Structural Sequence Detectability in Free Choice Interpreted Petri Nets

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***Abstract*—This paper is concerned with the structural sequence detectability problem in Free Choice Interpreted Petri nets, i.e. with the possibility of recovering the firing transition sequence in Free Choice Interpreted Petri nets using the output information when the initial marking is unknown. Based on the Free Choice Interpreted Petri net structure, three relationships are proposed which are devoted to capture the confusion over the transitions. These relationships depend on interpreted Petri nets structures such as $T-invariants$, $P-Invariants$, attribution and distribution places. Thus, the approach herein presented exploits the interpreted Petri nets structural information in order to determine the structural sequence detectability of an interpreted Petri net.**

Keywords: Petri Nets, Structural Sequence Detectability.

I. Introduction

Discrete Event Systems (DES's) have deserved a lot of attention by the scientific community since they can model the discrete behavior of robotic systems, supply chains, transport systems, digital communication systems, information systems, etc. The study of several properties has been reported in the literature. For instance, fault diagnosis is addressed in [5], [9], [3], [2], [10]; controllability is studied in [6], [7], [13]; observability in [1], [16],[11] and identification is presented in [12], among other properties that are reported in the literature. The characterization of the previous mentioned properties relies on the event sequence reconstruction, using for this purpose the information provided by the system sensors (herein named *the output Petri net information*). Thus the reconstruction of firing transition sequences using the output Petri net information is an important problem because it allows enlarging the class of diagnosable, observable, or identifiable Petri nets that can be characterized.

A similar property, named invertibility has been studied in finite state automata (FA) [15], where the event sequence is reconstructed after the occurrence of certain events and then it is lost again. Thus invertibility is a kind of resilient structural sequence detectability. Also structural sequence detectability was addressed in [17]. That work, however, is focused on Petri nets (PN) where the initial state is known and observable places cannot generate the same output information.

We deal in this work with the sequence detectability problem in Interpreted Petri nets (IPN), i.e. with the problem of inferring the fired transition sequence from the knowledge of the output IPN information. The definition of this problem could depend on an initial state or initial IPN output information. Unfortunately, this consideration is not enough to detect firing transition sequences after the occurrence of a fault (diagnosability case) where the reached state could be any one, or when the initial state is unknown (observability case). The more realistic case of this problem is concerned when the initial state and initial IPN output information is unknown.

Hence, this work focuses on the study of the sequence detectability problem when the initial state and initial IPN output information is unknown, this case is named the structural sequence detectability property in IPN and we focus on the Free Choice (FC) class. Our main goal is to avoid the enumeration of all possible firing transition sequences to characterize the structural sequence detectability. Instead of that, we analyze the IPN topological properties guaranteeing the structural sequence detectability. For instance, if two indistinguishable events t_i and t_j, enabled from a valid and unknown initial state (possibly from the same or from different initial states), could lead to the same state then two indistinguishable sequences can be generated $\sigma_1 = t_i\alpha$ and $\sigma_2 = t_j\alpha$, where α is an arbitrarily long firing transition sequence. Moreover, algorithms based on linear programming problems and Nerode's relationship are proposed to determine if the IPN presents the structures generating the indistinguishable firing transition sequences.

This paper is organized as follows. Section II presents the basic concepts and notation of PN and IPN. In Section III the concept of structural sequence detectability is formally defined. Section IV presents the characterization of the structural sequence detectability property in IPN belonging to live and safe FC class. Section V presents algorithms to test the conditions that a structural sequence detectable IPN must fulfill. Finally, some conclusions and future work are presented.

II. Background

This section introduces some basic PN and IPN concepts. An interested reader can consult [14] and [4] for further information on PN.

Definition 1. *A Petri net (PN) structure is a bipartite digraph defined by the 3-tuple $N = (P, T, W)$, where:*

- $P = \{p_1, p_2, ..., p_n\}$ *is a finite set of n places,*
- $T = \{t_1, t_2, ..., t_m\}$ *is a finite set of m transitions,*
- $P \cup T \neq \varnothing$ *and* $P \cap T = \varnothing$,
- $W : (P \times T) \cup (T \times P) \to \{0, 1\}$ *is a weight arc function.*

A marking is a function $M : P \to \{0, 1, 2, 3, ...\}$ that assigns to each place a nonnegative integer number, named the number of tokens residing inside each place. M_0 is the initial marking. A PN with a given initial marking is denoted by (N, M_0).

Pictorially, places are depicted by circles, transitions by boxes, arcs by arrows and tokens by black dots or integer numbers residing inside each place.

The $n \times m$ incidence matrix C of N is defined by $C(i, j) = W(t_j, p_i) - W(p_i, t_j)$. If $W(p_i, t_j)$ or $W(t_j, p_i)$ is not defined for a specific place p_i and transition t_j, then it is assumed as zero.

Let $x, y \in P \cup T$, the set of input nodes of x, ${}^\bullet x = \{y | W(y, x) = 1\}$ and the set of output nodes of x, $x^\bullet = \{y | W(x, y) = 1\}$ represent the input and output nodes from node x, respectively. These sets can be extended to a set of input (output) nodes of a set of nodes, i.e. ${}^\bullet\{x_1, ..., x_n\} = \{y | W(y, x_1) = 1 \vee ... \vee W(y, x_n) = 1\}$ ($\{x_1, ..., x_n\}^\bullet = \{y | W(x_1, y) = 1 \vee ... \vee W(x_n, y) = 1\}$).

