Simulation Study of the Unified Bayesian-Regularization Techniques for Enhanced Radar Imaging

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Abstract - In this paper, we intend to present the results of extended simulation study of the family of the radar image (RI) formation algorithms that employ the recently developed and investigated fused Bayesian-regularization (FBR) paradigm for high-resolution reconstruction of the spatial spectrum pattern (SSP) of the wavefield sources distributed in the remotely sensed environment. The FBR methodology is based on the aggregation of the Bayesian minimum risk statistical optimal estimation strategy with the descriptive weighted constrained least squares optimization technique that involves the non trivial a priori information on the desired properties of the SSP to be reconstructed from the actually measured data signals. The advantages of the well designed RI experiments (that employ the FBR-based methods) over the cases of poorer designed experiments (that employ the matched spatial filtering as well as the constrained least squares estimators) are investigated trough the simulation study.

Keywords: Radar/SAR Imaging, Method Fusion, Numerical Simulations.

I. INTRODUCTION

In this paper, we intend to present the results of extended simulation study of the family of the radar image (RI) formation algorithms that employ the recently developed and investigated fused Bayesian-regularization (FBR) paradigm for high-resolution reconstruction of the spatial spectrum pattern (SSP) of the wavefield sources distributed in the remotely sensed environment.

The FBR methodology is based on the aggregation of the Bayesian minimum risk statistical optimal estimation strategy [1], [2], [7] with the descriptive weighted constrained least squares optimization technique [1] that involves the non trivial a priori information on the desired properties of the SSP to be reconstructed from the actually measured data signals. Those may employ the specific metrics properties of the image space, boundary value conditions, calibration constraints, bench marks on the image scene [1], [3], [8], etc. In the applications related to passive and active radar remote sensing (RS), the unified FBR method was developed in our previous studies [4] – [12].

In this paper, we are going to present the results of extended simulation studies of the family of the FBR-based SSP estimation algorithms tested in the framework of the RI formation/reconstruction experiment. The use of MATLAB as simulation tools provided the computational efficiency and flexibility in performing all simulation experiments.

The family of the FBR-based SSP estimation (RI reconstruction) techniques that we investigate in these study trough computer simulations comprises the following basic estimators:

1. The simplest matched spatial filtering (MSF) algorithm for RI formation.
2. The descriptive constrained least squares (CLS) RI reconstruction algorithm.
3. The modified descriptive weighted constrained least squares (WCLS) algorithm.
4. The adaptive spatial filtering (ASF) algorithm.
5. The general FBR estimator for the SSP and its robustified version (RFBR).
6. The aggregated FBR-MVDR algorithm for reconstructive RS imagery.

The aim of the simulation experiment was to investigate the performances of these above listed six FBR-based SSP estimators.

II. SUMMARY OF THE FBR-BASED SSP ESTIMATORS

The family of the SSP estimation (reconstruction) algorithms that employ the FBR technique derived in [1], [4], [9] comprises the following estimators.

1. *General FBR estimator* of the SSP (in the vector form) is defined as follows,

$$\hat{\mathbf{B}}_{FBR} = \{\mathbf{FYF}^{+}\}_{\text{diag}} = \mathbf{K}_{\mathbf{A},\alpha}\mathbf{S}^{+}\mathbf{R}_{\mathbf{N}}^{-1}\,\mathbf{Y}\,\mathbf{R}_{\mathbf{N}}^{-1}\,\mathbf{SK}_{\mathbf{A},\alpha}\}_{\text{diag}} = \{\mathbf{K}_{\mathbf{A},\alpha}\operatorname*{aver}_{j\in J}\{\mathbf{Q}_{(j)}\mathbf{Q}^{+}_{(j)}\}\mathbf{K}_{\mathbf{A},\alpha}\}_{\text{diag}}\,, \qquad (1)$$

where

$$\mathbf{F} = \mathbf{K}_{\mathbf{A},\alpha}\mathbf{S}^{+}\mathbf{R}_{\mathbf{N}}^{-1}$$

is referred to as the image formation operator (IFO) in which

$$\mathbf{K}_{\mathbf{A},\alpha} = (\mathbf{S}^{+}\mathbf{R}_{\mathbf{N}}^{-1}\mathbf{S} + \alpha\mathbf{A}^{-1})^{-1}$$

represents the so-called reconstructive operator where α is the regularization parameter and $\mathbf{A}$ is the weight matrix. Parameter α and matrix $\mathbf{A}$ comprise the regularization degrees of freedom of the general FBR estimator (1). In (1),

$$\hat{\mathbf{B}}_{FBR} = \{\,\hat{\mathbf{D}}\,\}_{\text{diag}}$$

defines the estimate of the K-D SSP vector $\mathbf{B}=\{<\mathbf{EE}^{+}>\}_{\text{diag}}$ and

