UNIFIED BAYESIAN-EXPERIMENT DESIGN REGULARIZATION TECHNIQUE FOR HIGH-RESOLUTION RECONSTRUCTION OF THE REMOTE SENSING IMAGERY

Yuriy V. Shkvarko, *Senior Member, IEEE*, and **Ivan E. Villalon-Turrubiates**, *Member, IEEE*

CINVESTAV del IPN, Unidad Guadalajara
Prol. Av. López Mateos Sur 590, Apartado Postal 31-438, C.P. 45090, Guadalajara Jal., MEXICO
Tel: (+5233) 31345570+2041, Fax: (+5233) 31345579, E-mail: shkvarko@gdl.cinvestav.mx

ABSTRACT

In this paper, the problem of estimating from a finite set of measurements of the radar remotely sensed complex data signals, the power spatial spectrum pattern (SSP) of the wavefield sources distributed in the environment is cast in the framework of Bayesian minimum risk (MR) paradigm unified with the experiment design (ED) regularization technique. The fused MR-ED regularization of the ill-posed nonlinear inverse problem of the SSP reconstruction is performed via incorporating into the MR estimation strategy the projection-regularization ED constraints. The simulation examples are incorporated to illustrate the efficiency of the proposed unified MR-ED technique.

Keywords: Signal processing, image reconstruction, regularization, neural networks.

1. INTRODUCTION

In conventional applications related to remote sensing imagery [1]-[4], the power image formation problems are stated and treated as problems of estimating the SSP of the backscattered wavefield sources from a finite set of the available time-space measurements of the complex observation data fields. Such reconstructed SSPs are referred to as desired power images of the remotely sensed environmental scenes.

Two nonparametric approaches to the solution of such a class of problems are usually addressed as classical. The first one is based on the pre-estimation of the data field correlation function (CF) from a set of independent realizations of the data field and solution of an inverse problem of restoration of the SSP from the CF estimates via inverting the Van-Zittert-Zernike formula [2] - [4]. The second one is the celebrated kernel spectral estimation or smoothed periodogram method [2], [4], [6] traditionally applied to the one-dimensional spatial uniformly sampled data with the Fourier transform signal formation operator (SFO). These both classical nonparametric approaches do not employ the statistically optimal Bayesian estimation theory-based treatment of the problem. Moreover, in various problems related to the SSP estimation, the signals are contaminated with colored noise, the data recording method is not restricted to a uniform sampling, and modulated signal waveforms are used that specify the corresponding models of the SFO.

The key distinguishing feature of a new paradigm considered in the present study is as follows: the inverse problem of estimating the SSP of the random backscattered wavefield from the available measurements of a finite number of independent realizations of the data field is stated and treated in the framework of Bayesian minimum risk (MR) strategy aggregated with the robust experiment design (ED) descriptive regularization technique. The fused MR-ED regularization of the ill-posed nonlinear inverse problem of the SSP reconstruction is performed via incorporating into the MR estimation strategy the model-level and system-level ED considerations, e.g. metrics structures imposed in the corresponding observation and solution spaces and system-level constraints specified by an employed data recording method. To reduce the computational load of the MR-ED-optimal estimator, the robust numerical implementation scheme is proposed; due to incorporating the ED considerations the proposed robustification is radically distinct from the previously reported developments of the fused Bayesian-regularization approaches undertaken in the recent studies [8], [9].

2. MR-ED METHOD

2.1. ED projection formalism for data representation

Viewing it as an approximation problem [2], [8] leads one to a projection concept for a reduction of the data wavefield $u(\mathbf{y})$ observed in a given space-time domain

$Y \ni \mathbf{y}$ to the M-D vector $\mathbf{U}$ of sampled spatial-temporal data recordings. The M-D observations in the terms of projections [2], [9] can be expressed as

$$u_{(M)}(\mathbf{y}) = (P_{\mathrm{U}(M)}u)(\mathbf{y}) = \Sigma U_m \phi_m(\mathbf{y}) \quad (1)$$

