Intelligent Processing for SAR Imagery for Environmental Management

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***Abstract* – A new intelligent computational paradigm based on the use of Kalman filtering technique [4] modified to reconstruct the dynamic behavior of the physical and electrical characteristics provided via reconstructive SAR imagery. As a matter of particular study, we develop and report the Kalman filter-based algorithm for high-resolution intelligent filtration of the dynamic behavior of the hydrological indexes of the particular tested remotely sensed scenes. The simulation results verify the efficiency of the proposed approach as required for decision support in environmental resources management.**

Keywords: Environmental Remote Sensing, Resource Management, Decision Support.

I. INTRODUCTION

Modern applied theory of reconstructive signal and image processing for environmental monitoring and resource management [8] is now a mature and well developed research field, presented and detailed in many works ([1], [2], [3] are only some indicative examples). Although the existing theory offers a manifold of statistical and descriptive regularization techniques to tackle with the particular environmental monitoring problems, in many application areas there still remain some unresolved crucial theoretical and data processing problems related particularly to the extraction and enhancement of environmental characteristics for decision support in environmental management and end-user computing aspects that incorporate the high-precision filtering techniques for evaluation and prediction the dynamic behavior of the particular extracted environmental processes.

In this study, we undertake an attempt to develop and verify via computational simulations a new intelligent filtering method that provides the possibility to track, filter and predict the dynamical behavior of a physical characteristics extracted from the remotely sensed scenes provided with the real-world high-resolution SAR data as it is required for decision support in environmental resources management. The proposed methodology aggregates the Kalman filtering technique [4] with the high-resolution algorithms for enhanced SAR imagery [1], [5]. In the simulations, we tested the data provided with the spaceborne SAR with fractionally synthesized array [1], [2].

II. MATHEMATICAL MODEL OF THE LINEAR DYNAMIC PROBLEM

Consider the following model of the **Equation of Observation** (EO) in continuous time [6]

$$u(t) = S(\lambda(t)) + n(t) \tag{1}$$

where $n(t)$ is the White Gaussian Noise and $t \in T$, starting at t_0 (initial instant of time). Regarding the signal process, the following linear amplitude-modulated model $S(\lambda(t))$ is considered,

$$S(\lambda(t)) = \lambda(t) S_0(t) \tag{2}$$

where $S_0(t)$ is the deterministic "carrier" signal of a given model, and $\lambda(t)$ is the unknown stochastic information process to be estimated via processing (filtration) of the observation data signal $u(t)$. Regarding $\lambda(t)$, it is considered that it satisfies some dynamical model specified by the following linear differential equation

$$\frac{d^N \lambda(t)}{dt^N} + \alpha_{N-1} \frac{d^{N-1}\lambda(t)}{dt^{N-1}} + \ldots + \alpha_0 \lambda(t) = \beta_{N-1} \frac{d^{N-1} x(t)}{dt^{N-1}} + \ldots + \beta_0 x(t)\,. \tag{3}$$

The stochastic model can be redefined as follows: the differential equation (3) may be transformed into a system of *Linear Differential Equations of order 1* via performing replacement of variables [6], and may be represented in a canonical vector-matrix form

$$\frac{d\mathbf{z}(t)}{dt} = \mathbf{F}\mathbf{z}(t) + \mathbf{G}\mathbf{x}(t) \quad , \qquad \boldsymbol{\lambda}(t) = \mathbf{C}\mathbf{z}(t)\ . \tag{4}$$

where

$$\mathbf{F}=\begin{bmatrix} -\alpha_{N-1} & 1 & 0 & \dots & 0 \\ -\alpha_{N-2} & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ -\alpha_1 & 0 & 0 & \dots & 1 \\ -\alpha_0 & 0 & 0 & \dots & 0 \end{bmatrix}, \qquad \mathbf{G}=\begin{bmatrix} \beta_{N-1} \\ \beta_{N-2} \\ \dots \\ \beta_0 \end{bmatrix}, \qquad \mathbf{C}=\begin{bmatrix} 1 & 0 & \dots & 0 \end{bmatrix}.$$