$$\mathbf{Y} = \operatorname*{aver}_{j\in J}\{\mathbf{U}_{(j)}\mathbf{U}^{+}_{(j)}\} = \hat{\mathbf{R}}_{\mathbf{U}}$$

is the estimate of the M-by-M data correlation matrix. Here, $\mathbf{U}_{(j)}$ represents the jth realization of the M-D complex measurement data vector

$$\mathbf{U} = \mathbf{SE} + \mathbf{N}$$

where $\mathbf{E}$ is the original K-D vector of the discrete-form approximation of the random complex object scattering function (SF), K-by-M matrix $\mathbf{S}$ is referred to as the linear signal formation operator (SFO) and $\mathbf{N}$ is the observation noise. Also, in (1),

$$\mathbf{Q}_{(j)} = \{\mathbf{S}^{+}\mathbf{R}_{\mathbf{N}}^{-1}\mathbf{U}_{(j)}\}$$

defines an output of the matched spatial filtering (MSF) algorithm with noise whitening that assumes the given noise correlation matrix $\mathbf{R}_{\mathbf{N}}$, (in this study we accept the robust white noise model, i.e. $\mathbf{R}_{\mathbf{N}}^{-1} = (1/N_0)\mathbf{I}$, with the noise intensity N_0 pre-estimated by some means [2]). The robustified version (*RFBR*) of the general FBR estimator (1) is constructed as an iterative scheme for solving (1) with optimally adjusted $\mathbf{A} = \hat{\mathbf{D}}^{-1}$ [4]. While performing the iterations, $\mathbf{A}^{(i)}$ at the current iteration $i = 0, 1, \ldots$ is approximated by the estimate $\hat{\mathbf{D}}^{-1}$ obtained at the previous iteration of (1) with the initial guess $\hat{\mathbf{D}}_0 = B_0\mathbf{I}$, i.e. approximated by the average gray level B_0 in all image pixels [6], [10].

2. *Descriptive CLS estimator* is constructed as modification of (1) for the following re-adjustments: $\mathbf{A} = \mathbf{I}$ and $\alpha = N_0/B_0$, i.e. the inverse of the signal-to-noise ratio (SNR), where B_0 is the prior average gray level of the SSP. In that case, the IFO $\mathbf{F}$ is recognized to be the Tikhonov's CLS spatial filter

$$\mathbf{F}_{CLS} = (\mathbf{S}^{+}\mathbf{S} + \alpha\,\mathbf{I}\,)^{-1}\mathbf{S}^{+}. \qquad (2)$$

3. *Descriptive WCLS estimator* constructed as a *modified* version of (2) for the following re-adjustments of the degrees of freedom: $\mathbf{A}=\mathbf{M}_{\mathbf{B}}$; $\alpha=N_0/B_0$,

$$\mathbf{F}_{WCLS} = (\mathbf{S}^{+}\mathbf{S} + \alpha\,\mathbf{M}_{\mathbf{B}})^{-1}\mathbf{S}^{+} \qquad (3)$$

where $\mathbf{M}_{\mathbf{B}}$ represents the Tikhonov's stabilizer of the second order constructed numerically in [4].

4. The simplest *matched spatial filtering* (*MSF*) SSP estimation algorithm is given by the simplified version of (2) for an assumption, $\alpha >> ||\mathbf{S}^{+}\mathbf{S}||$, which yields

$$\mathbf{F}_{MSF} \approx const \cdot \mathbf{S}^{+}, \qquad (4)$$

hence, the rough image is formed applying the adjoint SFO $\mathbf{S}^+$.

5. The *adaptive spatial filtering* (*ASF*) algorithm is constructed as modification of (1) for the case of an arbitrary zero-mean noise with the correlation matrix $\mathbf{R_N}$, the equal importance of the systematic and noise error measures [1], i.e. α = 1, and the solution dependent weight matrix $\mathbf{A} = \hat{\mathbf{D}}^{-1}$. In this case, the IFO is recognized to be the adaptive spatial filter

$$\mathbf{F}_{ASF} = (\mathbf{S}^+ \mathbf{R}_\mathbf{N}^{-1} \mathbf{S} + \hat{\mathbf{D}}^{-1})^{-1} \mathbf{S}^+ \mathbf{R}_\mathbf{N}^{-1} . \qquad (5)$$

6. Aggregated *FBR-MVDR* estimator constructed as $\hat{\mathbf{B}}_{FBR-MVDR} = \{\mathbf{F}_{FBR\text{-}MVDR}\mathbf{Y}\mathbf{F}^+_{FBR\text{-}MVDR}\}_{\text{diag}}$ with the IFO given by [4]

$$\mathbf{F}_{FBR\text{-}MVDR} = (\mathbf{S}^+\mathbf{S} + N_0\, \hat{\mathbf{D}}^{-1})^{-1} \mathbf{S}^+ . \qquad (6)$$

Such $\mathbf{F}_{FBR\text{-}MVDR}$ is recognized to be the IFO that minimizes the Bayesian risk [4], [11] of estimates $\hat{\mathbf{B}}$. It is obvious that the MVDR, ASF and RFBR estimators may be considered as particular cases of the uniform FBR image formation algorithm (1) under the model assumptions specified above. Hence, by controlling the regularization degrees of freedom, **A**, α, one can proceed from the general FBR estimator (1) to the variety of different image formation algorithms, from the simplest matched spatial filtering to the adaptive beamforming techniques.

