A Simple Recurrent Neural Network for Solution of Linear Programming: Application to a Microgrid

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***Abstract*—The aim of this paper is to present a simple new class of recurrent neural networks, which solves linear programming. It is considered as a sliding mode control problem, where the network structure is based on the Karush-Kuhn-Tucker (KKT) optimality conditions, and the KKT multipliers are the control inputs to be implemented with finite time stabilizing terms based on the unit control, instead of common used activation functions. Thus, the main feature of the proposed network is the fixed number of parameters despite of the optimization problem dimension, which means, the network can be easily scaled from a small to a higher dimension problem. The applicability of the proposed scheme is tested on real-time optimization of an electrical Microgrid prototype.**

I. INTRODUCTION

Optimization methods have been widely applied in science and engineering. The optimization goal is to determine the decision variables values, which maximize or minimize an objective function, sometimes, subject to constraints. As a quite important application for those schemes, the successful operation of a Microgrid requires the solution of different optimization problems in order to obtain the best performance of the system. The Microgrids are active distribution networks, which include different renewable and non-conventional energy sources and various loads [1]. The use of conventional power sources in the current electrical network operation has been related with problems as gradual depletion of fossil fuel resources, poor energy efficiency and environmental pollution, increasing the interest on this class of networks. For most of the cases, the sources integrated to the Microgrid are natural gas, biogas, wind power, solar photovoltaic cells, fuel cells, combined heat and power systems, micro-turbines and Stirling engines. Due to the network operation conditions, an important number of these optimization procedures are performed off-line. On the other hand, these grids present problems as the time varying load demand and the no-conventional/renewable sources availability, requiring to solve large-scale real-time optimization procedures, most of them in the form of linear programming. For such applications, sequential algorithms as the classical simplex or the interior point methods are often proposed. However, those traditional approaches may not be efficient since the computing time required for a solution is greatly dependent on the problem dimension and structure.

Introduced in [2], the use of dynamical systems which can solve real-time optimization is a promising alternative. Taking advantage of the features presented by some discontinuous systems, a major contribution to this class of solutions is the use of systems with sliding modes, that is an integral manifold with finite reaching time [3], as proposed on [4], providing finite time convergence to the problem solution. Further extensions of the mentioned schemes were presented for linear programming [5]–[7], for nonlinear programming [8], finite time approaches [9]–[11] and, fixed time stability [12]. Some of these systems were presented as the solution to a controller design problem [13] (including the case of sliding mode control [14]), in the form of circuits [15], [16] or under the computational paradigm of the so-called artificial neural networks (ANN) where are known as recurrent neural networks (RNN) [17]. The main features, due to its inherent massive parallelism, are that RNN can solve optimization problems in running time at the orders of magnitude much faster than those of the most popular optimization algorithms executed on general-purpose digital computers [18] and the unusual flexibility because the system constantly seeks new solutions as the parameters of the problem are varied [2]. Usually, the network structure is proposed based on the Karush-Kuhn-Tucker (KKT) optimality conditions [19], [20],by using the KKT multipliers as activation functions.

All the mentioned approaches present high performance. Although, it is necessary to tune the network parameters such that the optimizer trajectories converge to the optimization solution. For most of the cases, the number of networks parameter increases linearly with the optimization problem dimension, since for every decision variable there is an individual selection of each activation function.

This paper is aimed to expose a new simple RNN for the solution of linear programming. It is included a design method for the activation function that enforces a sliding mode in which the optimization problem is solved without the individual selection of each activation function. The network structure is based on the Karush-Kuhn-Tucker (KKT) optimality conditions and, instead of common used activation functions, a multivariable function is proposed, based on the unit control presented in [21], [22]. The proposed approach have very attractive features as: finite time convergence to the optimization problem solution and a fixed number of network parameter (four for this case), regardless of the optimization problem dimension. Offering the characteristic of scalability that allows the on-line solution of problems with low and higher dimension without major changes in the network. This RNN is applied to determine the optimal amounts of power supplied by each energy source in a Microgrid prototype. As mentioned above, in contrast to the publications which use recurrent neural networks for Microgrid optimization [23], [24], the proposed approach provides finite convergence time to the solution and the tuning of only four network parameters, independent to the Microgrid optimization statement.

