Sliding Mode Control for Antilock Brake System

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***Abstract*—A Sliding Mode (SM) Block Control is proposed to control an Antilock Brake System (ABS). The control problem is to achieve reference tracking for the slip rate, such that, the friction between tyre and road surface is good enough to control the car. The closed-loop system is robust in presence of matched and unmatched perturbations. To show the performance of the proposed control strategy, a simulation study is carried on, where results show good behavior of the ABS under variations in the road friction.**

***Keywords*—Brake Control, Antilock Braking Systems (ABS), Sliding Mode Control, Automotive Control.**

I. INTRODUCTION

The ABS control problem consists in imposing a desired vehicle motion and as a consequence, provides adequate vehicle stability. The main difficulty arising in the ABS design is due to its high nonlinearities and uncertainties presented in the mathematical model. Therefore, the ABS has become an attractive research area in nonlinear systems control framework. There are several works reported in the literature using the sliding mode technique [1], [2], [3], [4], [5]. In this work we design a new controller on the basis of sliding mode (SM) [6]. In order to achieve robustness to matched, and unmatched perturbations, and ensure output tracking. Theorically, this SM control can guarantee the robustness of the system through the entire response starting from the initial time instance. In spite of the mentioned above works we consider a real situation: the control input can take only two values ”0” or ”1” that corresponds to the control valve position.

The work is organized as follows. The mathematical model for the longitudinal movement of a vehicle, including the brake system is presented in Section 2. In Section 3 a SM controller for ABS is designed. The simulation results are presented in Section 4 to verify the robustness and performance of the proposed control strategy. Finally, some conclusions are presented in Section 5.

II. MATHEMATICAL MODEL

In this section, the dynamic model of a vehicle is showed. Here we use a quarter of vehicle model, this model considers the pneumatic brake system, the wheel motion and the vehicle motion. We study the task of controlling the wheels rotation, such that, the longitudinal force due to the contact of the wheel with the road, is near from the maximum value in the period of time valid for the model. This effect is reached as a result of the ABS valve throttling.

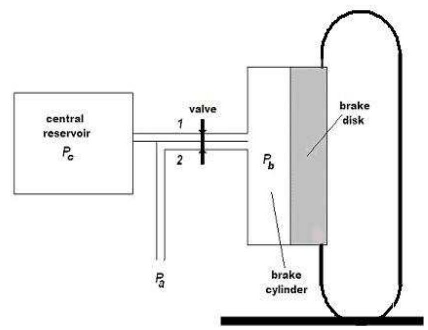

Fig. 1. Pneumatic brake system

A. Pneumatic brake system equations

The specific configuration of this system considers brake disks, which hold the wheels, as a result of the increment of the air pressure in the brake cylinder (Fig. 1). The entrance of the air trough the pipes from the central reservoir and the expulsion from the brake cylinder to the atmosphere is regulated by a common valve. This valve allows only one pipe to be open, when 1 is open 2 is closed and vice versa. The time response of the valve is considered small, compared with the time constant of the pneumatic system.

Lets consider Figure 1, we suppose the brake torque T_b is proportional to the pressure P_b in the brake cylinder

$$T_b = k_b P_b \tag{1}$$

with $k_b > 0$. For the brake system we use an approximated model of pressure changes in the brake cylinder due to the opening of the valve with a first order relation [7], this relation can be represented as

$$\tau \frac{dP_b}{dt} + P_b = P_c u \tag{2}$$

where τ is the time constant of the pipelines, P_c is the pressure inside the central reservoir, u is the valve input signal. We suppose that opening and closing of the valve is momentary and the parameters of the equation (2) are given by the following rules:

- When pipe 1 is opened and 2 is closed then $u = 1$ and $\tau = T_{in}$
- When pipe 2 is opened and 1 is closed then $u = 0$ and $\tau = T_{out}$

when pipe 2 is open the pressure into the brake cylinder is the atmospheric pressure P_a which is considered equal to zero.

B. Wheel motion equations

To describe the wheels motion we will use a partial mathematical model of the dynamic system [8], [9], [10] and [11].

