximations (5) and (6) to equations (1) and (2) together with (3) and (4), and then incorporating as additional states z_1 and λ , we get:

$$\begin{bmatrix} \dot{Q}_1 \\ \dot{H}_2 \\ \dot{Q}_2 \\ \dot{z}_1 \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} -\frac{Aq}{z_1}(H_2 - u_1) - \mu Q_1 |Q_1| \\ -\frac{b^2}{Agz_1}(Q_2 - Q_1 + \lambda \sqrt{H_2}) \\ -\frac{Aq}{L-z_1}(u_2 - H_2) - \mu Q_2 |Q_2| \\ 0 \\ 0 \end{bmatrix} \quad (7)$$

Here, the input vector is $u = [H_1 \ H_3]^T = [u_1 \ u_2]^T$, and the output vector is $y = [Q_1 \ Q_2]^T$. Even if it is very simplified, this model can be useful for actual leak isolation, as this will be seen from the results hereafter.

III. ALGEBRAIC OBSERVER SCHEME

A. Method Description

The pipeline equations (7) correspond to a nonlinear MIMO system of the general form:

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \quad (8)$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the input, $y \in \mathbb{R}^p$ is the output and f, g, h are sufficiently differentiable function vectors.

For system (8), it is possible to define the following vector of output derivatives:

$$V(t) = \begin{pmatrix} y_1(t) \\ \dot{y}_1(t) \\ \vdots \\ y_1^{(k)}(t) \\ \vdots \\ y_p(t) \\ \dot{y}_p(t) \\ \vdots \\ y_p^{(k)}(t) \\ \vdots \end{pmatrix} \quad (9)$$

From model (8), $V(t)$ can be expressed as a function of $x, u, \dot{u}, \dots, u^{(k)}, \dots$ as:

$$V(t) = \Gamma(x, u, \dot{u}, \dots, u^{(k)}, \dots) \quad (10)$$

Observability somehow means that this relationship is invertible, and that one can find elements among the components of Γ defining an invertible map with respect to x [13]. Let us denote by $\bar{\Gamma}(x, u, t)$ this map, and consider the vector $\bar{V}(t)$ conformed by any n independent elements selected from $V(t)$. Then:

$$x = \bar{\Gamma}^{-1}(\bar{V}(t), u, t). \quad (11)$$

B. Estimation of Algebraic Derivatives

Let $\gamma(t)$ be an analytical function around $t = 0$ and defined for $t > 0$. In order to estimate its derivative, the truncated Taylor series expansion of $\gamma(t)$ around $t = 0$ is

$$\gamma(t) = \sum_{i=0}^N a_i \frac{t^i}{i!} + O(t^N) \quad (12)$$

where $a_i = \left. \frac{d^i \gamma(t)}{dt^i} \right|_{t=0}$. This implies that $\gamma(t)$ can be approximated by the polynomial $p_N(t) = \sum_{i=0}^N a_i \frac{t^i}{i!}$.

From (12), the i -th order time derivative calculation of $\gamma(t)$ can be seen as a problem of parameter estimation for $p_N(t)$. It is possible to calculate a_i , $i = 0, \dots, N$ independently, this reduces sensitivity to noise and numerical computation errors, which often appears in simultaneous

estimation methods. In addition, the independent calculation allows the use of higher order polynomials without the calculation of all its coefficients. The method will be described with an example, a complete explanation is given in [20].

Let $p_2(t)$ a second order polynomial approximation of $\gamma(t)$,

$$p_2(t) = a_0 + a_1 t + \frac{a_2}{2} t^2. \quad (13)$$

Transforming it into Laplace domain yields:

$$P_2(s) = \frac{a_0}{s} + \frac{a_1}{s^2} + \frac{a_2}{s^3}. \quad (14)$$

A detailed explanation for the calculation of a_1 is presented as follows:

- In order to annihilate a_2 , both sides of (14) are multiplied by s^3 and the derivative with respect to s is computed:

$$3s^2 P_2(s) + s^3 \frac{dP_2(s)}{ds} = 2sa_0 + a_1. \quad (15)$$

- Now, in order to annihilate a_0 , both sides of (15) are divided by s and the derivative with respect to s is calculated:

$$3P_2(s) + 5s \frac{dP_2(s)}{ds} + s^2 \frac{d^2 P_2(s)}{ds^2} = -\frac{a_1}{s^2}. \quad (16)$$