In the following, Section II describes the proposed RNN for the solution of linear programs, including stability analysis and an academic example. Sections III and IV present the Microgrid laboratory prototype connection and control structure, respectively. Section V explains the problem of optimal power amount to be supplied by each energy source in a Microgrid prototype. Section VI shows the proposed RNN. The real-time results are presented in section VII. Finally, in Section VIII the conclusions are presented.

II. A RNN for Linear Programming Problem

A. Linear Programming Problem Statement

Let the following linear programming problem:

$$\begin{cases} \min_x & \mathbf{c}^T x \\ \text{s.t.} & \mathbf{A}x = \mathbf{b} \\ & l \leq x \leq h \end{cases} \tag{1}$$

where $x = \begin{bmatrix} x_1 & \dots & x_n \end{bmatrix}^T \in \mathbb{R}^n$ are the decision variables, $\mathbf{c} \in \mathbb{R}^n$ is a cost vector, $\mathbf{A}$ is an $m \times n$ matrix such that $\text{rank}(\mathbf{A}) = m$ and $m \leq n$; $\mathbf{b}$ is a vector in $\mathbb{R}^m$ and, $l = \begin{bmatrix} l_1 & \dots & l_n \end{bmatrix}$, $h = \begin{bmatrix} h_1 & \dots & h_n \end{bmatrix} \in \mathbb{R}^n$.

Let $y = \begin{bmatrix} y_1 & \dots & y_m \end{bmatrix}^T \in \mathbb{R}^m$ and $z = \begin{bmatrix} z_1 & \dots & z_n \end{bmatrix}^T \in \mathbb{R}^n$. Hence, the Lagrangian of (1) is

$$L(x, y, z) = \mathbf{c}^T x + z^T x + y^T (\mathbf{A}x - \mathbf{b}). \tag{2}$$

The KKT conditions establish that x^* is a solution for (1) if and only if x^*, y and z in (1)-(2) are such that

$$\nabla_x L(x^*, y, z) = \mathbf{c} + z + \mathbf{A}^T y = 0 \tag{3}$$

$$\mathbf{A}x^* - \mathbf{b} = 0 \tag{4}$$

$$z_i x_i^* = 0 \text{ if } l_i < x_i^* < h_i,\ \forall i = 1, \dots, n. \tag{5}$$

B. RNN Design with Finite Time Convergence

Following the KKT approach, a recurrent neural network which solves the problem (1) in finite time is proposed. For this purpose, let $\Omega_e = \{x \in \mathbb{R}^n : \mathbf{A}x - \mathbf{b} = 0\}$ and $\Omega_d = \{x \in \mathbb{R}^n : l \leq x \leq h\}$. According to (1), $x^* \in \Omega$ where $\Omega = \text{int}(\Omega_d \cap \Omega_e)$.

From (3), let

$$\dot{x} = -\mathbf{c} + \mathbf{A}^T y + z, \tag{6}$$

then, y and z must be designed such that Ω is an attractive set, fulfilling conditions (3)-(5). For this case, in addition to condition (5), z is considered such that

$$\begin{cases} z_i \geq 0 & \text{if } x_i \geq h_i \\ z_i \leq 0 & \text{if } x_i \leq l_i \end{cases}, \tag{7}$$

and the variable $\sigma \in \mathbb{R}^m$ is defined as $\sigma = \mathbf{A}x - \mathbf{b}$.

In order to obtain finite time stability to the solution x^*, the terms y and z are proposed in (6) as the multivariable activation functions $y = \phi(\sigma)$ and $z = \varphi(x, l, h)$ defined as

$$\phi(\sigma) = \begin{cases} -k_1 \frac{\sigma}{\|\sigma\|} - k_2 \sigma & \text{if } l \leq x \leq h \\ 0 & \text{if } x < l \text{ or } x > h \end{cases} \tag{8}$$

and $\varphi(x, l, h) = \begin{bmatrix} \varphi_1(x, l_1, h_1) & \dots & \varphi_n(x, l_n, h_n) \end{bmatrix}^T$, with $\varphi_i(x, l_i, h_i)$ of the form