Let N be a PN. Vectors X_i (Y_i) such that $CX_i = 0$, X_i entries are non negative integers ($Y_i^T C = 0$, Y_i entries are non negative integers) are named $T - invariants$ ($P - invariants$). The support of a $T - invariant$ X_i ($P - invariant$ Y_i), denoted by $\langle X_i \rangle$ ($\langle Y_i \rangle$) , is the transition set $T_i = \{t_j | X_i(j) > 0\}$ (place set $P_i = \{p_j | Y_i(j) > 0\}$). The subnet $\mathcal{T}_i = \{(P_i, T_i, W_i), M_{0i}\}$ of N induced by the $T - invariant$ X_i is a $T - component$ if $P_i = ({}^\bullet\langle X_i \rangle \cup \langle X_i \rangle^\bullet)$, $T_i = \langle X_i \rangle$, W_i is the weight arc function restricted to P_i and T_i, and M_{0i} is the initial marking, restricted to P_i. In a similar way, the subnet $\mathcal{P}_i = \{(P_i, T_i, W_i), M_{0i}\}$ of N induced by the $P - invariant$ Y_i^T is a $P - component$ if $T_i = ({}^\bullet\langle Y_i \rangle \cup \langle Y_i \rangle^\bullet)$, $P_i = \langle Y_i \rangle$, W_i is the weight arc function restricted to P_i and T_i; M_{0i} is the initial marking restricted to P_i.

A $P - invariant$ ($T - invariant$) is said to be minimal if the greatest common divisor of its entries is 1 and it is no linear combination of others $P - invariants$ ($T - invariants$). A transition t_j

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is said to be enabled at marking M_k if each input place p_i of t_j (i.e. each place p_i such that $W(p_i, t_j) = 1$) is marked with one token; this is denoted by $M_k\,[t_j(k+1)\rangle$. The firing of an enabled transition t_j removes one token from each input place p_i of t_j, and adds one token to each output place p_k of t_j, reaching a new marking M_{k+1}. This fact is represented by $M_k\,[t_j(k+1)\rangle\, M_{k+1}$. The new marking M_{k+1} can be computed using the state equation:

$$M_{k+1} = M_k + C\overrightarrow{t_j}$$

where $\overrightarrow{t_j}(i) = 1$ if $i = j$ and $\overrightarrow{t_j}(i) = 0$ otherwise.

Notation $M_0\,[t_a\rangle\, M_1$ can be extended to a transition sequence $M_0\,[\sigma\rangle\, M_q$, where $\sigma = t_a t_b ... t_r$ and $M_0\,[t_a\rangle\, M_1\,[t_b\rangle\, M_2 ...\,[t_r\rangle\, M_q$. In this case M_q is named *reachable marking* from M_0. Moreover, M_q is said to be reachable from M_0. The notation $\overrightarrow{\sigma}$ is the Parikh vector of σ, i.e. the $i-th$ entry of $\overrightarrow{\sigma}$ is the number of times that t_i appears in σ. The reachability set of (N, M_0), denoted by $R(N, M_0)$, is the set of all possible reachable markings from M_0, firing only enabled transition sequences.

Definition 2. *An Interpreted Petri net* (IPN) *structure is the pair* $Q = (N, \Phi)$ *where:*

- N is a PN structure together with an initial marking M_0.
- There exists a $q \times n$ matrix Φ of integer numbers, such that $y_k = \Phi M_k$ is mapping the marking M_k into the $q - dimensional$ observation vector. The vector y_k is named the output information of the IPN. In this work we focus on cases where each column of matrix Φ is an elementary or null vector.

Transitions t_i and t_j have identical behavior (redundant) if $C(\bullet, i) = C(\bullet, j)$, where $C(\bullet, i)$ denotes the column of C corresponding to transition t_i. If t_i and t_j have identical behavior, then trivially the IPN Q has firing transition sequences that cannot be distinguished from each other. Therefore, we focus on nets that do not present this kind of transitions. The IPN state equation is:

$$M_{k+1} = M_k + C\overrightarrow{t_j}; \qquad y_k = \Phi M_k$$

notice that the output IPN information is included.

Definition 3. *A firing transition sequence of an* $IPN(Q, M_0)$ *is a sequence* $\sigma = t_i t_j \ldots t_k \ldots$ *such that* $M_0\,[t_i\rangle\, M_1\,[t_j\rangle \ldots M_{n-1}\,[t_k\rangle \ldots$*. The set of all firing transition sequences is called the firing language* $\pounds(Q, M_0) = \{\sigma |\ \sigma = t_i t_j \ldots t_k \ldots$ *and* $M_0\,[t_i\rangle\, M_1\,[t_j\rangle \ldots M_{n-1}\,[t_k\rangle \ldots\}$.

Definition 4. *A sequence of observation vectors (output information) of* (Q, M_0) *is a sequence* $\omega = (y_0)(y_1) \ldots (y_n)$*,* $y_i = \Phi M_i$*.*

Definition 5. *A* PN (N, M_0) *is said to be live (or equivalently* M_0 *is a live marking of* N*) if, no matter what marking has been reached from* M_0*, it is possible to ultimately fire any transition of* N *by progressing through some further firing sequence ([14]). A* PN (N, M_0) *is safe if the maximum number of tokens in places is* 1 *for every* $M \in R(N, M_0)$*.*

Definition 6. *A Free Choice* (FC) *net is a strongly connected* IPN *subclass (i.e. for any pair of nodes* $x, y \in P \cup T$ *there exist directed paths from* x *to* y *and vice versa) such that if* $p_i^\bullet \cap p_j^\bullet \neq \varnothing$ *then* $p_i^\bullet = p_j^\bullet$*,* $\forall p_i, p_j \in P$ *([14]).*

As a notation we will use μ_0 to represent the set of the k initial markings $\mu_0 = \{M_0^1, M_0^2, ..., M_0^k\}$ such that (Q, M_0^i) becomes live and safe, $M_0^i \in \mu_0$. Notice that if $M_0 \in \mu_0$, then any reachable marking from M_0 also belongs to μ_0.

Notation (Q, μ_0) is used to emphasize that the IPN initial marking is unknown, but could be any one in μ_0. Testing if a marking M_0^j belongs to μ_0, in a Free Choice PN, can be performed using the Commoner's Theorem (see [4]). In this work we focus on the case when the set μ_0 is known. Also, the firing language can be extended to represent all possible firing transition sequences from μ_0 as $\pounds(Q, \mu_0) = \bigcup_{i=1}^{k} \pounds(Q, M_0^i)$. Notice that if $\sigma_1, \sigma_2 \in \pounds(Q, \mu_0)$, it means that there exist M_0^i, $M_0^j \in \mu_0$ such that $\sigma_1 \in \pounds(Q, M_0^i)$ and $\sigma_2 \in \pounds(Q, M_0^j)$.