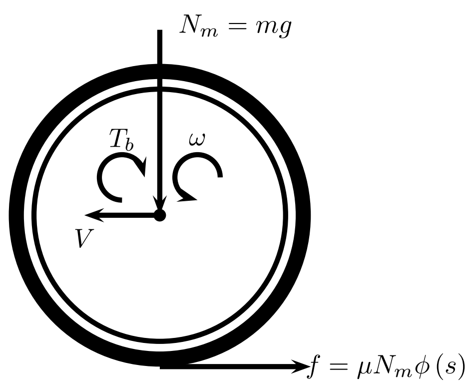

Fig. 2. Wheel forces and torques

Consider Fig. 2, the dynamics of the angular momentum change relative to the rotation axis are given by

$$J\frac{d\omega}{dt} = rf - T_b \tag{3}$$

where ω is the wheel angular velocity, J is the wheel inertia moment, r is the wheel radius and f is the contact force of the wheel.

The expression for longitudinal component of the contact force in the motion plane is

$$f = \nu N_m \phi(s) \tag{4}$$

where ν is the nominal friction coefficient between the wheel and the road, N_m is the normal reaction force in the wheel

$$N_m = mg$$

with m equal to the mass supported by the wheel and g is the gravity acceleration. The function $\phi(s)$ represents a friction/slip characteristic relation between the tyre and road surface. Here, we use the Pacejka model [12], defined as follows

$$\phi(s) = D\sin\left(C\arctan\left(Bs - E\left(Bs - \arctan\left(Bs\right)\right)\right)\right) \tag{5}$$

in general, this model produces a good approximation of the tyre/road friction interface. With the following parameters $B = 10$, $C = 1.9$, $D = 1$ and $E = 0.97$ that function represents the friction relation under a dry surface condition. A plot of this function is shown in Fig. 3

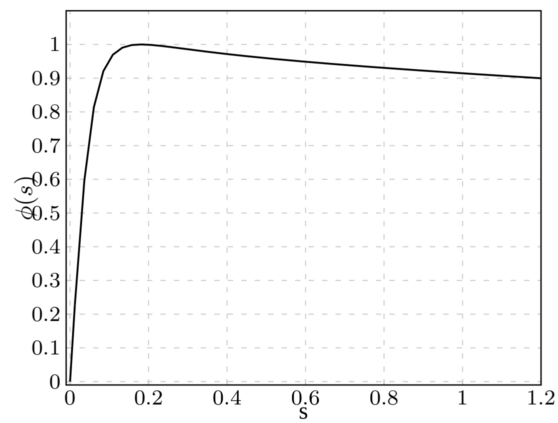

Fig. 3. Characteristic function $\phi(s)$

The slip rate s is defined as

$$s = \frac{V - r\omega}{V} \tag{6}$$

where V is the longitudinal velocity of the wheel mass center. The equations (3)-(5) characterize the wheel motion.

C. The vehicle motion equation

The vehicle longitudinal dynamics without lateral motion considered are represented as

$$M\frac{dV}{dt} = -F - F_a \tag{7}$$

where M is the vehicle mass; F_a is the aerodynamic drag force, which is proportional to the vehicle velocity and is defined as

$$F_a = \frac{1}{2}\rho C_d A_f \left(V + V_w\right)^2$$

where ρ is the air density, C_d is the aerodynamic coefficient, A_f is the frontal area of vehicle, V_w is the wind velocity; the contact force of the vehicle F is modeled of the form

$$F = \mu N_M$$

where N_M is the normal reaction force of the vehicle, $N_M = Mg$ with M equal to the vehicle mass and $\mu = \nu\phi(s)$.

The dynamic equations of the whole system (2)-(7) can be rewritten using the state variables

$$x = [x_1, x_2, x_3]^T = [\omega, P_b, V]^T$$

with initial conditions $x_0 = x(0)$ results the following form:

$$\begin{aligned} \dot{x}_1 &= a_1 f - a_2 x_2 \\ \dot{x}_2 &= -a_3 x_2 + bu \\ \dot{x}_3 &= -a_4 F - f_3(x_3) \end{aligned} \tag{8}$$

with output

$$y = s = h(x) = 1 - r\frac{x_1}{x_3}$$

where $a_1 = r/J$, $a_2 = k_b/J$, $a_3 = 1/\tau$, $a_4 = -1/M$, $b = P_c/\tau$ and $f_3(x_3) = d_1 (x_3 + V_w)^2$ with $d_1 = \frac{1}{2M}(\rho C_d A_f)$.