with coefficients $U_m = [u, h_m]_{\mathrm{U}}$; $m = 1, \ldots, M$, where $P_{\mathrm{U}(M)}$ denotes the projector onto the M-D observation subspace $\mathrm{U}_{(M)}$ that is uniquely defined by a set of the (spatial-temporal) basis functions $\{\phi_m(\mathbf{y})\}$ that span $\mathrm{U}_{(M)}$. Recall that functions $\{\phi_m(\mathbf{y})\}$ and $\{h_m(\mathbf{y})\}$ must compose the dual bases in $\mathrm{U}_{(M)}$, i.e. they must be mutually orthonormal, $[\phi_n, h_m] = \delta_{nm}$, where δ_{nm} is the Kroneker operator. Physically, the complex conjugate set $\{h_m^*(\mathbf{y})\}$ is specified by a composition of the antenna element tapering functions $\{\tau_l(\mathbf{p}); l = 1, \ldots, L\}$ (that we admit to be either identical or different for the different elements of the L-D array), and the pulse response functions $\{\chi_i(t)$; $i = 1, \ldots, I\}$ of the I sampling filters in the corresponding spatial receiving channels (as well identical or different) ordered by multi-index $m = (l, i) = 1, \ldots, M = L \times I$. In practice [2], [7], the antenna elements are distanced in space (do not overlap), i.e. the tapering functions $\{\tau_l(\mathbf{p})\}$ have the distanced supports in $P \ni \mathbf{p}$, thus they compose a set of orthogonal functions. The same assumption of orthogonality is usually valid for the sampling filters $\{\chi_i(t)\}$, $t \in T$, in which case the dual basis $\{\phi_m(\mathbf{y})\}$ is simply the properly normalized set of $\{h_m(\mathbf{y})\}$, i.e. $\{\phi_m(\mathbf{y}) = ||h_m(\mathbf{y})||^{-2}h_m(\mathbf{y}); m = 1, \ldots, M\}$.

Note that in the operator formalism, the projector, $P_{\mathrm{U}(M)}$, in (1) can be expressed as a linear integral operator with the functional kernel, $P_{\mathrm{U}(M)}(\mathbf{y}, \mathbf{y}') = \Sigma \phi_m(\mathbf{y})\, h_m^*(\mathbf{y}')$.

In analogy to (1), one can define the projection scheme for the K-D approximation of the scene scattering function over a given spatial image domain $X \ni \mathbf{x}$ as follows,

$$e_{(K)}(\mathbf{x}) = (P_{\mathrm{E}(K)}\, e)(\mathbf{x}) = \Sigma E_k \varphi_k(\mathbf{x}); \quad (2)$$

$E_k = [e, g_k]_{\mathrm{E}}$; $k = 1, \ldots, K$, where $P_{\mathrm{E}(K)}$ defines a projector onto the K-D image subspace $\mathrm{E}_{(K)}$ spanned by K basis functions $\{\varphi_k(\mathbf{x})\}$. The $\{\varphi_k(\mathbf{x})\}$ and $\{g_k(\mathbf{x})\}$ compose the dual bases in $\mathrm{E}_{(K)}$, and the linear integral projector operator is specified by its kernel $P_{\mathrm{E}(K)}(\mathbf{x}, \mathbf{x}') = \Sigma \varphi_k(\mathbf{x})\, g_k^*(\mathbf{x}')$.

2.2. Problem model

General model of the observation wavefield u is defined by specifying the stochastic equation of observation of an operator form [8]: $u = Se + n$; $e \in \mathrm{E}$; $u, n \in \mathrm{U}$; S: $\mathrm{E} \rightarrow \mathrm{U}$, in the Gilbert signal spaces E and U with the metric structures induced by the inner products, $[u_1, u_2]_{\mathrm{U}} = \int_Y u_1(\mathbf{y})u_2^*(\mathbf{y})d\mathbf{y}$, and $[e_1, e_2]_{\mathrm{E}} = \int_X e_1(\mathbf{x})e_2^*(\mathbf{x})d\mathbf{x}$, respectively. The operator model of the stochastic equation of observation (EO) in the conventional integral form [2], [4] may be rewritten as

$$u(\mathbf{y}) = (Se(\mathbf{x}))(\mathbf{y}) = \int_X S(\mathbf{y}, \mathbf{x})\, e(\mathbf{x})d\mathbf{x} + n(\mathbf{y}). \quad (3)$$

Using the presented above ED formalism, one can proceed from the operator-form EO (3) to its conventional vector form,

$$\mathbf{U} = \mathbf{SE} + \mathbf{N}\,, \quad (4)$$

in which $\mathbf{E}$, $\mathbf{N}_{\Sigma}$ and $\mathbf{U}$ are the zero-mean vectors composed of the coefficients E_k , N_m , and U_m. These are characterized by the correlation matrices $\mathbf{R_E} = \mathbf{D} = \mathbf{D}(\mathbf{B}) = \mathrm{diag}(\mathbf{B})$ (a diagonal matrix with vector $\mathbf{B}$ at its main diagonal), $\mathbf{R_N}$, and $\mathbf{R_U} = \mathbf{S}_0\mathbf{R}_{\mathrm{E}}\mathbf{S}_0^+ + \mathbf{R_N}$, respectively. (Recall that superscript $^+$ defines the Hermitian conjugate when stands with matrix or vector). The vector, $\mathbf{B}$, is composed of the elements $B_k = \langle E_k E_k^* \rangle$; $k = 1, \ldots, K$, and is referred to as a K-D vector-form approximation of the SSP.

