Chapter 3
Testing the Capital Asset Pricing Model using the Kalman Filter: Empirical Evidence from the Mexican Stock Market

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Abstract

The Capital Asset Pricing Model (CAPM) widely used for the valuation of financial assets may have periods of low explanation (low R-square). For those periods, the factor models have a low confidence. The Kalman filter is able to sort out the noise that often have the data, such as the high volatility of the time series in financial markets. This chapter presents empirical evidence of CAPM model calculation using the Kalman filter from the Mexican financial market data.

Keywords: CAPM; Kalman Filter; Factor Model; Asset Pricing.

1. Introduction

The Capital Asset Pricing Model (CAPM) developed by Sharpe (1964) and Lintner (1965) has been widely used to explain the portfolio behavior of financial assets. It can be helpful to assess whether a financial asset is undervalued or overvalued regarding market behavior and

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also, it can estimate future returns. The aforementioned helps to assign returns to evaluate investment projects or simply to evaluate the financial asset's performance.

In the literature, the emergence of new models to valuate financial assets cannot fail to mention the CAPM given its importance in the financial industry. Some examples seeking to improve the CAPM results by means of other factors, are those carried out by Fama & French (1996), who used portfolios with three factors, obtaining slightly higher results to explain the financial assets behavior[2]. Later, Carhart (1997) incorporated another factor to the previous model, composed of portfolios of gainers and losers stocks, yielding slightly better results. Bornholt (2007) incorporates another factor to CAPM related to risk premiums from financial asset and market[3], without obtaining significant results in comparison to the three or four model factors (Kristjanpoller & Liberona, 2010). There are studies seeking to improve the CAPM model efficiency without adding other factors. Fama & MacBeth (1973) proposed a two-step regression to improve the coefficients and so the residuals of the model[4]. Berglund & Knif (1999) stated that small businesses presenting little diversification in their financial assets may show changing periods in the leverage level and consequently changes in the value of its beta. This beta variability has been already documented in the literature (Jogannathan & Wang, 1996), and the most common approach is to estimate the beta using monthly data in periods of 5 years[5]. Another method is the one used by Virk & Butt (2016), "*the generalized method of moments*" *(GMM)* to allow variability in beta, the foregoing based on the two-steps regression of Fama & MacBeth (1973). Using Value Weighted Portfolios (VWP) and Equally Weighted Portfolios (EWP), the CAPM models, Fama & French (1996) and Carhart (1997) were compared, giving as a result the Carhart model as the best for both (EWP y VWP).

2. Coefficient of determination close to 30% (R-square)
3. Factor: reward beta.
4. Ver Shanken & Zhou (2007) where the two-step model of Fama & MacBeth (1973) is analyzed.
5. Berglund & Knif (1999) present a summary of the authors who study the variability of beta.

On the other hand, there are other studies where the Kalman (1960) is used to explain the behavior of different financial assets, since it has the special feature of filtering the noise of a non-linear series. For example, the CAPM is used by the Kalman filter to study the dynamics of systematic risk measured by beta coefficient (Eduardo Ortas ; José M. Moneva ; Manuel Salvador, 2010). Javaheri, Lautier, & Galli (2003) apply the Kalman filter to temporal structure models of interest rates and stochastic volatility. Dablemont, Van Bellengem, & Verleysen (2007) seek to determine when the price of a financial asset exceeds a target price (rise / fall in the price) through a neuronal model.

The present study follows the procedure used by Berglund & Knif (1999) through which the Kalman filter and the CAPM model capture the variability in beta. Unlike Berglund & Knif (1999), where rolling windows [6] were applied using weekly data. Also, the CAPM regression models are compared using different combination of procedures with the Kalman (1960) and Fama & MacBeth (1973) in the Mexican stock market. This procedure helps to identify where the CAPM has a greater certainty to obtain a higher coefficient of determination.