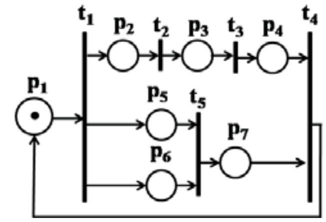

Fig. 1. A PN with a fork-join transition pair.

Since columns of matrix Φ are elementary or null vectors, then if $\Phi(\bullet, i) + \Phi(\bullet, j) = \Phi(\bullet, k)$ implies that one of the three vectors are the null one, i.e. column linear combinations means that the columns are equal with each other.

Throughout this work we will consider the following points:

1) This work focuses on pure (i.e. $\forall p \in P$, $p^\bullet \cap {}^\bullet p = \varnothing$) Free Choice nets where the initial marking $M_0 \in \mu_0$ is unknown.
2) Input places to the same transition must have associated output information, i.e., if $|{}^\bullet t_j| > 1$ then $\forall p_i \in {}^\bullet t_j$, $\Phi(\bullet, i) \neq \overrightarrow{0}$. This consideration guarantees that if two transitions are indistinguishable with each other (their firing result in the same change of the output information), then they have the same cardinality in their sets of input places.
3) For any transition t_j it is not allowed that for any $p_k \in {}^\bullet t_j$, $p_l \in t_j^\bullet$, the marking of these places mapped into the observation vector be equal, i.e. $\Phi(\bullet, k) = \Phi(\bullet, l)$. This consideration ensures that, if we decompose the IPN into $P - components$ $\{\mathcal{P}_1, ..., \mathcal{P}_w\}$ then the firing of t_j produces a change in the output IPN information (see [16]) in every $\mathcal{P}_k$.

Definition 7. *Let* C *be a set of* $T - components$ *of a net.* C *is a* $T - Cover$ *if every transition of the net belongs to a* $T - component$ *of* C*.*

Definition 8. *Let* SN *be any* $T - Cover$ *of a* FC *net (see [4]), transitions* t_i*,* t_j *(*t_i *and* t_j *could be the same) form a fork-join transition pair if* $|t_i^\bullet| > 1$*,* $|{}^\bullet t_j| > 1$ *and there does not exist a* $P - invariant$ Y *such that* $\exists p_k \in t_i^\bullet$*,* $Y(p_k) > 0$ *and* $\forall p_q \in {}^\bullet t_j$ $Y(p_q) = 0$*.*

For instance, consider the PN shown in Fig. 1, the transitions t_1 and t_5 do not form a fork-join transition pair since there exists a $P - invariant$ $Y^T = [\ 1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0\ ]$ where $Y(p_2) = 1$ $(p_2 \in t_1^\bullet)$ and $Y(p_5) = Y(p_6) = 0$ $(p_5, p_6 \in {}^\bullet t_j)$. Transitions t_1, t_4 form a fork-join transition pair.

III. Structural sequence detectability definition

This section introduces Forward, Reverse and Concurrence relationships on the transition set and the indistinguishable relationship on the transition and $T - component$ sets. These relationships will be useful to characterize the structural sequence detectability (SSD) in FC IPN subclass.

Definition 9. *Let* t_i*,* t_j *be two transitions. The firing of* t_i *is indistinguishable from the firing of* t_j *(*$t_i \approx_I t_j$*) if* $\Phi C(\bullet, i) = \Phi C(\bullet, j)$*.*

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In a similar way, two arbitrarily long firing transition sequences $\sigma_1 = t_1 \dots t_k \dots$, $\sigma_2 = t'_1 \dots t'_k \dots$, $|\sigma_1| = |\sigma_2|$ are indistinguishable from each other, $\sigma_1 \approx_I \sigma_2$, if $t_1 \approx_I t'_1, \dots, t_k \approx_I t'_k, \dots$.

Notice that indistinguishability over transitions is an equivalence relationship, thus it partitions the transition set. Transitions t_j belonging to a class such that $|[t_j]| = 1$ are transitions whose firing can be distinguished from any other transition firing. In the following, the set $Gr(\approx_I)$ will denote the set of transitions pairs (t_i, t_j) such that $t_i \approx_I t_j$ and $t_i \neq t_j$ (i.e. it is the indistinguishable transition relationship where the ref lexivity has been removed).

In live and safe PN, arbitrarily long sequences $\sigma_1, \sigma_2 \in \pounds(Q, \mu_0)$, such that $\sigma_1 \approx_I \sigma_2$ could be generated by firing $T-invariants$ (see [4]), the $T-components$ induced by these $T-invariants$ will be named indistinguishable $T-components$. The next definition formalizes this notion.

Definition 10. *Let $\mathcal{T}_i$, $\mathcal{T}_j$ be two $T-components$ induced by the $T-invariants$ X_i, X_j respectively. $T-components$ $\mathcal{T}_i$, $\mathcal{T}_j$ ($\mathcal{T}_i$ could be equal to $\mathcal{T}_j$) are indistinguishable ($\mathcal{T}_i \approx_I \mathcal{T}_j$) from each other if there exist two firing transition sequences $\sigma_i \neq \sigma_k$, $|\sigma_i| = |\sigma_k|$, $\sigma_i \approx_I \sigma_k$, such that $\overrightarrow{\sigma}_i = X_i$, $\overrightarrow{\sigma}_k = X_j$.*

However, not all arbitrarily long indistinguishable sequences are generated by indistinguishable $T-components$. The next transition relationships capture the IPN structures that can generate indistinguishable and arbitrarily long firing transition sequences.

Definition 11. *Transitions relationships:*

$a)$ Reverse relationship ($\approx^-$). Let Q be an IPN. The transitions t_i and t_j are reverse related ($t_i \approx^- t_j$) if $t_i \approx_I t_j$ and $^\bullet t_i = {^\bullet t_j}$.