We refer to the estimate $\hat{\mathbf{B}}$ as the discrete-form representation of the brightness image of the wavefield sources distributed in the environment remotely sensed with the array radar (SAR), in which case the continuous-form finite dimensional approximation of the estimate of the SSP distribution $\hat{B}_{(K)}(\mathbf{x})$ in the environment in a given spatial image domain $X \ni \mathbf{x}$ can be expressed as follows,

$$\hat{B}_{(K)}(\mathbf{x}) = \Sigma B_k |\varphi_k(\mathbf{x})|^2 = \boldsymbol{\varphi}^{\mathrm{T}}(\mathbf{x})\mathrm{diag}(\hat{\mathbf{B}})\boldsymbol{\varphi}(\mathbf{x})\,, \quad (5)$$

where $\boldsymbol{\varphi}(\mathbf{x})$ represents a K-D vector composed of the basis functions $\{\varphi_k(\mathbf{x})\}$.

2.3. Experiment design considerations

In the traditional remote sensing approach to image formation [3], the matched filter $S^+P_{\mathrm{U}(M)}u_{(M)}(\mathbf{y}) = \hat{e}_{(K)}$ is first applied to the data $u_{(M)}(\mathbf{y})$ to form the estimate $\hat{e}_{(K)}(\mathbf{x})$ of the complex scattering function $e_{(K)}(\mathbf{x})$ and the resulting image is formed as the averaged (over the $J \ni j$ independent data recordings) squared modulus of such the estimates, i.e. $\hat{B}_{(K)}(\mathbf{x}) = \mathrm{aver}\{|\hat{e}_{(K)}^{(j)}(\mathbf{x})|^2\}$. In that case, the degenerate (rank M) SFO $S_{(M)} = P_{\mathrm{U}(M)}S$ uniquely specifies the system ambiguity function (AF), i.e. the instrumental function of the imaging system that is defined as a kernel of the integral operator, $\Psi_{(M)} = S^+P_{\mathrm{U}(M)}[P_{\mathrm{U}(M)}R_nP_{\mathrm{U}(M)}]^{-1}P_{\mathrm{U}(M)}S$, where R_n is the correlation operator of the noise field in (1) and S^+ is the adjoint to S operator [2]. Therefore, it is reasonable for practical applications to treat the problem taking into account the ED-based principles [2], [8] on how to design the system instrumental function that has the "best shape" (e.g. the narrow main beam with the lowest possible level of sidelobes of the AF). Next, to satisfy the observability requirements [5], for any chosen $P_{U(M)}$ one should design the image subspace $E_{(K)}$ = span$\{\varphi_k(\mathbf{x})\}$ of dimension $K \leq M$ that is orthogonal to the null-space [10] of the degenerate SFO $S_{(M)} = P_{U(M)}S$. Hence, all conventional imaging techniques that ignore these ED-motivated requirements including the celebrated minimum variance distortionless response (MVDR) method should be considered as not properly regularized in the ED sense. In all such cases, some form of regularization of the image formation algorithms should be accomplished [2], [9].

2.4. MR-ED strategy

In the descriptive statistical formalism, the desired SSP vector $\hat{\mathbf{B}}$ is recognized to be the vector of a principal diagonal of anstimate of the correlation matrix $\mathbf{R_E}(\mathbf{B})$, i.e. $\hat{\mathbf{B}} = \{\hat{\mathbf{R}}_\mathbf{E}\}_{\text{diag}}$. Thus one can seek to estimate $\hat{\mathbf{B}} = \{\hat{\mathbf{R}}_\mathbf{E}\}_{\text{diag}}$ given the data correlation matrix $\mathbf{R_U}$ pre-estimated by some means [4,

$$\hat{\mathbf{R}}_\mathbf{U} = \mathbf{Y} = \underset{j\in J}{\text{aver}}\,\{\mathbf{U}_{(j)}\mathbf{U}^+_{(j)}\}, \qquad (7)$$

by determining the solution operator $\mathbf{F}$ such that

$$\hat{\mathbf{B}} = \{\hat{\mathbf{R}}_\mathbf{E}\}_{\text{diag}} = \{\mathbf{FYF}^+\}_{\text{diag}}\,. \qquad (8)$$