III. SLIDING MODES CONTROL FOR ABS

Based on system (8) the considered problem is to design an Sliding Mode Block controller that obtains reference tracking in despite of the perturbations in the system. Define s^* as the desired trajectory of the relative slip, which must maximize the function $\phi(s)$.

Throughout the development of the controller, we will use the following assumption:

A1) All the state variables are available for measurement.

A. Control Design

Let $s^*(t)$ be a twice differentiable function, but with unknown derivatives, now we define the output tracking error as $e_1 \triangleq s - s^*$ then its derivative is

$$\dot{e}_1 = c_1(x) + c_2(x)x_2 + f_y(x) - \dot{s}^* \tag{9}$$

where

$$\begin{aligned} c_1(x) &\triangleq c_1 = -r\left(\frac{a_1}{x_3}f + a_4 F\frac{x_1}{x_3^2}\right) \\ c_2(x) &\triangleq c_2 = r\frac{a_2}{x_3} \\ f_y(x) &\triangleq f_y = -rf_3(x_3)\frac{x_1}{x_3} + \Delta(\nu) \end{aligned}$$

$f_y(x)$ will be considered as an unmatched and bounded perturbation term

$$\| f_y(x,t) \| < \beta < \infty$$

The term $\Delta(\nu)$ contains the variations of the friction parameter ν.

Considering the variable x_2 as virtual control in (9) we determinate the desired value x_{2ref} as

$$x_{2ref} = -\frac{1}{c_2}[c_1 + k_0 e_0 + k_1 e_1] \tag{10}$$

where $k_0 > 0$, $k_1 > 0$ and e_0 is the integral of the tracking error e_1 that is

$$\dot{e}_0 = e_1 \tag{11}$$

The variable x_{2ref} is used to put the desired dynamic for e_1 and obtain the control aim. Now we define a new error variable $e_2 \triangleq \alpha_2(x,t)$ in the form

$$e_2 = x_{2ref} - x_2 \tag{12}$$

Using (8) and (10), straightforward calculations reveal

$$\dot{e}_2 = -a_3 e_2 - bu + f_{2e}(x) \tag{13}$$

where

$$f_{2e}(x) = a_3 x_{2ref} + \frac{\partial \alpha_2(x,t)}{\partial x_1}\dot{x}_1 + \cdots + \frac{\partial \alpha_2(x,t)}{\partial x_3}\dot{x}_3$$

To induce sliding mode on the sliding manifold $e_2 = 0$ we choose the control signal as

$$u = 0.5\text{sign}(e_2) + 0.5 \tag{14}$$

B. Stability analysis

Using the new variables e_0, e_1 and e_2 the extended closed loop system (9), (11) and (13) is presented as

$$\dot{e}_0 = e_1 \tag{15}$$
$$\dot{e}_1 = -k_0 e_0 - k_1 e_1 + c_2 e_2 + g_1(x,t) \tag{16}$$
$$\dot{e}_2 = -a_3 e_2 + f_{2e}(x) - 0.5b\text{sign}(e_2) - 0.5b \tag{17}$$

with $g_1(x,t) = f_y(x) - \dot{s}^*$.

The stability of (16) - (17) can be is studied step by step:
A) SM stability of the projection motion (17);
B) SM stability of the projection motion (15)-(16);

We use the following assumptions:

$$|g_1(x,t)| \leqslant \alpha_1 |e_1| + \beta_1 \tag{18}$$

$$|f_{2e}(x)| \leqslant \alpha_2 |e_2| + \beta_2 \tag{19}$$

with $\alpha_1 > 0$, $\alpha_2 > 0$, $\beta_1 > 0$, $\beta_2 > 0$, $a_3 > \alpha_2$ and $b > |f_{2e}(x)|$.