III. SIMULATIONS

In the simulations, we investigated the performances of the family of all the FBR-based methods summarized above in their applications to reconstructive RS imagery. We simulated conventional side-looking imaging radar (i.e. the radar array was constructed by the moving antenna as in [6]) with the SFO factored along two axes in the imaging plain. In the range direction (over the vertical axis), the radar ambiguity function was approximated by a triangular shape pulse [3] of three pixels width at a half-maximum level and in the azimuth direction (over the horizontal axis) the ambiguity function was approximated by a Gaussian bell of 8 pixels width [6], [12]. Figure 1 presents the initial image of the reported here scene formed applying the *MSF* method, i.e. $\hat{\mathbf{B}}_{MSF}$, contaminated with 8% additive white noise. All other images correspond to different reconstructive FBR-based methods as specified in the Figures.

IV. DISCUSSIONS AND CONCLUSION

In this study, we examined the behavior and performances of a family of the recently developed FBR-based SSP estimators in application to the reconstructive RS imagery. The advantages of the well designed RI experiments (that employ the FBR, ASF and FBR-MVDR algorithms) over the cases of poorer designed experiments (that employ the MSF, CLS and WCLS methods) were investigated and reported here for one test scene borrowed from the RS imagery. These results qualitatively demonstrate that with some proper adjustment of the degrees of freedom of the robustified FBR-based techniques (i.e. the RFBR, ASF and FBR-MVDR), one could approach the quality of the statistically optimal general FBR method avoiding the cumbersome adaptive computations. The resolution is substantially improved when each of three techniques (RFBR, ASF and FBR-MVDR) was applied to enhance the RS images, i.e. regions of interest are much better defined, and ringing effects are within the acceptable tolerance level. The iterative RFBR and the FBR-MVDR techniques somewhat overperform the ASF algorithm but require almost 10 times more computations than the ASF algorithm. The optimization of the adjustments of the regularization degrees of freedom could further enhance the performances of all three reported methods and reduce the computational load. Such optimization is a matter of the further studies.

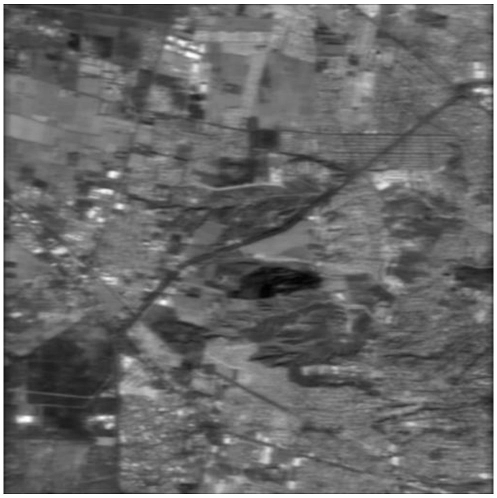
Fig. 1. Rough Image formed applying the matched spatial filtering (MSF) technique.


Fig. 2. Enhanced radar image formed applying the descriptive constrained least squares (CLS) algorithm.


Fig. 3. Enhanced radar image formed applying the modified descriptive weighted constrained least squares (WCLS) algorithm.

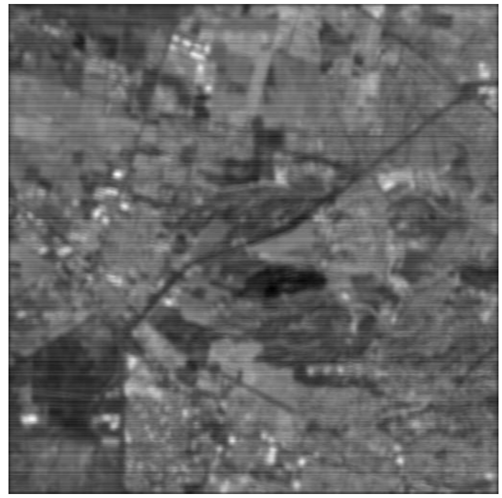
Fig. 4. Enhanced radar image formed applying the adaptive spatial filtering (ASF) algorithm.


Fig. 5. Enhances radar image formed applying the robust FBR estimator.


Fig. 6. Enhances radar image formed using the aggregated FBR-MVDR algorithm.

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