- By multiplying both sides by $s^{-\nu}$, $\nu \geq 3$, here $\nu = 3$,

$$\frac{3}{s^3} P_2(s) + \frac{5}{s^2} \frac{dP_2(s)}{ds} + \frac{1}{s} \frac{d^2 P_2(s)}{ds^2} = -\frac{a_1}{s^5}. \quad (17)$$

- Using the Cauchy rule for iterated integrals, the time domain expression for a_1 is

$$a_1 = -\frac{12}{T^4} \int_0^T (15t^2 - 16Tt + 3T^2) p_2(t) dt. \quad (18)$$

- Finally, $p_2(t)$ is replaced by $\gamma(t)$

$$a_1 = -\frac{12}{T^4} \int_0^T (15t^2 - 16Tt + 3T^2) \gamma(t) dt. \quad (19)$$

In a very similar way, the value of a_2 is given by

$$a_2 = \frac{60}{T^5} \int_0^T (6t^2 - 6Tt + T^2) \gamma(t) dt. \quad (20)$$

A filtered approximation for $\gamma(t)$ is given by a_0 in the form

$$a_0 = \frac{3}{T^3} \int_0^T (10t^2 - 12Tt + 3T^2) \gamma(t) dt. \quad (21)$$

For the integrals, a moving window of length T is used. A quite short time window is sufficient to obtain accurate estimations. In addition, the iterated integrals work as low pass filters which smooth highly fluctuating noises. Therefore, ones does not need any knowledge on the statistical properties of the noise.

C. Observer Design for the LDI System

Let $x \doteq [Q_1 \ H_2 \ Q_2 \ z_1 \ \lambda]^T = [x_1 \ x_2 \ x_3 \ x_4 \ x_5]^T$ by considering unidirectional flow (i.e. $x_1 > 0$ and $x_3 > 0$), the equation (7) can be written in compact form (8) with $f(x)$, $g(x)$ and $h(x)$ as:

$$f(x) = \begin{bmatrix} -\frac{Ag}{x_4} x_2 - \mu x_1^2 \\ -\frac{b^2}{Agx_4} (x_3 - x_1 + x_5 \sqrt{x_2}) \\ \frac{Ag}{L-x_4} x_2 - \mu x_3^2 \\ 0 \\ 0 \end{bmatrix}$$

$$g(x) = \begin{bmatrix} \frac{Ag}{x_4} & 0 \\ 0 & 0 \\ 0 & -\frac{Ag}{L-x_4} \\ 0 & 0 \end{bmatrix}$$

$$h(x) = \begin{bmatrix} h_1(x) \\ h_2(x) \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix}$$

Here it is easy to check that an invertible map $\bar{\Gamma}$ can be formed with the output time derivative vector as: $\bar{\Gamma} = [y_1(t) \ \dot{y}_1(t) \ \ddot{y}_1(t) \ y_2(t) \ \dot{y}_2(t)]$. Using $f(x)$, $g(x)$ and $h(x)$ yields: $\bar{\Gamma} = [x_1, -\frac{Ag}{x_4}(x_2 - u_1) - \mu x_1^2, 2\mu^2 x_1^3 + \frac{b^2}{x_4^2}(x_3 - x_1 + x_5 \sqrt{x_2}) + \frac{Ag}{x_4}(\dot{u}_1 + 2\mu x_1 x_2 - 2\mu x_1 u_1), x_3, \frac{Ag}{L-x_4}(u_2 - x_2) - \mu x_3^2]^T = [\bar{\Gamma}_i]$ with $i = 1, \dots, 5$.