$b)$ Forward relationship ($\approx^+$). Let Q be an IPN. The transitions t_i and t_j are forward related ($t_i \approx^+ t_j$) if $t_i \approx_I t_j$ and $t_i^\bullet \cap t_j^\bullet \neq \varnothing$.

$c)$ Concurrence relationship ($\approx_p$). Let Q be an IPN, and $t_i, t_j \in T$, $i \neq j$, such that $t_i \approx_I t_j$. If $\nexists$ minimal $P-invariant$ Y_k such that $^\bullet t_i, ^\bullet t_j, t_i^\bullet, t_j^\bullet \subseteq \langle Y_k \rangle$ then $t_i \approx_p t_j$.

Two indistinguishable transitions t_i, t_j, $(t_i, t_j) \in Gr(\approx_I)$ evolve in concurrence with a $T-invariant$ X_k if the sequences $t_i\sigma_k$, $t_j\sigma_k$, where $\overrightarrow{\sigma}_k = X_k$ can be fired from markings $M_0, M'_0 \in \mu_0$.

Now the structural sequence detectability property in IPN is formalized in the following definition.

Definition 12. *Let (Q, μ_0) be an IPN. The IPN Q is said to be structurally sequence detectable if there exists $k < \infty$, $k \in \mathbb{N}$ such that any pair of firing transition sequences $\sigma_1, \sigma_2 \in \pounds(Q, \mu_0)$ with $\sigma_1 \neq \sigma_2$, $|\sigma_1| = |\sigma_2|$ and $\forall \alpha_1, \alpha_2$ such that $\sigma_1\alpha_1, \sigma_2\alpha_2 \in \pounds(Q, \mu_0)$, $|\sigma_1\alpha_1| = |\sigma_2\alpha_2| > k$ then $\sigma_1\alpha_1 \not\approx_I \sigma_2\alpha_2$.*

Example 13. *Let (Q, μ_0) be the safe and pure PN shown in Fig. 2, where*

$$\Phi = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Thus the firing transition sequences $\sigma_1, \sigma_2 \in \pounds(Q, \mu_0)$, where $\sigma_1 = t_2\beta$ fireable at

$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix}^T$, and $\sigma_2 = t_7\beta \in \pounds(Q, \mu_0)$ fireable at

$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \end{bmatrix}^T$, where $|\beta|$ is arbitrarily long, then $\sigma_1 \approx_I \sigma_2$, thus Q is not SSD.

Notice that in this example the transitions t_2 and t_5 cannot be simultaneously enabled, because the initial marking $M_0^k = [0\ 1\ 0\ 1\ 0]^T \notin \mu_0$. As a notation, in the IPN (see Fig. 2 as an example), we associate the symbol Φ_j to places p_k if $\Phi(j,k) = 1$.

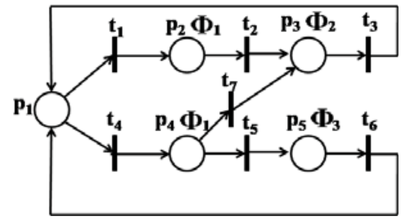

Fig. 2. A non SSD FC considering the initial marking unknown.

If $M_k [t_i\rangle M_{k+1}$ then the output information change, obtained when t_i is fired, is computed by $\Phi M_{k+1} - \Phi M_k = \Phi C(\bullet, i)$.

Example 14. *In the FC Q shown in Fig. 2, it can be seen that $t_2 \approx^+ t_7$, $t_1 \approx^- t_4$ and $t_4t_7t_3 \dots t_4t_7t_3 \approx_I t_1t_2t_3 \dots t_1t_2t_3$.*

IV. SSD CHARACTERIZATION IN FC

The next theorem characterizes the structural sequence detectability property in PN.

Theorem 15. *Let (Q, μ_0) be a live, safe and pure PN belonging to the FC class. Then Q is SSD iff the following conditions are fulfilled:*

1) *$\forall t_i, t_j \in T$, $i \neq j$, $t_i \not\approx^+ t_j$,*
2) *$\forall t_i, t_j \in T$, $i \neq j$, $t_i \not\approx^- t_j$,*
3) *$\forall t_i, t_j \in T$, $i \neq j$, $t_i \not\approx_p t_j$,*
4) *if $(t_i, t_j) \in Gr(\approx_I)$ then t_i, t_j are not evolving in concurrence with $T-invariants$.*
5) *there are no indistinguishable $T-components$.*

Proof. The proof is based on the contrapositive statement of Theorem 15.

($\rightarrow$) If there exist $t_i, t_j \in T$, $i \neq j$, $t_i \approx^+ t_j$ in a PN, then by Definition 11 $b)$ $t_i^\bullet = t_j^\bullet$. Since Q is live and safe, then $\mu_0 \neq \varnothing$, thus there exists a marking $M_0 \in \mu_0$ such that t_i is enabled or it is enabled in a reachable marking M from M_0. Then we can choose M as the new initial marking by redefining $M_0 = M$. A new marking $M'_0 \in \mu_0$ can be built as follows, $M'_0(p) = M_0(p)$ if $p \notin {^\bullet t_i} \cup {^\bullet t_j}$, $M'_0(p) = 0$ if $p \in {^\bullet t_i}$, $M'_0(p) = 1$ if $p \in {^\bullet t_j}$. Thus $M_0 [t_i\rangle M_1 [\sigma_1\rangle$ and $M'_0 [t_j\rangle M_1 [\sigma_1\rangle$, σ_1 is an arbitrarily long sequence (there is no integer k such that $|\sigma_1| < k$). Sequences $t_i\sigma_1$, $t_j\sigma_1$ are indistinguishable from each other, thus Q is not SSD.