$$\varphi_i(x, l_i, h_i) = \begin{cases} -k_3 \frac{x_i - l_i}{\|x - l\|} - k_4 (x_i - l_i) & \text{if } x_i \leq l_i \\ 0 & \text{if } l_i < x_i < h_i \\ -k_3 \frac{x_i - h_i}{\|x - h\|} - k_4 (x_i - h_i) & \text{if } x_i \geq h_i \end{cases}, \tag{9}$$

and $k_1, \dots, k_4$ positive scalars. Therefore, with the structure given by (6) and the activation functions (8) and (9), the proposed RNN has the form

$$\dot{x} = -\mathbf{c} + \mathbf{A}^T \phi(\sigma) + \varphi(x, l, h). \tag{10}$$

Note that, in contrast to the RNN presented in the literature, this scheme only needs the tuning of four variables in spite of the problem dimensions.

C. Stability Analysis

In order to analyze the stability of the presented network (10), the following Lyapunov candidate is proposed

$$V = \frac{1}{2} \left(\sigma^T (\mathbf{A}\mathbf{A}^T)^{-1} \sigma + x^T x \right) \tag{11}$$

where it is highlighted the existence of $(\mathbf{A}\mathbf{A}^T)^{-1}$ due to $\mathbf{A}$ is a full rank matrix. The derivative of the Lyapunov function (11) is given by

$$\dot{V} = -\sigma^T (\mathbf{A}\mathbf{A}^T)^{-1} \mathbf{A}\mathbf{c} + \sigma^T \phi(\sigma) + \sigma^T (\mathbf{A}\mathbf{A}^T)^{-1} \mathbf{A} \varphi(x, l, h) - x^T \mathbf{c} + x^T \mathbf{A}^T \phi(\sigma) + x^T \varphi(x, l, h).$$

From (8) and (9), $\dot{V}$ can be written as

$$\dot{V} = \begin{cases} -x^T \mathbf{c} + x^T \varphi(x, l, h) & \text{if } x < l \text{ or } x > h \\ -\sigma^T (\mathbf{A}\mathbf{A}^T)^{-1} \mathbf{A}\mathbf{c} + \sigma^T \phi(\sigma) & \text{if } l \leq x \leq h \end{cases} \tag{12}$$

thus

$$\dot{V} = \begin{cases} -x^T \mathbf{c} - k_3 \|x\| - k_4 \|x\|^2 & \text{if } x < l \text{ or } x > h \\ -\sigma^T (\mathbf{A}\mathbf{A}^T)^{-1} \mathbf{A}\mathbf{c} - k_1 \|\sigma\| - k_2 \|\sigma\|^2 & \text{if } l \leq x \leq h. \end{cases}$$

Hence, with $k_1 > \left\| (\mathbf{A}\mathbf{A}^T)^{-1} \mathbf{A}\mathbf{c} \right\|$, $k_2 > 0$, $k_3 > \|\mathbf{c}\|$ and $k_4 > 0$, the RNN (10) converges in finite time $t_f > 0$.

At this point, it is clear that the conditions (4) and (5) are satisfied. Now, by using the *equivalent control method* [22] as solution of $\dot{x} = 0$ in (6) for $t > t_f$, it follows that $z = \varphi(x, l, h) = 0$ and $\mathbf{c} + \mathbf{A}^T \{\phi(\sigma)\}_{eq} = 0$, satisfying the condition (3).

D. An Academic Example

Let the following linear programming problem [11]:

$$\begin{cases} \min_x & 4x_1 + x_2 + 2x_3 \\ \text{s.t} & x_1 - 2x_2 + x_3 = 2 \\ & -x_1 + 2x_2 + x_3 = 1 \\ & -5 \leq x_1, x_2, x_3 \leq 5 \end{cases}. \tag{13}$$

The proposed neural network (6), with the parameters $k_1 = 1$, $k_2 = 3$, $k_3 = 0.5$ and $k_4 = 1.$, gives the results shown in Fig. 1.