A) The system (17) can be presented as follows:
CASE 1, $e_2 < 0$, then

$$\dot{e}_2 = -a_3 e_2 + f_{2e}(x) \tag{20}$$

CASE 2, $e_2 > 0$, then

$$\dot{e}_2 = -a_3 e_2 + f_{2e}(x) - b \tag{21}$$

we use the Lyapunov candidate function $V_2 = \frac{1}{2}e_2^2$ to analyze the stability conditions. The derivative of V_2 with respect to time in Case 1 is

$$\dot{V}_2 = e_2\left(-a_3 e_2 + f_{2e}(x)\right) \tag{22}$$

under condition (19) we have

$$\dot{V}_2 \leq |e_2|\left(-a_3|e_2| + \alpha_2|e_2| + \beta_2\right)$$

In this case, the solution of (17) is ultimately bounded by [13]

$$|e_2(t)| \leqslant \delta_0, \quad \delta_0 = \frac{\beta_2}{a_3 - \alpha_2} \tag{23}$$

that is similar in case 2.

B) To analyze stability of the sliding mode equations (15)-(16) with $e_2 = 0$, that system can be regarded as a linear system with nonvanishing perturbation in the form:

$$\dot{\xi} = A\xi + D(\xi) \tag{24}$$

where

$$\xi = \begin{bmatrix} e_1 & e_2 \end{bmatrix}^T; A = \begin{bmatrix} 0 & 1 \\ -k_0 & -k_1 \end{bmatrix}; D = \begin{bmatrix} 0 \\ g_1(t) \end{bmatrix}$$

Now we use the following Lyapunov candidate function:

$$V_1 = \frac{1}{2}\xi^T P \xi \tag{25}$$

with P positive definite. With the correct selection of the elements k_0 and k_1 the matrix A is Hurwitz, then exists one unique solution ($P > 0$) to the Lyapunov equation

$$A^T P + PA = -Q$$

where $Q = Q^T$, $Q > 0$.

Lyapunov equation satisfies:

$$\lambda_{min}(P)\left\|\xi\right\|_2^2 \leq \xi^T P \xi \leq \lambda_{max}(P)\left\|\xi\right\|_2^2 \tag{26}$$
$$\frac{\partial V_1}{\partial \xi}A\xi = -\xi^T Q \xi \leq -\lambda_{min}(Q)\left\|\xi\right\|_2^2$$

and the perturbation term is bounded by $\|D(\xi)\| \leq \alpha_1\left\|\xi\right\|_2 + \beta_1$.
Derivating (25) we obtain

$$\dot{V}_1 = -\xi^T Q \xi - 2\xi^T P D(\xi) \tag{27}$$

substituting the bounds (26) in (27), we have

$$\begin{aligned}\dot{V}_1 &= -\xi^T Q \xi - 2\xi^T P D(\xi) \\ &\leq -\lambda_{min}(Q)\left\|\xi\right\|_2^2 + 2\lambda_{max}(P)\left\|\xi\right\|_2\left(\alpha_1\left\|\xi\right\|_2 + \beta_1\right) \\ &\leq \left(-\lambda_{min}(Q) + 2\alpha_1\lambda_{max}(P)\right)\left\|\xi\right\|_2^2 + 2\beta_1\lambda_{max}(P)\left\|\xi\right\|_2 \\ &= -\alpha\left(1-\theta\right)\left\|\xi\right\|_2^2 - \alpha\theta\left\|\xi\right\|_2^2 + \beta\left\|\xi\right\|_2\end{aligned}$$

where $\alpha = \lambda_{min}(Q) - 2\alpha_1\lambda_{max}(P)$ and $\beta = 2\beta_1\lambda_{max}(P)$, then

$$\dot{V}_1 \leq -\alpha\left(1-\theta\right)\left\|\xi\right\|_2^2 \tag{28}$$

for $\forall \left\|\xi\right\|_2 > \frac{\beta}{\alpha\theta} = \delta$.
Thus, the nominal system $\dot{\xi} = A\xi$ has an exponentially stable equilibrium point $\xi = 0$, the solution $\xi(t)$ of (24) is ultimately bounded and the ultimate bound is given by

$$\left\|\xi\right\|_2 \leq \delta\frac{\sqrt{\lambda_{max}(P)}}{\sqrt{\lambda_{min}(P)}} \tag{29}$$

Finally, considering the absolute value of the wind speed in (8), the remaining dynamics x_3 is locally stable.