If $t_i \approx^- t_j$, then $^\bullet t_i = {^\bullet t_j}$. Since Q is live and bounded, then there exists M_k reachable from any $M_0 \in \mu_0$ (see the definition of home spaces in [4]) enabling t_i such that M_k can be reached infinitely often, otherwise if there is no such M_k then Q is blocked or there exists an infinite number of reachable markings, a contradiction. M_k is also enabling t_j since $^\bullet t_i = {^\bullet t_j}$. Then there exists a fireable sequence σ from M_0 reaching a marking M_k where place $^\bullet t_i = {^\bullet t_j}$ is marked. Since M_k is reached infinitely often, then there exists a sequence β such that $M_k [\beta\rangle M_k$. Then the sequence $\sigma\beta^k t_i$, $M_0 [\sigma\rangle M_k [\beta^k\rangle M_k [t_i\rangle$, is indistinguishable from $\sigma\beta^k t_j$, $M_0 [\sigma\rangle M_k [\beta^k\rangle M_k [t_j\rangle$, where k is an arbitrary positive integer. Hence Q is not SSD.

If $t_i \approx_p t_j$, then t_i, t_j belong to different $P-components$. Since Q is a live net, then there exists an initial marking $M_0 \in \mu_0$ such that t_i, t_j are enabled. Thus the sequence $t_it_j\sigma_1$, ($M_0 [t_it_j\rangle M_2 [\sigma_1\rangle$) is indistinguishable from $t_jt_i\sigma_1$, ($M_0 [t_jt_i\rangle M_2 [\sigma_1\rangle$), where σ_1 is arbitrarily long and the net is not SSD.

If there is $(t_i, t_j) \in Gr(\approx_I)$ and t_i, t_j are evolving in concurrence with a $T-invariant$ X_z, then there exists $M_0, M'_0 \in \mu_0$ enabling the sequences $t_i\sigma^k$, $t_j\sigma^k$ respectively, $\overrightarrow{\sigma} = X_z$. Since $(t_i, t_j) \in$

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$Gr(\approx_I)$, then $t_i\sigma^k$ is indistinguishable from $t_j\sigma^k$, where k is an arbitrary positive integer, thus the net is not SSD.

If there exist indistinguishable $T-components$ X_i, X_j then there are two arbitrarily long sequences $\overrightarrow{\sigma_i} = kX_i$, $\overrightarrow{\sigma_j} = kX_j$ (where k is an arbitrary positive integer) that are indistinguishable from each other, thus Q is not SSD.

($\leftarrow$) Assume that Q is not SSD, then there exist two arbitrarily long firing transition sequences $\sigma_1, \sigma_2 \in \pounds(Q, \mu_0)$, $\sigma_1 \neq \sigma_2$, enabled from M_0, $M_0' \in \mu_0$ such that $\sigma_1 \approx_I \sigma_2$ (i.e. there exists no $k < \infty$ such that the sequences are distinguishable from each other). It could be the case where σ_1 is completely different from σ_2, they have common subsequences or they are equal. The last case is not important for the structural sequence detectability study because we need different sequences. Thus we will focus on the first two cases.

1) If they are completely different from each other, since $\sigma_1 \approx_I \sigma_2$, both of them are arbitrarily long firing transition sequences and the IPN is live and safe, then these sequences are being generated by the indistinguishable $T-invariants$ $\overrightarrow{\sigma_1}$ and $\overrightarrow{\sigma_2}$ (see [4]), then there exist indistinguishable $T-components$.

2) If they share a common subsequence, then the following two cases are possible:

A) We analyze the previous transitions t_a, t_b to the common subsequence α_1 ($\sigma_1 = \beta_1 t_a \alpha_1 ...$, $\sigma_2 = \beta_2 t_b \alpha_1 ...$). Notice that $|\beta_1| = |\beta_2|$ and β_1, β_2 must be finite, otherwise this case must be analyzed as case 1.

I) If t_a, t_b belong to the same $P-component$. Two cases arise:

i) t_a, t_b are enabled simultaneously, since the IPN is safe, then there exists $p_z \in P$ such that $p_z \in {}^\bullet t_a \cap {}^\bullet t_b$, (otherwise there are tokens residing in the input places to t_a and t_b simultaneously and this $P-component$ is not safe, a contradiction), and since the IPN is FC then ${}^\bullet t_a = {}^\bullet t_b$, thus $t_a \approx^- t_b$. Moreover, let t_x be the first transition in α_1, then the subsequences $t_a t_x$ and $t_b t_x$ are obtained. Since by hypothesis we are not allowing $C(\bullet, a) = C(\bullet, b)$, then α_1 is enabled before the firing of t_a or t_b. If α_1 is arbitrarily long then t_a, t_b are evolving in concurrence with the $T-invariant$ $\overrightarrow{\alpha_1}$.

ii) t_a, t_b are not enabled simultaneously. Let t_x be the first transition in α_1, then the subsequences $t_a t_x$ and $t_b t_x$ are obtained. If t_x belongs to the same $P-component$ that t_a, t_b then the input place to t_x is a common output place to t_a and t_b, thus $t_a \approx^+ t_b$. If t_x does not belong to the same $P-component$ that t_a, t_b, then two cases are possible. If α_1 can be fired infinitely often, then t_a, t_b are transitions evolving in concurrence with the $T-invariant$ $\overrightarrow{\alpha_1}$. If α_1 cannot be fired infinitely often, then the tokens residing in $t_a^\bullet$ or $t_b^\bullet$ (but not both since the net is safe) are required to fire a transition t_q fired after α_1. Thus there are two transition sequences, starting from t_a and t_b, marking the same place p_z before t_q is fired (otherwise both sequences include t_q and t_a, t_b do not belong to the same $P-component$, a contradiction), then these tokens should mark a place p_z before these tokens enable t_q. Then p_z has two input transitions t_a', t_b' and $t_a' \approx^+ t_b'$.

II) If t_a, t_b belong to different $P-components$. By hypothesis there exist two markings M_y, M_z such that $M_0\,[\beta_1\rangle\,M_y$, $M_0'\,[\beta_2\rangle\,M_z$ enabling t_a, t_b, then $t_a \approx_p t_b$.