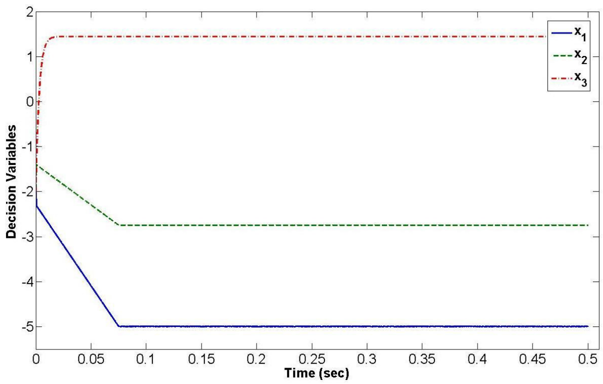

Fig. 1. Transient behavior of the x variables.

Here, it can be observed that the network converges to the optimal solution $x^* = [-5, -2.75, 1.5]$.

III. Electrical Microgrid Prototype

The Microgrid laboratory prototype contains a DC voltage bus as a common interjection point for a batteries bank, which stores surplus power, a photovoltaic cell bank and a load test bench. From the operational point of view, the micro-sources must be equipped with power electronic interfaces and control to provide the required flexibility for ensuring its operation as a single aggregated system, and to maintain the specified power quality and energy output. This flexibility would allow the Microgrid to present itself to the main utility power system as a single controlled unit, which meets local energy needs for reliability and security.

The Microgrid prototype connection scheme can be seen in Fig.2, and a properly picture is displayed in Fig.3. All these devices are developed by Lab-Volt[1].

IV. Prototype Control Structure

Divide and conquer are the basis of a multi-agent system. In this kind of system, there are special agents (SA) which verify the related inputs and outputs signals for an specific task, i.e. wind power energy production. A central module (CM) serves as a link point for communications between agents. SA has autonomy over easily to solve problems, i.e. short circuits. CM takes decision over important issues detected by SA, i.e. bus fail, or the interconnection of the Microgrid, i.e. changing to island mode, as [25] explains. Many Microgrids have applied MAS to reduce runtime for an interconnected system, whose agents are connected to it, as presented in [26].

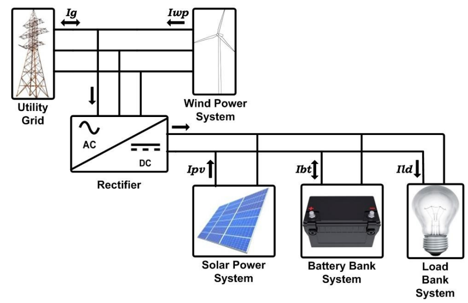

Fig. 2. Microgrid prototype connection scheme.

Fig. 3. Microgrid prototype.

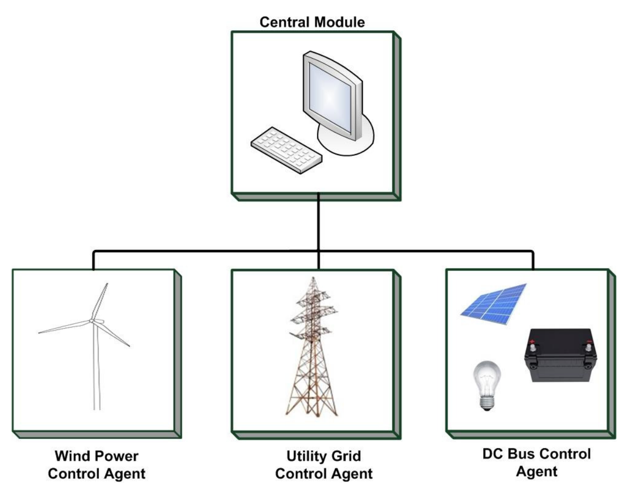

Fig. 4. Microgrid prototype control scheme.

The Microgrid laboratory prototype control structure is based under a MAS design (Fig.4); this kind of scheme allows us to add other energy sources in the future. By now, only two agents have been developed and the CM tasks are executed by a common PC using serial and USB communication (Fig.5).

[1]Lab-Volt,675, rue du Carbon G2N 2K7 Quebec, Quebec Canada.