Therefore, the system state in terms of the output derivatives (11) is written as

$$x_1 = y_1$$

$$x_2 = \frac{\dot{y}_2 + \mu y_2^2}{Ag} \left[\frac{L(\dot{y}_1 + \mu y_1^2) - Ag(\dot{y}_2 + \mu y_2^2)}{(\dot{y}_1 + \mu y_1^2) - (\dot{y}_2 + \mu y_2^2)} \right] + u_2$$

$$x_3 = y_2 \quad (22)$$

$$x_4 = \frac{L(\dot{y}_2 + \mu y_2^2) - Ag(u_1 - u_2)}{(\dot{y}_2 + \mu y_2^2) - (\dot{y}_1 + \mu y_1^2)}$$

$$x_5 = \frac{x_4^2}{b^2 \sqrt{x_2}} \left(\dot{y}_1 + 2\mu y_1 \dot{y}_1 - \frac{Ag}{x_4} \dot{u}_1 \right) + \frac{1}{\sqrt{x_2}} (y_1 - y_2).$$

Now, the estate estimation problem can be reduced to the real time estimation $\hat{V}(t)$ of $\bar{V}(t)$ in (11) (note that the system output time derivative is unknown). Thereby, an estimation of the estate can be given by:

$$\hat{x} = \bar{\Gamma}^{-1} \left(\hat{V}(t), u, t \right) \quad (23)$$

Thus, from (23), the estimated values of the state variables in (22), \hat{x}_i , $i = 1, \dots, 5$, are obtained by replacing the output, the inputs and their derivatives by their algebraic estimations. The first derivatives of the outputs are estimated as in (19), $\hat{y}_i = -\frac{12}{T^4} \int_0^T (15t^2 - 16Tt + 3T^2) y_i(t) dt$, with $i = 1, 2$. Also, based on (19), the first derivative estimation of the input u_1 , \hat{u}_1 , is obtained. In a similar way, the second derivative estimation of the output y_1 , $\hat{\dot{y}}_1$, is calculated using (20). Finally, u_i and y_i leads to the estimates \hat{u}_i and \hat{y}_i as in (21) with $i = 1, 2$.

IV. SIMULATION AND REAL-TIME RESULTS

The present section shows simulation and experimental results in order to assess the algebraic observer performance. In both cases, the pipeline prototype located at the Research and Advanced Studies Center in Guadalajara, Mexico (CINVESTAV-Guadalajara) was considered. This pipeline is composed by plastic pipes and it includes two water-flow (FT) and two pressure-head (PT) sensors at the beginning and the end of the pipe, as illustrated in Fig. 2. The length of the pipeline between sensor is 68.4 m. The prototype also has a 5 hp centrifugal pump and three valves, which serve to emulate the effect of a leak, at 16.8, 33.3 and 49.8 meters respectively. More information about the pipeline prototype can be found in [18].

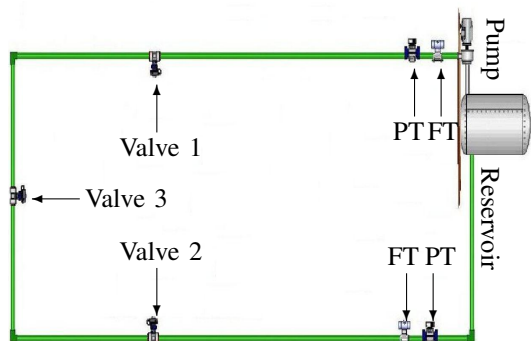


Fig. 2. Schematic diagram of the pipeline prototype.

In order to generate the synthetic data (and to deduce the observer), the mathematical model (7) was used. This model assumes a straight pipeline, and the prototype is not straight. Therefore, it is necessary to find an equivalent straight pipeline of this prototype. To do that, we follow [21], to find the lengths of the equivalent pipeline, which are named Equivalent Straight Lengths (ESL). These lengths will be used in the model to design the LDI.

Table I shows the main parameters of the pipeline including their length expressed in Equivalent Straight Coordinates.