Considering that $\xi(t)=x(t)$ is white noise, the statistics are $\langle \xi(t) \rangle = 0$ and $\langle \xi(t)\xi^*(t') \rangle = P_\xi(t)\sigma(t-t')$ [6], where $P_\xi(t)$ is the **Disperse Function** that represents the dynamics of the process variance developed in a continuous time. Accepting the model of the information process and output of a Linear Dynamic formation system defined above, the Equation of Observation can be defined as follows

$$\mathbf{u}(t)=\mathbf{S_0}(t)\mathbf{C}(t)\mathbf{z}(t)+\mathbf{n}(t)=\mathbf{H}(t)\mathbf{z}(t)+\mathbf{n}(t) \tag{5}$$

where $\mathbf{H}(t)=\mathbf{S_0}(t)\mathbf{C}(t)$. This model allows formal generalization of an arbitrary m-channel observation $\mathbf{u}(t)$. The aim of the Linear Dynamic Filtration is to find an optimal estimate of the information process $\boldsymbol{\lambda}(t)$ in current time t ($t_0 \to t$) via processing the information data vector $\mathbf{z}(t)$ taking in account the a-priori dynamic model of $\boldsymbol{\lambda}(t)$. In other words, one have to design the optimal dynamic filter that when applied to the observation vector $\mathbf{u}(t)$ provides the optimal estimation of the desired process that satisfies the a-priori dynamic model specified by the stochastic dynamic state equation [6]

$$\hat{\boldsymbol{\lambda}}(t)=\mathbf{C}(t)\hat{\mathbf{z}}(t). \tag{6}$$

The **Canonical Discrete Form of a LDS** represented in *state variables* is [6]

$$\begin{cases} \mathbf{z}(k+1)=\boldsymbol{\Phi}(k)\mathbf{z}(k)+\boldsymbol{\Gamma}(k)\mathbf{x}(k), \\ \boldsymbol{\lambda}(k)=\mathbf{C}(k)\mathbf{z}(k) \end{cases} \tag{7}$$

where $\boldsymbol{\Phi}(k)=\mathbf{F}(t_k)\Delta t+\mathbf{I}$ and $\boldsymbol{\Gamma}(k)=\mathbf{G}(t_k)\Delta t$. In this case, the Eq. (5) in discrete time becomes

$$\mathbf{u}(k)=\mathbf{H}(k)\mathbf{z}(k)+\mathbf{n}(k). \tag{8}$$

The statistical characteristics of the a-priori information in discrete time are [6]

- Model Noise (initializing or generating model) $\{\boldsymbol{\xi}(k)\}$: $\langle \boldsymbol{\xi}(k) \rangle = \mathbf{0}$; $\langle \boldsymbol{\xi}(k)\boldsymbol{\xi}^*(j) \rangle = \mathbf{P}_\xi(k,j)$.
- Observation Noise $\{\mathbf{n}(k)\}$: $\langle \mathbf{n}(k) \rangle = \mathbf{0}$; $\langle \mathbf{n}(k)\mathbf{n}^*(j) \rangle = \mathbf{P_n}(k,j)$.
- Random State Vector $\{\mathbf{z}(k)\}$: $\langle \mathbf{z}(0) \rangle = \mathbf{m}_z(0)$; $\langle \mathbf{z}(0)\mathbf{z}^*(0) \rangle = \mathbf{P_z}(0)$.

The **Disperse Matrix** $\mathbf{P}_Z(0)$ (initial state) satisfies the following *Disperse Dynamic* equation

$$\mathbf{P_z}(k+1)=\langle \mathbf{z}(k+1)\mathbf{z}^*(k+1) \rangle = \boldsymbol{\Phi}(k)\mathbf{P_z}(k)\boldsymbol{\Phi}^+(k)+\boldsymbol{\Gamma}(k)\mathbf{P}_\xi(k)\boldsymbol{\Gamma}^+(k). \tag{9}$$

III. STRATEGY OF THE OPTIMAL DYNAMIC KALMAN FILTER

The Kalman filter is an estimator used to estimate the state of a Linear Dynamic System (LDS) perturbed by white Gaussian noise using measurements that are linear functions of the system state corrupted by additive white Gaussian noise. The mathematical model used in the derivation of the Kalman filter is a reasonable representation for many problems of practical interest, including control problems as well as estimation problems. The Kalman filter model is also used for the analysis of measurements and estimation problems [4]. The optimal strategy is to design an optimal decision procedure (optimal filter) that, when applied to all registered observations, provides an optimal solution to the state vector $\mathbf{z}(k)$ subjected to it's a-priori defined dynamic model given by the **Statistic Dynamic Equation** (SDE). The Optimal Estimate is defined as optimal in the sense of the Bayesian Minimum Risk Strategy (BMR) [6]

$$\hat{\mathbf{z}}_{opt}(k)=\langle \mathbf{z}(k) | u(0),u(1),\dots,u(k) \rangle. \tag{10}$$

In discrete time, the design procedure is based on the concept of mathematical induction, that is, suppose that after k observations $\{u(0),u(1),...,u(k)\}$, one had produced the desired optimal estimate defined for the ultimate step

$$\hat{\mathbf{z}}(k) = \underset{opt}{\hat{\mathbf{z}}(k)}. \tag{11}$$

The problem is as follows: using this estimate $\underset{opt}{\hat{\mathbf{z}}(k)}$ is necessary to design the algorithm for producing the optimal estimate $\mathbf{z}(k+1)$ incorporating new measurements $u(k+1)$ according to the **State Dynamic Equation** (SDE), this estimate must satisfy the dynamic equation

$$\mathbf{z}(k+1) = \mathbf{\Phi}(k)\mathbf{z}(k) + \mathbf{\Gamma}(k)\xi(k). \tag{12}$$