Fig. 5. Microgrid agents.

A. Wind Power Control Agent

The Wind Power Control Agent (WPCA) extracts the maximum power available from the wind, as presented for isolated wind control unit in [27]. Using a data acquisition and control module (Fig.6), we can get different voltages, current and angular speed data from WPS.

Fig. 6. Data acquisition and control module.

For the generator speed control, the WPCA has an internal controller which allows speed tracking. By means of a PI controller, the generator speed reference ω_r^{ref} is forced to track a wind generator power reference P_w^{ref}. The controller is $\omega_r^{ref} = K_p e_P + K_i \int_t e_P d\tau$, where e_P is the error power tracking defined as $e_P = P_w^{ref} - P_w$, P_w is the wind generator power, K_p is proportional constant and K_i is the integral constant.

Fig. 7. DC bus control agent.

B. DC Bus Control Agent

In order to verify, at any moment, voltage magnitude and quality in the DC bus, a DC bus control agent (DCBCA) has been developed according to the Microgrid laboratory prototype requirements (Fig.7). In order to obtain an efficient energy distribution, the DCBCA monitors the voltages present in the utility grid and the output current values. This agent has the authority to disconnect any load in short circuit. A user interface is connected to the DC bus acquisition and control module in order to display the most important signal values for pertinent analysis. This module serves as a link between the different types of energy sources.

C. Utility Grid Control Agent

Utility Grid Control Agent (UGCA) has the task to measure power and voltage delivered by utility grid. These values are send to CM to ensure Microgrid functionality.

V. Optimization Statement

A. Wind Power System (WPS)

This system includes a three phase induction motor with a dynamometer to emulate the wind power impacting in the propellers. The wind generated power P_W is

$$P_W = \begin{cases} 0 & V_t \leq V_{W_{min}} \\ 0 & V_t \geq V_{W_{max}} \\ P_m & V_{W_{min}} \leq V_t \leq V_{W_{max}} \end{cases} \tag{14}$$

where $t = 1, 2, \ldots, T$, V_t is the generator speed at time t, $V_{W_{min}}$is the generator minimum allowed speed, $V_{W_{max}}$ is the generator maximum allowed speed and P_m is the calculated WPS power. For this case, $V_{W_{min}}$ is $1840 rpm$ because at this speed the motor begins to act as a generator and $V_{W_{max}}$ is $2000 rpm$ for safe dynamometer functionality. With this speed values, the wind generator $P_{W_{min}}$ is equal to 0 and $P_{W_{max}}$ is 240 Watts.

B. Solar Power System

Solar power systems (SPS) is implemented by means of a two cells photovoltaic work bench. SPS power contribution P_S is restricted as

$$0 \leq P_{S_t} \leq P_{S_{max}} \tag{15}$$

where P_{S_t} is the SPS power at time t and $P_{S_{max}}$ is the SPS maximum power. The SPS maximum power $P_{S_{max}}$ for the Microgrid laboratory is 1.2 Watts.

C. Battery Bank System

Microgrid power surplus is stored on a Battery Bank System (BBS), which includes two lead-acid batteries. Battery power P_B must satisfy the constraint

$$P_{B_{min}} \leq P_{B_t} \leq P_{B_{max}} \tag{16}$$

where P_{B_t} is the BBS power at time t, $P_{B_{min}}$ is the BBS minimum allowed power and $P_{B_{max}}$ is the BBS maximum allowed power. The BBS maximum and minimum power are fixed to increase the batteries lifespan as long as possible. For this purpose $P_{B_{min}}$ is established as a 10 percent of its charge and $P_{B_{max}}$ as 60%. Therefore if the batteries has a power rate of 2.5 watts, $P_{B_{min}}$ is taken as 0.25 watts and $P_{B_{max}}$ as 1.5 watts.

D. Utility Grid System

The Microgrid has a junction point with the utility grid system all the time as can be seen in Fig.2.

The power from this system P_G is formulated as:

$$0 \leq P_{G_t} \leq P_{G_{max}} \tag{17}$$

where P_{G_t} is the utility power at time t, $P_{G_{max}}$is the utility maximum allowed power. The utility power can be considered has an infinite source; however for this Microgrid $P_{G_{max}}$ bound is set to 250 watts.