B) We analyze the next transitions t_a, t_b to the common subsequence α_1 ($\sigma_1 = \beta_1 \alpha_1 t_a ...$, $\sigma_2 = \beta_2 \alpha_1 t_b ...$). Notice that β_1, β_2 must be finite, otherwise this case must be analyzed as case 1.

I) If α_1 is arbitrarily long, then this case must be analyzed as case 2.A).

II) If α_1 is finite, then the subsequences $t_a ...$ and $t_b ...$ are arbitrarily long, since σ_1, σ_2 are arbitrarily long. If these two subsequences are completely different from each other, then they are analyzed as case 1, otherwise they must be analyzed using case 2. □

The $IPNs$ depicted in Fig. 3 illustrate the five conditions that lead to non structural sequence detectability. The IPN 3.a) captures the forward relationship, in this case $t_3 \approx^+ t_6$. The IPN in Fig. 3.b) captures the reverse relationship, in this case $t_1 \approx^- t_4$. The IPN in Fig. 3.c) captures the concurrence relationship, in this case $t_2 \approx_p t_3$. In the IPN in Fig. 3.d) there are indistinguishable transitions t_2, t_4 that are evolving in concurrence with the $T-invariant$ $X = \overrightarrow{t_8 t_9}$ (i.e. there are enough tokens to simultaneously fire transition t_2 or t_4 and the transition sequence $t_8 t_9$). In the $IPNs$ in Fig. 3 e) and f) there are indistinguishable $T-components$. In Fig. 3.e) $X_1 = \overrightarrow{t_5 t_6}$ and $X_2 = \overrightarrow{t_7 t_8}$ are the $T-invariants$ generating these $T-components$. In Fig. 3.f) the unique $T-component$ is indistinguishable with respect to itself, for instance the permutations $\sigma_1 = (t_1 t_2 t_3 t_4)^k$ and $\sigma_2 = (t_3 t_4 t_1 t_2)^k$, where k is an arbitrary positive integer, are indistinguishable from each other. As it was shown in the previous examples, the five conditions are independent from each other. In fact the 2^5 combinations are possible.

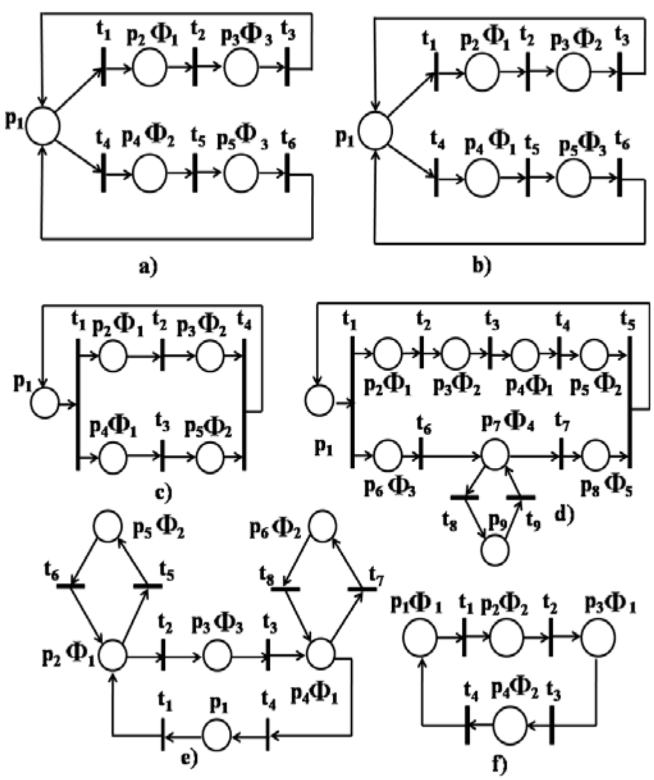

Fig. 3. This figure illustrates the conditions of Theorem 15.

V. ALGORITHMS

The previous section characterizes IPN exhibiting the structural sequence detectability property. Now, this section presents algorithms to test if a Free Choice IPN exhibits this property. As stated in Theorem 15, structural sequence detectable IPN do not exhibit transitions $\approx^+$, $\approx^-$ and $\approx_p$ related. Testing if IPN transitions are $\approx^+$ or $\approx^-$ related is a straightforward task. It consists of locating attribution places (places p_i such that $|{}^\bullet p_i| > 1$) or distribution places (places p_i such that $|p_i^\bullet| > 1$) and testing if their input or output transitions are indistinguishable from each other. Relationship $\approx_p$ is tested in the following way. For any $(t_i, t_j) \in Gr(\approx_I)$, compute the existence of a minimal $P-invariant$ Y such that $Y(p_i) = 1$ for any $p_i \in {}^\bullet t_i$ and $Y(p_j) = 1$ for any $p_j \in {}^\bullet t_j$, i.e. if the input places to both transitions belong to the same minimal $P-invariant$. If

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so, then both transitions cannot fire concurrently, thus $t_i \not\approx_p t_j$, else $t_i \approx_p t_j$.

In order to test if there exists a $(t_i, t_j) \in Gr(\approx_I)$ such that t_i, t_j are evolving in concurrence with $T - invariants$ we propose the following algorithm.

Algorithm 16. *It computes if there exists $(t_i, t_j) \in Gr(\approx_I)$ with t_i, t_j evolving in concurrence with $T - invariants$*

Input: A Free Choice IPN $Q = \{P, T, \Phi, W\}$

Output: A $(t_i, t_j) \in Gr(\approx_I)$ such that t_i, t_j are evolving in concurrence with $T - invariants$ or empty if such (t_i, t_j) does not exist.