According to the dynamicl model, the anticipated mean value becomes

$$\mathbf{m}_z(k+1) = \langle \mathbf{z}(k+1) \rangle = \langle \mathbf{z}(k+1) | \hat{\mathbf{z}}(k) \rangle. \tag{13}$$

Thus, $\mathbf{m}_\mathbf{z}(k+1)$ must be considered as a-priori conditional mean-value of the stat vector for the next (k+1) estimation step, according to the $\mathbf{z}(k+1)$ model

$$\mathbf{m}_z(k+1) = \mathbf{\Phi}\langle \mathbf{z}(k) | u(0),u(1),...,u(k) \rangle + \mathbf{\Gamma}\langle \xi(k) \rangle = \mathbf{\Phi}\hat{\mathbf{z}}(k). \tag{14}$$

That is why the prognosis of the mean-value of the next step becomes $\mathbf{m}_z(k+1) = \mathbf{\Phi}\hat{\mathbf{z}}(k)$. Now it is possible to reduce the estimate strategy to the one-step optimization procedure:

$$\begin{aligned} \hat{\mathbf{z}}(k+1) &= \langle \mathbf{z}(k+1) | u(0),u(1),...,u(k),u(k+1) \rangle = \langle \mathbf{z}(k+1) | \hat{\mathbf{z}}(k); u(k+1) \rangle \ ; \\ \hat{\mathbf{z}}(k+1) &= \langle \mathbf{z}(k+1) | u(k+1); \mathbf{m}_z(k+1) \rangle \ . \end{aligned} \tag{15}$$

For the ultimate (k+1) step of measurements, the **Equation of Observation** becomes [6]

$$u(k+1) = \mathbf{H}(k+1)\mathbf{z}(k+1) + n(k+1) \tag{16}$$

with the summarized a-priori information given by Eq. (14). Applying the Bayesian-Wiener time [6],

$$\hat{\mathbf{z}}(k+1) = \mathbf{m}_z(k+1) + \mathbf{W}(k+1)\left[u(k+1) - \mathbf{H}(k+1)\mathbf{m}_z(k+1)\right] \tag{17}$$

where the Dynamic Filter Operator is specified as follows,

$$\begin{gathered} \mathbf{W}(k+1) = \mathbf{K}(k+1)\mathbf{H}^+(k+1)\mathbf{P}_N^{-1}(k+1)\,; \\ \mathbf{K}(k+1) = \left[\mathbf{\Psi}(k+1) + \mathbf{P}_z^{-1}(k+1)\right]^{-1}; \\ \mathbf{\Psi}(k+1) = \mathbf{H}^+(k+1)\mathbf{P}_N^{-1}(k+1)\mathbf{H}(k+1)\,; \\ \mathbf{P}_N^{-1}(k+1) = \frac{1}{\mathbf{P}_N(k+1)}\,. \end{gathered} \tag{18}$$

The Figure 1 shows the Optimal Procedure of the discrete Kalman filter technique in a flow diagram form. The Optimal Procedure is defined by the Stochastic Dynamic State Equation [6]

$$\hat{\mathbf{z}}(k+1) = \mathbf{\Phi}(k)\hat{\mathbf{z}}(k) + \mathbf{W}(k+1)\left[\mathbf{u}(k+1) - \mathbf{H}(k+1)\mathbf{\Phi}(k)\hat{\mathbf{z}}(k)\right] \tag{19}$$

The model of the problem is applied considering that $\mathbf{H}$(k) is the Signal Formation Operator (SFO) that corresponds to the SAR imaging system [1]. The particular SFO was modeled by the *sinc*-type spectral ambiguity function [9]. The $\mathbf{z}$(k) is the observation data vector from the image, $\mathbf{u}$(k) is the observation data vector contaminated by additive Gaussian noise, and $\boldsymbol{\lambda}$(k) is the dynamically filtered information process.