The main goal is to optimize the power of the Microgrid based on the available energy sources and the required output power for the load P_L, which can be expressed in the form (1) as follows:

$$\begin{cases} \text{Minimize} & P_G - P_W - P_S - P_B \\ \text{s.t} & P_G + P_W + P_S + P_B = P_L \\ & 0 \leq P_{G_t} \leq P_{G_{max}} \\ & P_{W_{min}} \leq P_{W_t} \leq P_{W_{max}} \\ & 0 \leq P_{S_t} \leq P_{S_{max}} \\ & P_{B_{min}} \leq P_{B_t} \leq P_{B_{max}} \end{cases} \tag{18}$$

Microgrid constraints according the equations (14) to (17) are based on the prototype inherent features.

VI. RNN Optimizer

By defining the utility grid power and maximizing WPS, SPS and BBS power as $\mathbf{c}^T = [2 \ \ -1 \ \ -1 \ \ -1]^T$, $x = [P_G \ P_W \ P_S \ P_B]^T$, $\mathbf{A} = [1 \ 1 \ 1 \ 1]$, $\mathbf{b} = [P_L]$, $l = [0 \ P_{W_{min}} \ 0 \ P_{B_{min}}]$ and $h = [P_{G_{max}} \ P_{W_{max}} \ P_{S_{max}} \ P_{B_{max}}]$, the linear programming formulation (18) is written as

$$\begin{cases} \min_x & \mathbf{c}^T x \\ \text{s.t} & \mathbf{A}x = \mathbf{b} \\ & l \leq x \leq h. \end{cases}$$

In order to solve this problem, the proposed RNN optimizer is used by defining $\sigma = x_1 + x_2 + x_3 + x_4 - P_L$, with the parameters $k_1 = 50$, $k_2 = 200$, $k_3 = 0.1$ and $k_4 = 0.1$.

VII. Real-Time Results

Real time test are done based on Microgrid constraints defined in section V. DCBCA and WPCA measured values are send to CM. The RNN uses the load power calculated as the vector $\mathbf{b}$ of the equation (1), and set the power references for the system agents.

RNN obtained reference for DCBCA are a continuous value; however this agent can only turn on or turn off the batteries and the solar cells from the Microgrid; for this reason a high limit for activation and a low limit for deactivation is established.i.e. if BBS high limit is overcome then this module is connected to the Microgrid; on the other hand if the reference power is lower than the low limit, the module is disconnected. This same logic is applied to SPS control. WPCA PI controller use the RNN power WPS reference to change the dynamometer speed and accomplish the power set for this module. To test RNN on the Microgrid laboratory prototype, at the beggining we let the Microgrid to stabilize the power according to the reference provided by RNN. At 30s a 145Ω resistive load is connected to the DC Bus; then at 60s and 90s a same value resistor in parallel configuration is plug-in; then a fourth 19Ω load is connected at 120s; this last one represent a high disturbance to the system. The loads are disconnected at the same order to show the transient behavior of the Microgrid, as Fig.8 displays.

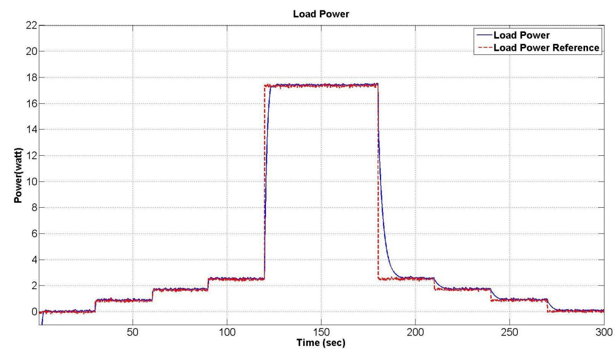

Fig. 8. Load power and RNN references sum.

In Fig.9, the utility grid behavior is shown, it can be seen that the power is close to the $P_{G_{min}}$ which is set as 0 watts.