Begin

1) *Compute the set $P_{FJ} = \{(p_i, t_F, t_J) | \ p_i$ is residing inside of a fork-join transition pair $t_F, t_J, |p_i^\bullet| > 1\}$.*
2) *Compute the set $P_{FJ}^{X_a} = \{(p_i, t_F, t_J, X_a) | (p_i, t_F, t_J) \in P_{FJ}$ and there exists a $T - invariant$ X_a such that $X_a(t_F) = X_a(t_J) = 0$ and $X_a(t_k) = 1$, where $t_k \in {}^\bullet p_i\}$ i.e. there exists a $T - invariant$ X_a that is also residing inside the fork-join transition pair t_F, t_J.*
3) *Compute if there exist two indistinguishable transitions that evolve in concurrence with $T - invariants$.*
4) *Return the t_i, t_j evolving in concurrence with $T - invariants$ or empty if such t_i, t_j do not exist.*

End

In order to perform step 3 of the previous algorithm we have the following facts. Notice that every $(p_i, t_F, t_J, X_a) \in P_{FJ}^{X_a}$ contains the $T - invariant$ X_a and distribution place p_i that are residing inside the fork-join transition pair (t_F, t_J). When p_i is marked the $T - invariant$ X_a is enabled. The indistinguishable transitions t_i, t_j, $(t_i, t_j) \in Gr(\approx_I)$ evolve in concurrence with X_a if the places $p_a \in {}^\bullet t_i$, $p_b \in {}^\bullet t_j$ can be marked simultaneously with the distribution place p_i, and p_i lies in a different minimal $P - invariant$ from those minimal $P - invariants$ containing p_a and p_b.

The next linear programming problems ($LPP's$) find out if this p_i lies in a different $P - invariant$ from those containing the input places p_a and p_b. Notice that the computation of a minimal $P - invariant$ containing p_i and not containing p_a and p_b implies that $Y^T C = 0$, $Y \geq 0$, $Y(p_i) \geq 1$ and $Y(p_a) = Y(p_b) = 0$ for those input places p_a and p_b to t_i and t_j. Notice that the $LPP's$ find out rational vectors Y in the left kernel of the incidence matrix, so they are no $P - invariants$ since their entries are not non negative integers. Fortunately, the existence of these rational vectors Y implies the existence of $P - invariants$ (since Y is rational valued vector, then it can be multiplied by an appropriate integer value and the $P - invariant$ is obtained). Thus, abusing of the language, we call these vectors Y as $P - invariants$. Computing the existence of the minimal $P - invariant$ is performed in three steps. First a $P - invariant$ Y_1 containing place p_i and the input places $p_a \in {}^\bullet t_i$ is computed. Afterwards a $P - invariant$ Y_2 containing place p_i and the input places $p_b \in {}^\bullet t_j$ is computed. If both p_i and p_a (p_i and p_b) belong to a minimal $P - invariant$, then Y_1 (Y_2) is a rational representation of this minimal $P - invariant$, otherwise it is a linear combination of the minimal $P - invariants$ (those containing p_i and p_a or p_i and p_b).

The $P - invariant$ $Y_G = Y_1 + Y_2$ is computed. Notice that Y_G is a linear combination of $P - invariants$ (hence Y_G is not minimal) and Y_G satisfies that it contains place p_i and the places $p_a \in {}^\bullet t_i$, $p_b \in {}^\bullet t_j$. Then a new $P - invariant$ Y_3, included in Y_G, containing the places $p_a \in {}^\bullet t_i$, $p_b \in {}^\bullet t_j$ where $Y_3(p_i) = 0$ is computed. If such Y_3 is found then t_i and t_j evolve in concurrence with $T - invariants$. These facts are considered in the following $LPP's$.

For each $(p_i, t_F, t_J, X_a) \in P_{FJ}^{X_a}$ do
select p_i,
For each $(t_i, t_j) \in Gr(\approx_I)$ do
select t_i, t_j

Min $\sum_{i=1}^{\|P\|} Y_1(i)$	Min $\sum_{i=1}^{\|P\|} Y_2(i)$
s.t.	s.t.
$Y_1^T C = 0$	$Y_2^T C = 0$
$Y_1(p_i) \geq 1$	$Y_2(p_i) \geq 1$
$Y_1(p_k) \geq 0$, $p_k \in P - \{p_i\}$	$Y_2(p_k) \geq 0$, $p_k \in P - \{p_i\}$
$\sum Y_1(p_a) \geq 1$, $p_a \in {}^\bullet t_i$	$\sum Y_2(p_b) \geq 1$, $p_b \in {}^\bullet t_j$

end for
end for

If Y_1 or Y_2 is not empty, then there exist minimal $P - invariants$ containing places p_i, $p_a \in {}^\bullet t_i$, $p_b \in {}^\bullet t_j$ or linear combinations of $P - invariants$ containing p_i, $p_a \in {}^\bullet t_i$, $p_b \in {}^\bullet t_j$. The next linear programming problem determines what is the case. If Y_1 and Y_2 are empty (both problems have no solution), then there are not $P - invariants$ containing places p_i, ${}^\bullet t_i$, ${}^\bullet t_j$.

If both problems have a solution, then compute $Y_G = Y_1 + Y_2$ and

Min $\sum_{i=1}^{\|P\|} Y_3(i)$
s.t.
$Y_3^T C = 0$
$Y_3(p_i) = 0$, p_i is the selected distribution place
$Y_3(p_k) \geq 0$, p_k is any place of P different from p_i
$\sum Y_3(p_a) \geq 1$, $p_a \in {}^\bullet t_i$
$\sum Y_3(p_b) \geq 1$, $p_b \in {}^\bullet t_j$
$Y_3(i) \leq Y_G(i)$

If there exists Y_3, then place p_i is in a different minimal $P - invariant$ from those containing $p_a \in {}^\bullet t_i$, $p_b \in {}^\bullet t_j$. If Y_1 and Y_2 are empty or there exists Y_3 then there exists a $(t_i, t_j) \in Gr(\approx_I)$ such that t_i, t_j are evolving in concurrence with $T - invariants$, else there exists no such $(t_i, t_j) \in Gr(\approx_I)$ evolving in concurrence with $T - invariants$.