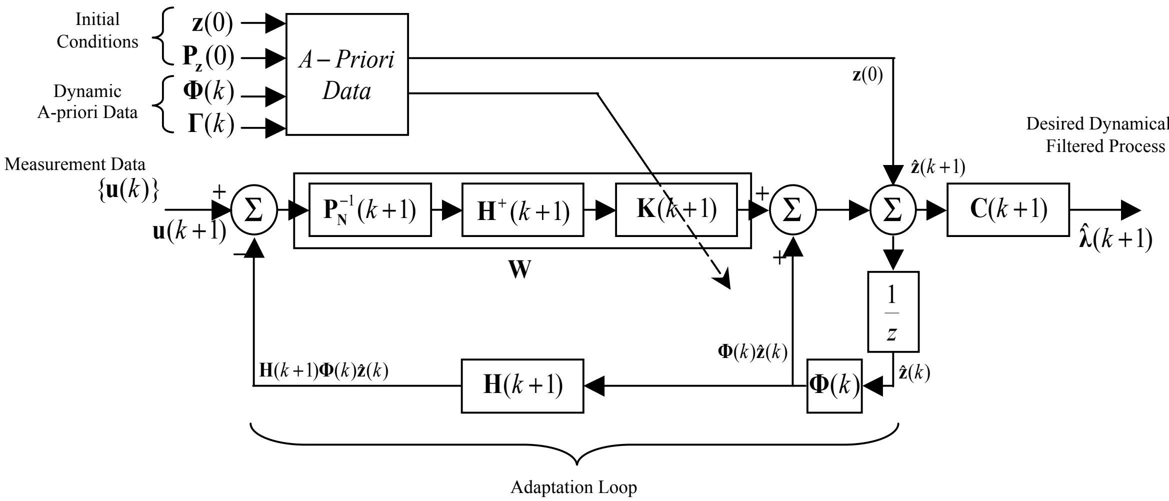

Fig. 1: Implementation Signal Flow Diagram

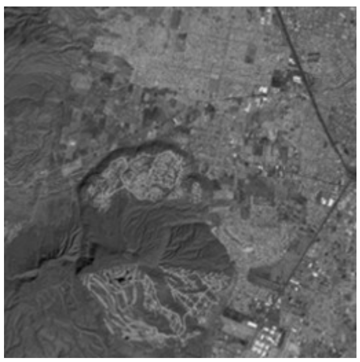

Fig. 2. Tested SAR image

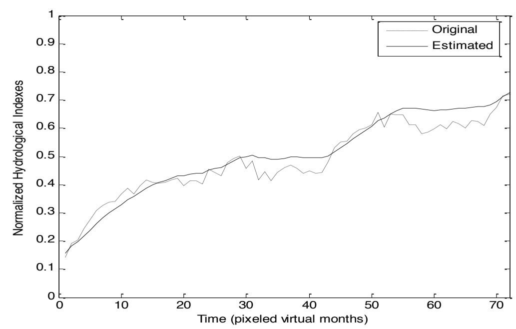

Fig. 3. Dynamics of hydrological indexes (in the normalized virtual time)

The data dynamics was approximated by the following model

$$\begin{aligned} \mathbf{z}(0) &= \mathbf{0}; \\ \mathbf{P}_{\mathbf{z}}(0) &= \mathbf{N}_{\mathbf{0}}\mathbf{I}; \\ \mathbf{\Phi}(k) &= \mathbf{F}(t_k)\Delta t + \mathbf{I}; \\ \mathbf{\Gamma}(k) &= \mathbf{G}(t_k)\Delta t. \end{aligned} \tag{20}$$

IV. SIMULATIONS AND CONCLUDING REMARKS

In the simulations, we considered the SAR with partially/fractionally synthesized array [1], [2] as a prime remote sensing imaging system. Figure 2 shows the 2-*D* 256-by-256 pixel format original scene image provided by the carrier SAR sensor system in 2005. This data was borrowed from the real-world remotely sensed SAR imagery of the tested scene of the Guadalajara region (Forest of Primavera) in Mexico. To study the dynamics of the particular hydrological indexes [3] of these scenes that were considered as the particular physical characteristics of interest, the experimental data covered the period of expertise from the year 2000 up to the year 2005, respectively. Figure 3 shows the results obtained with the application of the Kalman technique algorithm summarized in the previous section for enhanced filtering of the dynamics of the hydrological indexes [3] of the tested scenes, studied in the normalized virtual time [7] related to the physical time of the dynamics of the characteristics under our particular study. In the reported simulations we applied the a priori dynamic scene information modeled by Eq. (19).

This study intends to establish the foundation to assist in understanding the basic theoretical aspects of how to aggregate the enhanced SAR imaging techniques with Kalman filtering for high-precision intelligent filtration of the dynamical behavior of the physical characteristics of the remotely monitored scenes for decision support in environmental resources management. In our particular study, the dynamics of the hydrological indexes of the SAR maps of the particular tested terrestrial zones (Guadalajara region) were processed. The reported results can be also expanded to other fields related to the study of the dynamical behavior of different physical characteristics provided by remote sensing systems of other particular applications. The reported results of simulation study are indicative of a usefulness of the proposed approach for monitoring the physical environmental characteristics, and those could provide a valuable support in different environmental resource management applications.

V. REFERENCES

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