To optimize the search of $\mathbf{F}$ we propose here the following MR-ED *descriptive regularization* strategy

$$\mathbf{F} \to \min_{\mathbf{F}}\{\,\mathcal{R}(\mathbf{F})\}, \qquad (9)$$

$$\mathcal{R}(\mathbf{F}) = \text{trace}\{(\mathbf{FS}-\mathbf{I})\mathbf{A}(\mathbf{FS}-\mathbf{I})^+\} + \alpha\,\text{trace}\{\mathbf{FR_N F}^+\}$$

that implies the minimization of aweighted sum of the systematic and fluctuation errors in the desired estimate $\hat{\mathbf{B}}$, where the selection (adjustment) of the regularization parameter α and the weight matrix $\mathbf{A}$ provides the additional ED rees of freedom incorporating any descriptive properties of a solution if those are known a priori [5, [8. It is easy to recognize that strategy (9) is structurally similar to the statistical MR linear estimation strategy [2], [6] because in the both cases the balance between the gained spatial resolution and the noise energy in the resulting estimate is to be optimized.

2.5. General form of solution operator

Routinely solving the minimization problem (9) we obtain

$$\mathbf{F} = \mathbf{K}_{\mathbf{A},\alpha}\mathbf{S}^+\mathbf{R}_\mathbf{N}^{-1}\,, \qquad (10)$$

where $$\mathbf{K}_{\mathbf{A},\alpha} = (\mathbf{S}^+\mathbf{R}_\mathbf{N}^{-1}\mathbf{S} + \alpha\mathbf{A}^{-1})^{-1} \qquad (11)$$

and the desired SSP estimate is given by

$$\hat{\mathbf{B}}_{MR-ED} = \{\mathbf{K}_{\mathbf{A},\alpha}\mathbf{S}^+\mathbf{R}_\mathbf{N}^{-1}\mathbf{Y}\,\mathbf{R}_\mathbf{N}^{-1}\mathbf{S}\mathbf{K}_{\mathbf{A},\alpha}\}_{\text{diag}}$$
$$= \{\mathbf{K}_{\mathbf{A},\alpha}\,\underset{j\in J}{\text{aver}}\,\{\mathbf{Q}_{(j)}\mathbf{Q}^+_{(j)}\}\mathbf{K}_{\mathbf{A},\alpha}\}_{\text{diag}}\,, \qquad (12)$$

where $\mathbf{Q}_{(j)} = \{\mathbf{S}^+\mathbf{R}_\mathbf{N}^{-1}\mathbf{U}_{(j)}\}$ is recognized to be an output of the matched spatial processing algorithm with noise whitening [6]. In the case of white noise, $\mathbf{R}_\mathbf{N}^{-1} = (1/N_0)\mathbf{I}$.

For such solution operator, the objective function $\mathcal{R}(\mathbf{F})$ attains its minimal possible value, $\mathcal{R}_{\min}(\mathbf{F}) = \text{tr}\{\mathbf{K}_{\mathbf{A},\alpha}\}$.

2.6. MR-ED-robustified algorithms

2.6.1. Robust spatial filtering (RSF)

Putting $\mathbf{A} = \mathbf{I}$ and $\alpha = N_0/B_0$, where B_0 is the prior average gray level of the SSP, the $\mathbf{F}$ can be reduced to the following Tikhonov-type robust spatial filter

$$\mathbf{F}_{RSF} = \mathbf{F}^{(1)} = (\mathbf{S}^+\mathbf{S} + (N_0/B_0)\mathbf{I}\,)^{-1}\mathbf{S}^+. \qquad (13)$$

2.6.2. Matched spatial filtering (MSF)

In the previous scenario for $\alpha >> ||\mathbf{S}^+\mathbf{S}||$, the $\mathbf{F}$ becomes

$$\mathbf{F}_{MSF} = \mathbf{F}^{(2)} \approx const \cdot \mathbf{S}^+ \qquad (14)$$

i.e. reduces to the conventional MSF operator.