IV. Simulation Results

To show the effectiveness of the proposed control law, simulations have been carried out on one wheel model design example, the system parameters used are listed in Table 1.

TABLE 1
Values of Parameters

Parameter	Value	Parameter	Value
A_f	6.6	V_w	-6
P_c	8	v	0.5
M	1800	B	10
J	18.9	C	1.9
R	0.535	D	1
m	450	E	0.97
ρ	1.225	g	9.81
C_d	0.65	P_a	0

In order to maximize the friction force, we suppose that slip tracks a constant signal during the simulations.

$$y_{ref} = 0.203$$

which produces a value close to the maximum of the function $\phi(s)$. The parameters used in the control law are $k_0 = 700$ and $k_1 = 120$.

On the other hand, to show robustness property of the control algorithm in presence of parametric variations we introduce a change of the friction coeficient ν which produces diferent contact forces, namely $\hat{F}$ and $\hat{f}$. Then, $\nu = 0.5$ for $t < 1$ s, $\nu = 0.52$ for $t \in [1, 2.5)$ s, and $\nu = 0.5$ for $t \geq 2.5$ s. It is worth mentioning that just the nominal values were considered in the control design.

In Figure 4 the slip performance trough the simulation is showed, Figure 5 shows the friction function behavior $\phi(s)$ during the braking process

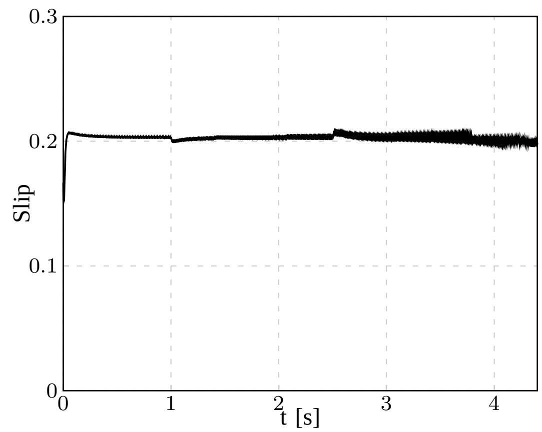

Fig. 4. Slip performance in the braking process

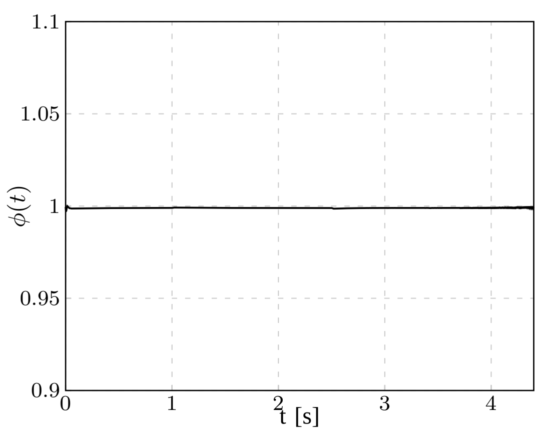

Fig. 5. Performance of $\phi(s)$ in the braking process

while Figures 6 and 7 summarize the behavior of the error variables e_1 and e_2 respectively.