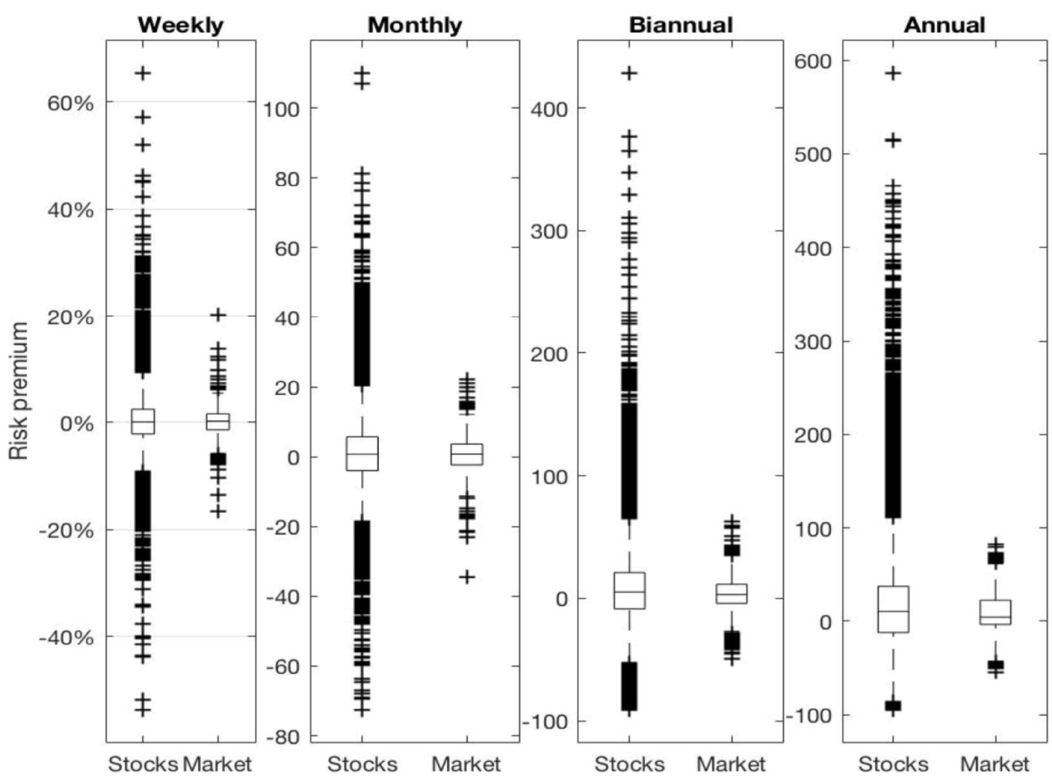
2. Data

For this study, weekly data was computed from January 1996 to June 2016; over 60 financial assets were located in low, medium and high rates of capitalization in Mexico. Under the assumption that the financial assets have the largest trading volume in its category in order to avoid problems of abrupt increases in prices in the absence of observations in certain periods. Not all financial assets are traded from 1996, since it depends on the procedure to be used (see next section); some financial assets are excluded by not having enough observations[7].

6. The returns are into overlapping cycles. For example, if the variability of returns is monthly, the first return is expected from January 1st to February 1st, the second return is estimated from 2nd day of January to 2nd day of February, and so on.
7. 45 out of 60 are active. In addition, CETES were used as the risk-free rate from 2003 (maximum records). Banco de México (2016). Economic information system. August 1st, 2016, Bank of México, Website: http://www.banxico.org.mx/

In Fig. 1, risk premium from financial assets and market index (IPC) can be observed. In order to increase the amount of observations, taking into consideration the prices between each yield (risk premium), rolling window is used, based on weekly prices. For the following yield periods: weekly, monthly, biannual and annual, it can be observed a higher volatility in premium stocks (risk) in comparison to the market premium

Figure 1
Boxplot risk premiums (using rolling windows): 2003-2016



Using the Kolmogorov-Smirnov test in order to prove the normality hypothesis in each of the risk premiums groups, the hypothesis stating the data have a normal standard distribution with a significance level of 5% is accepted (see Fig. 2 and 3).

Figure 2
Histograms of risk premium of financial assets
(using rolling windows): 2003-2016