5.6.3. Adaptive spatial filtering (ASF)

Consider now the case of an arbitrary zero-mean noise with correlation matrix $\mathbf{R_N}$, equal importance of two error measures in (9), i.e. $\alpha = 1$, and the solution dependent weight matrix $\mathbf{A} = \hat{\mathbf{D}} = \text{diag}(\hat{\mathbf{B}})$. In this case, the MR-ED solution operator defines the adaptive spatial filter

$$\mathbf{F}_{ASF} = \mathbf{F}^{(3)} = \mathbf{H} = (\mathbf{S}^+\mathbf{R}_\mathbf{N}^{-1}\mathbf{S} + \hat{\mathbf{D}}^{-1})^{-1}\mathbf{S}^+\mathbf{R}_\mathbf{N}^{-1}\,. \qquad (15)$$

3. SIMULATIONS AND CONCLUDING DISCUSSIONS

We simulated conventional side-looking imaging radar (i.e. the array was synthesized by moving antenna) with the SFO factored along two axes in the image plane: the azimuth (horizontal axis) and the range (vertical axis). We considered a triangular shape of the imaging radar range ambiguity function of 5 pixels width, and a $\sin(x)/x$ shape of the side-looking radar antenna radiation pattern of 15 pixels width at 0.5 from the peak level. Simulation results are presented in Figures 1 – 4. The figure notes specify each particular employed imaging method. All scenes are presented in the same 512-by-512 pixel image format. The advantage of reconstructive imaging using the MR-ED-optimal ASF estimator (Fig. 4) and its robustified suboptimal RSF version (Fig. 3) over the case of conventional MSF technique (Fig. 2) is evident. The spatial resolution is substantially improved with both (RSF and ASF) techniques; the regions of interest and distributed scene boundaries are much better defined.

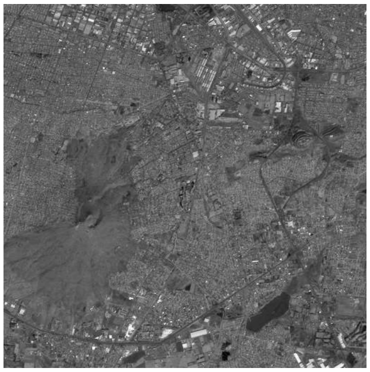

Fig. 1. Original scene (not observable in the radar imaging experiment)

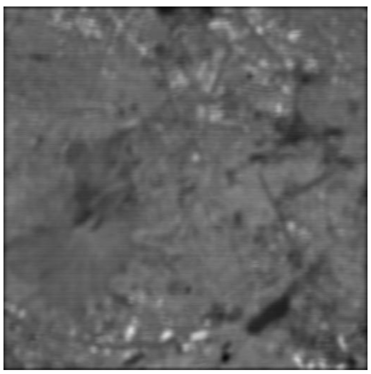

Fig. 2. Rough radar image formed using conventional MSF technique

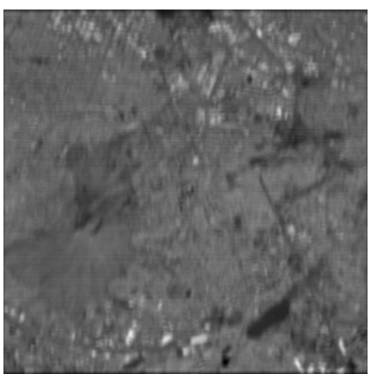

Fig. 3. Enhanced scene image formed applying the RSF method

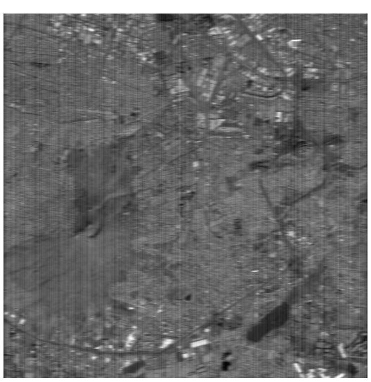

Fig. 4. Scene image reconstructed applying the ASF method

The presented study revealed also the way for deriving the suboptimal RSF technique with substantially decreased computational load. Being a structural simplification of the optimal ASF estimator, the RSF technique permits efficient non-adaptive numerical implementation in both iterative and concise direct computational forms. The proposed robust and adaptive nonlinear estimators contain also some design parameters viewed as the system-level degrees of freedom, which with an adequate selection can improve the performance of the corresponding techniques. The proposed methodology could be considered as an alternative approach to the existing ones that employ the descriptive regularization paradigm [1] - [4] as well as the MR method for SAR image enhancement recently developed in [8], [9]. The provided simulation examples illustrate the overall performance improvements attainable with the proposed methods. The simulations were performed over the typical environmental scene borrowed from the real-world remote sensing imagery.

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