Proposition 17. *Given an IPN of the FC class, the existence of a $(t_i, t_j) \in Gr(\approx_I)$ such that t_i, t_j are evolving in concurrence with $T - invariants$ can be determined using Algorithm 16.*

Proof. In live and bounded (safe in this case) Free Choice IPN two places are marked in the same marking iff they belong to different $P - invariants$ [4]. According to the $S - coverability$ and $T - coverability$ theorems [4], the different $P - invariants$ are generated by the transitions t_a such that $|t_a^\bullet| > 1$ (i.e. the fork transitions). Thus, if two places are going to be marked simultaneously, they must be in different downstream paths from a fork transition. $T - coverability$ theorems ensure that the downstream paths from a fork transition are joined by a transition t_b such that $|{}^\bullet t_b| > 1$ and the fork-join transition pair is formed. Thus, the algorithm computes the distribution places $|p_i^\bullet| > 1$ residing inside a fork-join transition pair. Distribution places have more than one output transitions, thus they are the only places candidate to generate $T - invariants$ inside of a fork-join transition pair (other places have only one output transition and their $T - invariants$ must include the join transition). Thus the algorithm searches for places p_i such that $|p_i^\bullet| > 1$ and at least one output transition of this place is included in a $T - invariant$ not containing the join transition. The set of such places p_i is the computed set $P_{FJ}^{X_a}$ in step 2). Now, $T - invariants$ X_a can be fired in concurrence with transitions in different downstream paths from the fork transition t_F. Now, the transitions in different downstream paths from the fork transition t_F must include indistinguishable transitions t_i, t_j to ensure that the Free Choice IPN is not SSD, otherwise it is SSD. Thus the algorithm computes (using the two linear programming problems) if there is a minimal $P-invariant$ containing p_i, $^\bullet t_i$, $^\bullet t_j$ or the addition of some minimal disjoint $P-invariants$ containing p_i, $^\bullet t_i$, $^\bullet t_j$. If no such $P-invariants$ are found then there exists a $(t_i, t_j) \in Gr(\approx_I)$ such that t_i, t_j are evolving in concurrence with $T-invariants$. If such $P-invariant$ is found, then the third linear programming problem uses Y_G to determine if places p_i, $^\bullet t_i$, $^\bullet t_j$ are in different minimal $P-invariants$ not sharing places, if such $P-invariant$ exists then places p_i, $^\bullet t_i$, $^\bullet t_j$ are in different minimal $P-invariants$ not sharing places. □

Previous $LPP's$ can be solved using the Simplex algorithm, it has not polynomial complexity but in almost all cases performs very fast.

Algorithm 18. *It determines the existence of an indistinguishable $T-component$ with respect to itself*

Input: A Free Choice IPN $Q = \{P, T, \Phi, W\}$

Output: If there exist an indistinguishable $T-component$ with respect to itself.

1) *Compute the set $T_{\approx_I}$, where $T_{\approx_I}$ represents the domain of relation $\approx_I$ (i.e. the set of indistinguishable transitions).*
2) *Compute a $T-component$ $\mathcal{T}_i = \{(P_i, T_i, W_i)\}$, where $T_i \subseteq T_{\approx_I}$.*
3) *Using Nerode's relationship (see [8]) verify if each transition of $\mathcal{T}_i$ can bisimulate another one, i.e. any pair of states reached after a given string of transitions should have the same future behavior in terms of a post-language of transitions.*
4) *If each transition is bisimulable then $\mathcal{T}_i \approx_I \mathcal{T}_i$, else $\mathcal{T}_i \not\approx_I \mathcal{T}_i$.*

Algorithm 19. *Determine the existence of indistinguishable $T-components$.*

Input: A Free Choice IPN $Q = \{P, T, \Phi, W\}$

Output: If there exist two indistinguishable $T-components$ from each other.

1) *Compute the set $T_{\approx_I}$.*
2) *Compute a $T-component$ $\mathcal{T}_i = \{(P_i, T_i, W_i)\}$, such that $\mathcal{T}_i \subseteq T_{\approx_I}$.*
3) *Compute a $T-component$ $\mathcal{T}_j = \{(P_j, T_j, W_j)\}$, such that $\mathcal{T}_j \subseteq T_{\approx_I}$ and $X_i \neq X_j$ (the $T-invariants$ that generate the $T-components$ are different.)*
4) *Using Nerode's relationship verify if each transition of the $T-component$ $\mathcal{T}_i$ can bisimulate another one of the $T-component$ $\mathcal{T}_j$.*
5) *If each transition of the $T-component$ $\mathcal{T}_i$ bisimulate a transition of the $T-component$ $\mathcal{T}_j$ then $\mathcal{T}_i \approx_I \mathcal{T}_j$, else $\mathcal{T}_i \not\approx_I \mathcal{T}_j$.*

The complexity of both algorithms, for testing condition $5)$ of Theorem 15, is NP. Nevertheless, the performance of these algorithms can be improved as follows. In Algorithm 18 the number of tested $T-invariants$ is reduced by only testing $T-invariants$ X_i where the greatest common divisor of the vector's entries $\Phi Post X_i$ is greater than one, where $Post(i,j) = W(t_i, p_j)$. In the Algorithm 19 the number of tested $T-invariants$ is reduced by adding the constraint that the two $T-invariants$ X_i and X_j must generate the same output information (i.e. they have the same natural projection). It is important to remark that there exist polynomial algorithms [2] that only test a sufficient condition for the non existence of indistinguishable $T-components$.

VI. Conclusions

This paper characterized the structural sequence detectability property in DES modeled by live, safe and pure Free Choice IPN. It has been shown that structural sequence detectability property can be characterized using the IPN structure, instead of enumerating all the firing transition sequences. These results are useful to enlarge the class of observable and diagnosable $IPN's$.

Currently we are working on extending our results in several ways. First, we are extending the structural sequence detectability characterization to more complex IPN classes. Also, we are trying to relax some work hypothesis in order to cover a broader set of nets. Furthermore, the same proposed $IPN's$ structures are being used to deal with the marking reconstruction characterization.

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