TABLE I
PIPELINE PROTOTYPE PARAMETERS

Parameter	Symbol	Value	Units
ESL between sensors	L	100.15	[m]
ESL for the first valve	l_1	22.42	[m]
ESL for the second valve	l_2	48.03	[m]
ESL for the third valve	l_3	75.36	[m]
Internal diameter	D	6.54×10^{-2}	[m]
Pressure wave speed	b	375.88	[m/s]
Friction factor	τ	1.68×10^{-2}	[-]

The simulation as well as the real experiments were performed as follows: first, a leak located at $z_l = l_3$, the

third valve (i.e. $z_l = 75.36$), was induced at time $t = 500$ s. A simple leak alarm given by $|Q_{in} - Q_{out}| > \delta$ (Q_{in} represents the inflow measured whereas Q_{out} is the outflow measured and δ is a chosen threshold) was implemented in order to start the observer at the time of the leak occurrence, denoted as t_l . At this time, the leak isolation begins. The threshold is defined experimentally according to the noise in the system.

All initial conditions for the simulation, as well as the real-time implementation, were initialized equal to zeros, i.e. $\hat{y}_1(0) = \hat{y}_1(0) = 0$, $\hat{y}_1(0) = \hat{y}_1(0) = 0$, $\hat{y}_1(0) = \hat{y}_1(0) = 0$; $\hat{y}_2(0) = \hat{y}_2(0) = 0$, $\hat{y}_2(0) = \hat{y}_2(0) = 0$, $\hat{y}_2(0) = \hat{y}_2(0) = 0$ and $\hat{u}_1(0) = 0$. Here, $\hat{\bullet}(\cdot)$ denote the estimated variables with the synthetic data and $\hat{\bullet}(\cdot)$ the variables estimated with real data. Finally, the T value in (19), (20) and (21) was fixed equal to 5s.

Figure 3 shows the evolution of the pressure head at the beginning and at the end of the duct (the signal inputs u_1 and u_2). This figure depicts both: the synthetic data (\hat{H}_{in} and \hat{H}_{out} , dotted line) and the real data (H_{in} and H_{out} , continuous line). In the same manner, Fig. 4 shows the flow rate at the extremes of the pipe: real data in continuous line (Q_{in} and Q_{out}) and the synthetic data in dotted line (\hat{Q}_{in} and \hat{Q}_{out}). Fig. 5 presents the pressure head at the leak point: \hat{H}_2 represents the pressure estimated with synthetic data (dotted line) and \hat{H}_2 is the pressure head at the leak point estimated with real data (continuous line); the dashed line represents the “ideal” pressure given by the model (denoted by H_2). In Fig. 6, the dotted line represents the leak position in the ESL coordinates, estimated with synthetic data (\hat{z}_1), and the continuous line shows the leak position in the same coordinates, estimated with real data (\hat{z}_1). In the same figure, the dashed line shows the real leak position in the ESL coordinates (z_1). Fig. 7 depicts the lambda parameter evolution. This Figure shows the accurate estimation with synthetic (dotted line) and real (continuous line) data ($\hat{\lambda}$ and $\hat{\lambda}$, respectively) with its real magnitude represented by the dashed line (denoted by λ).

V. CONCLUSIONS

This work presents a methodology based on an algebraic observer for leak detection and isolation in a plastic pipeline using only pressure and flow-rate sensor at the ends of the duct. The method was tested with synthetic data and real data taken from a pipeline prototype. In the experimental results, it is possible to verify that the pressure head dynamics as well as the leak position and its magnitude were tracked in an accurate way by the simulated observer. This methodology could afford a good solution for analytic redundancy fault detection problem.

As future work, the algorithm as well as the leak alarm will be refined to achieve better performance. In particular the extension of the method to the location of multiple leaks will be investigated.

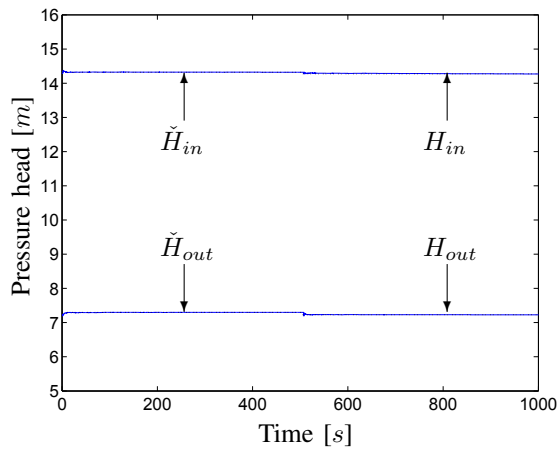


Fig. 3. Pressure head at inlet and outlet of the pipe (synthetic and real data).