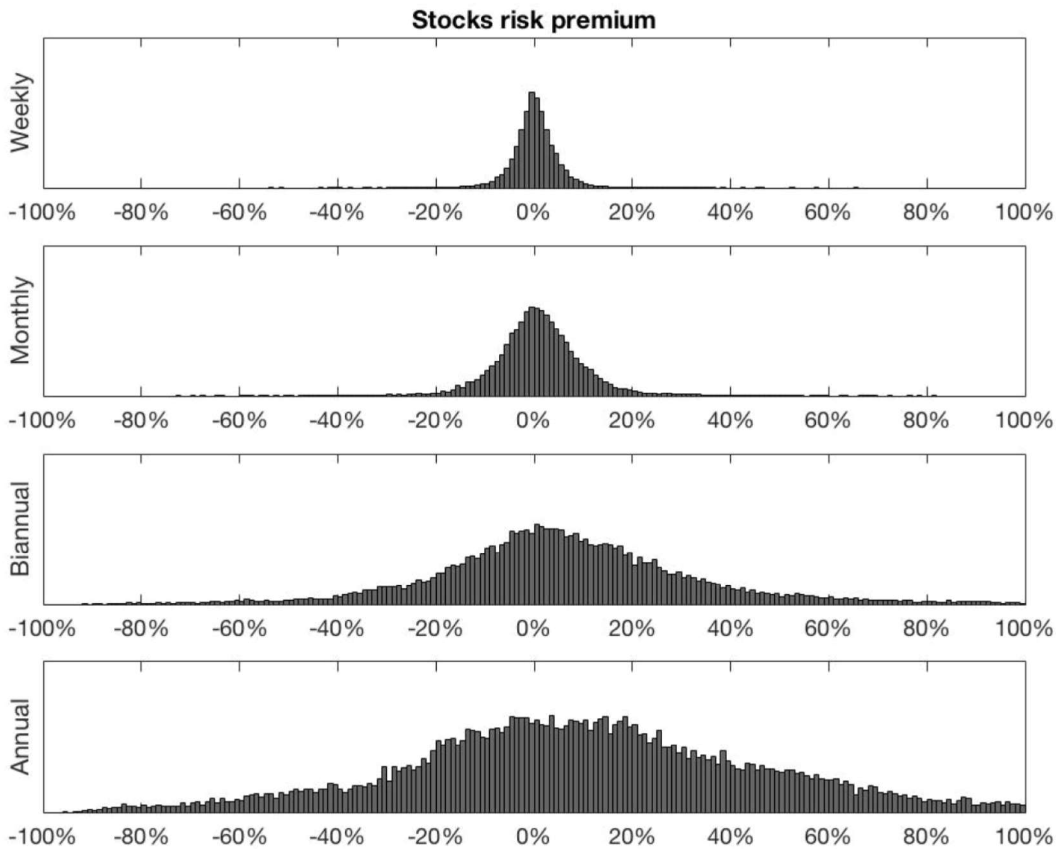
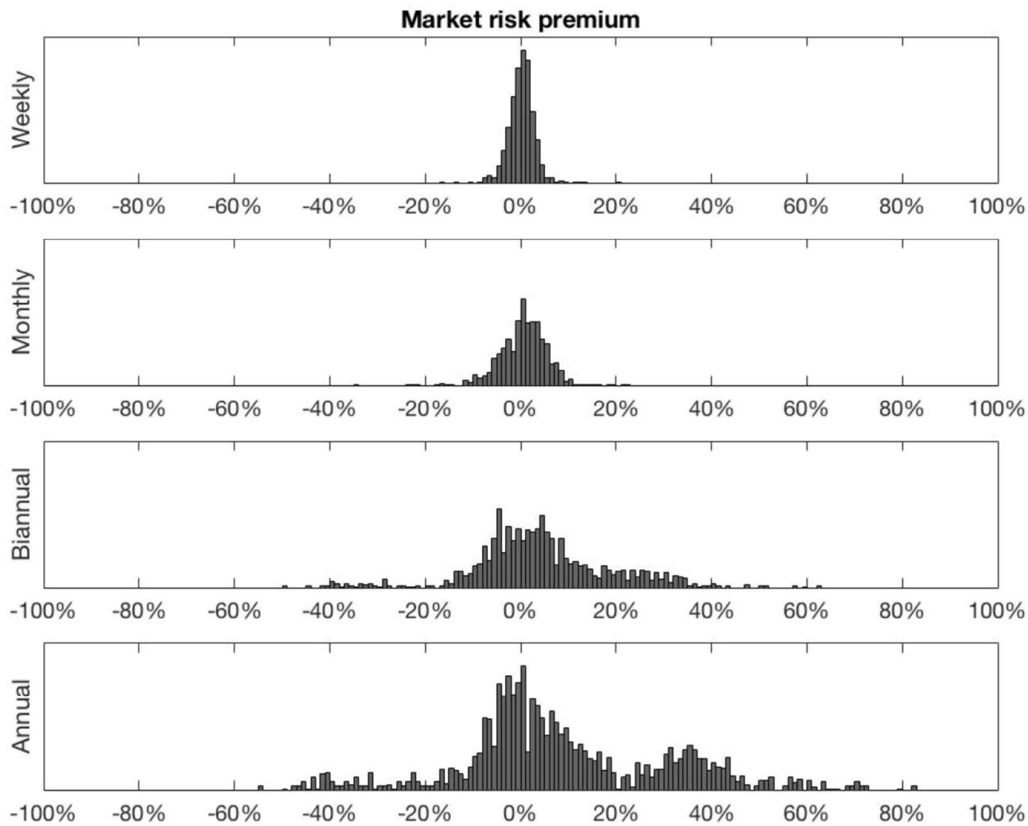


Figure 3
Histograms of risk premium of IPC
(using rolling windows): 2003-2016



3. Methodology

The objective of this methodology is to select the procedure where you can get greater certainty in obtaining a higher R-square in the CAPM, among the proposed methods within the time frame of this study. For which different periods of observations are used (rolling window of 1, 3 y 5 years) with different yields to one week, one month, six months and one year[8]. The higher the term, the lower volatility would be expected in yields, therefore a lower volatility in β, by increasing the R-square. The observation periods and the different yields are combined with

8. 52 weeks were taken for one year, 156 weeks for 3 years and 260 weeks for 5 years.