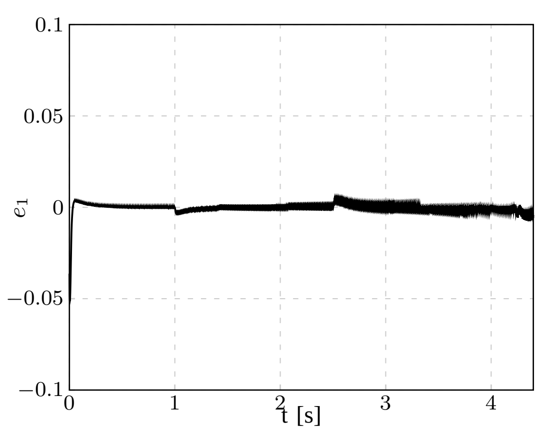

Fig. 6. Tracking error $e_1 = s - s^*$

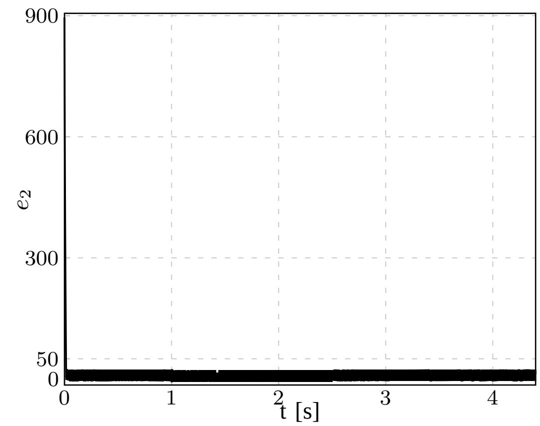

Fig. 7. Error variable $e_2 = x_{2ref} - x_2$

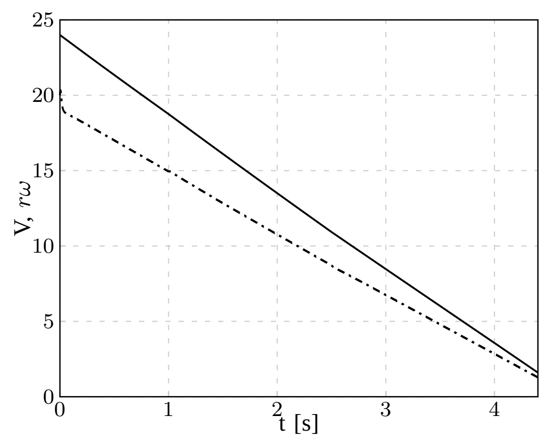

Fig. 8. Longitudinal speed V (solid) and linear wheel speed $r\omega$ (dashed)

In Figure 8 the longitudinal speed V and the linear wheel speed $r\omega$ are showed; it is worth noting that the slip controller should be turn off when the longitudinal speed V is close to zero. Figure 9 the control action is shown.

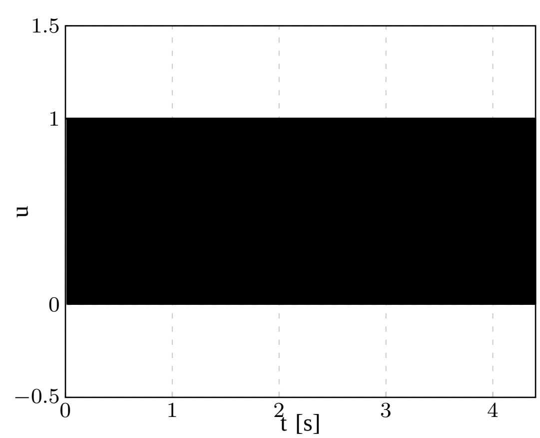

Fig. 9. Control input u

Finally, in Figure 10 the nominal F, and the $\hat{F}$ contact force are shown.

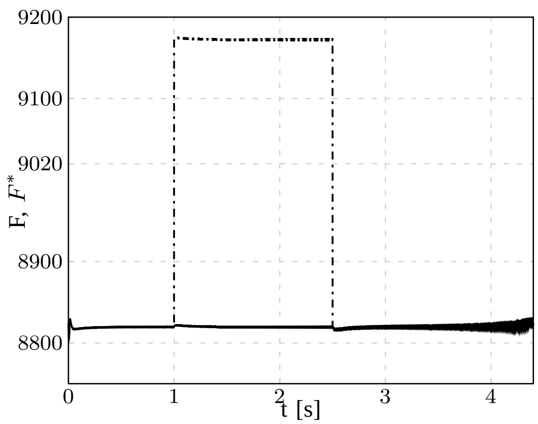

Fig. 10. Nominal F (solid) and $\hat{F}$ (dashed) vehicle contact forces

V. Conclusion

In this work an sliding mode block control for ABS has been proposed. The simulation results show good performance and robustness of the closed-loop system in presence of both the matched and unmatched perturbations, namely, parametric variations and neglected dynamics.

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