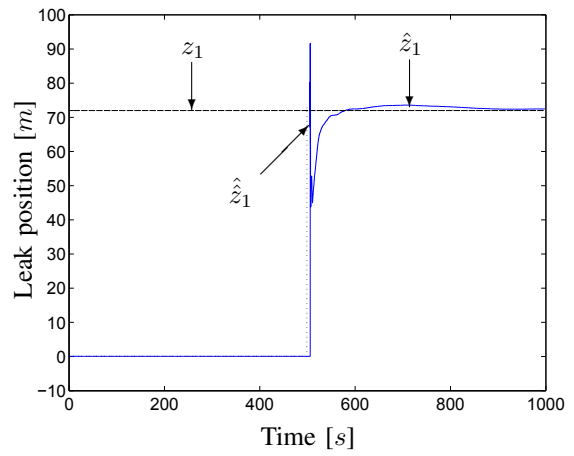


Fig. 6. Leak position in ESL coordinates (Estimation with synthetic data and estimation with real data).

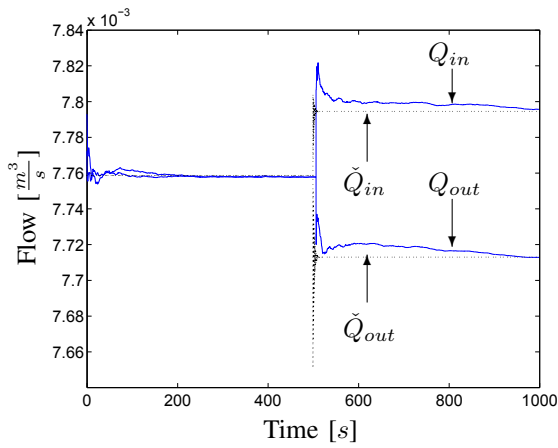


Fig. 4. Flow rate at inlet and outlet of the pipe (synthetic and real data).

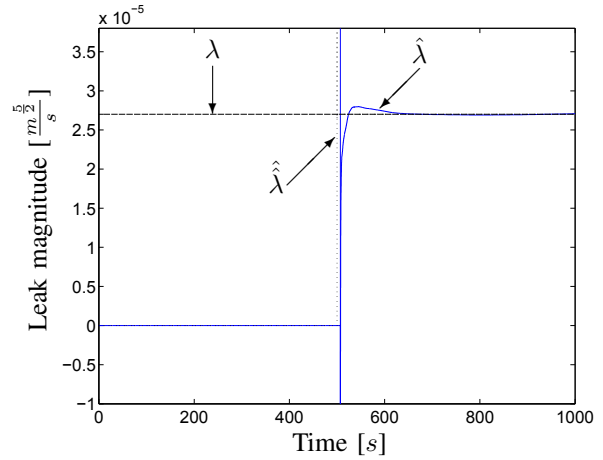


Fig. 7. Leak magnitude (Estimation with synthetic data and estimation with real data).

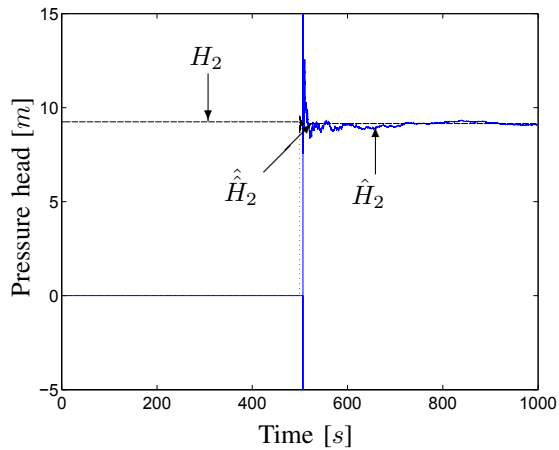


Fig. 5. Pressure head estimation at the leak point (Estimation with synthetic data and estimation with real data).

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