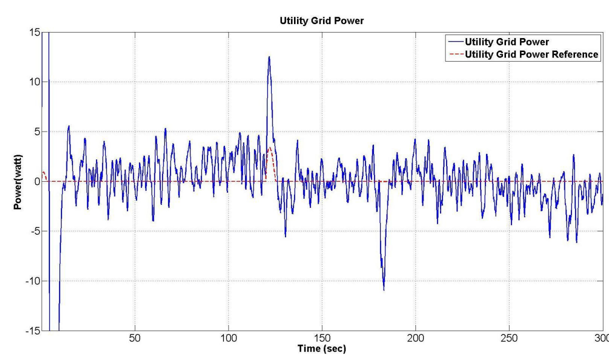

Fig. 9. Utility grid power and RNN utility grid power reference.

In Fig.10, the WPS power and power reference is displayed. It can be seen that this module reaches the reference power all the time and is bounded within the fixed limits.

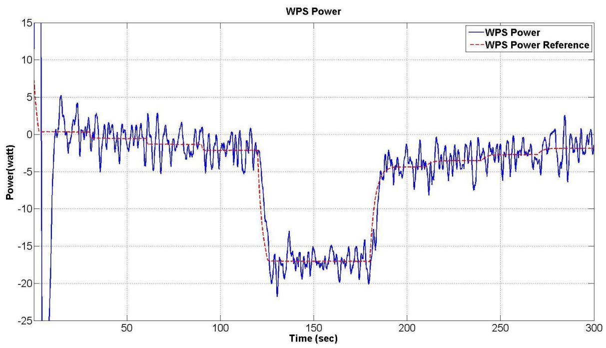

Fig. 10. Wind power and RNN wind power reference.

Fig.11 shows how SPS reaches their power references even though a related controller for this module has not been developed yet. BBS tracking error is higher than the other modules because a related controller for this module is not yet developed too, Fig. 12.

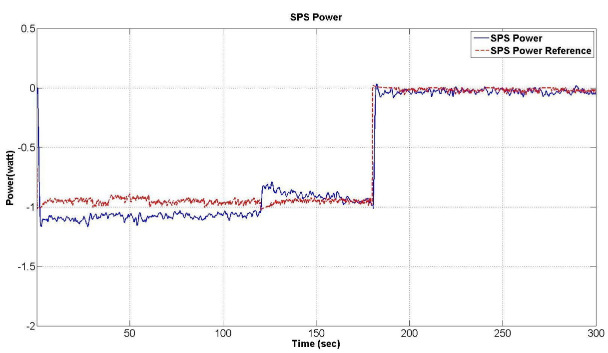

Fig. 11. Solar power and RNN solar power reference.

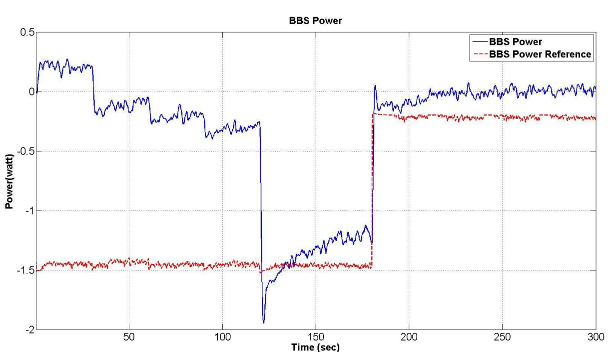

Fig. 12. Battery power and RNN battery power reference.

VIII. Conclusions

In this paper a novel optimization algorithm is applied in Real-time. A recurrent neural network is used to obtain the optimal solution to a Microgrid laboratory prototype source assignment problem. The references obtained are implemented to control a Microgrid power distribution, in order to minimize the consumed power from the utility grid, maximizing the use of the alternative power sources connected to it. Real-time results validates that the proposed optimization algorithm can be implemented for control an small power Microgrid. The optimization algorithm proposed in this paper can be easily extend to obtain the solution for a Multi-Microgrid system without considerably increase the computational load; then, this algorithm can be implemented in real-time for a larger scale high power systems. As future work, the optimization algorithm proposed will be extend to solve quadratic programing problems and to structures which provide fixed time convergence.

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