Kalman's filter and the Fama and MacBeth's procedures in order to identify the best procedure to follow. As it was mentioned in the introduction the most common practice is to estimate the regression model using monthly data over a 5-year period. The following is an introduction to the CAPM and the procedures used for calculation.

3.1 CAPM

The Capital Asset Pricing Model associates the risk premium of financial assets $(r_i - r_f)$ with market risk premium $(r_m - r_f)$, see equation (1). It would be expected to obtain an equal to zero or significantly different from zero, if the model explains the movement of a risk premium of a financial asset with only a factor β.

$$(r_i - r_f) = \alpha + \beta(r_m - r_f) + \varepsilon. \tag{1}$$

In case of obtaining a positive α, the financial asset has an abnormal yield average greater than the market yield. The term ε represents the model error and it is considered white homoscedastic noise; which will attempt to be reduced, in certain degree, from the series using the Kalman filter.

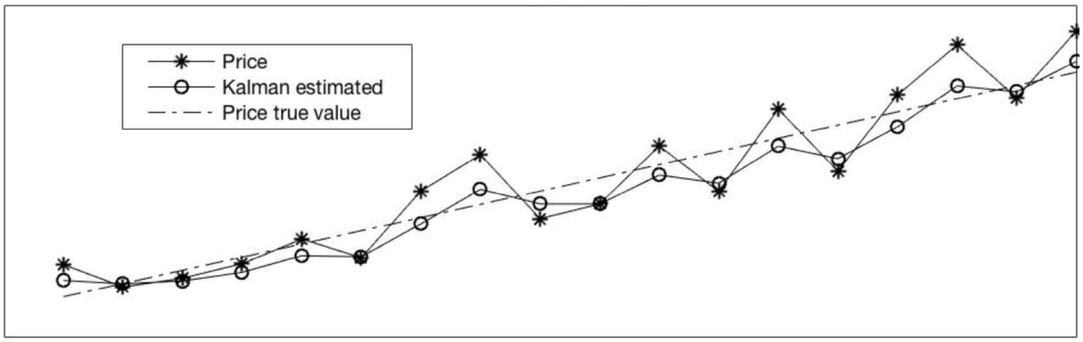
Should the investment period or another investment horizon chosen to evaluate performance of a financial asset is one year, it would be desirable to know the relationship between the market and the annual yields of the asset. The outcome would be the period of time of the yields to use.

3.2 The Kalman filter

The general idea of the Kalman filter is to remove the white noise in order to get a better approximation of the true value price from the series value. For instance, in Figure 4, it notes the true value of an asset price, the market price and the estimated price using the Kalman filter. It is expected that the latter are moved around the true value; Figure 4 shows the closest price to the true value price, which was obtained with the Kalman filter. Although in reality there is no true value.

However, a lower volatility would be expected in the price obtained through the Kalman filter in comparison with the market price.

Figure 4
Kalman filter estimated



The Kalman filter is a recursive algorithm of two states which is used for linear systems or by the extended Kalman filter and, unscented Kalman filter for nonlinear systems. These last two are an extension of the first. See Bhar (2010) for more details. The method for linear systems follows Javaheri et al. (2003) and it is derived as follows:

In order to calculate the price with Kalman's filter ($\hat{x}_t$, measurement domain), the algorithm performs two states: prediction and correction. From a dynamic process $\hat{x}_t$ where t is the time-step apply the following transition equation:

$$\hat{x}_t = f(\hat{x}_t, w_t).$$

For which, in our case there is a price Z_i with white noise:

$$Z_i = f(Z_{t-1}, u_t).$$

Where w and u are mutually uncorrelated, they also have a normal distribution with zero means and covariance R for the price with the Kalman's filter applied and Q for the financial asset price.

The prediction state equations

An initial value is assumed for $\hat{P}_{t-1}$, $\hat{x}_{t-1}$, R and Q on the following equations. $\bar{x}_t$ is the prediction price through the Kalman's filter which is corrected in second state.

$$\bar{\boldsymbol{x}}_{\boldsymbol{t}} = \hat{\boldsymbol{x}}_{\boldsymbol{t-1}},$$

$$\bar{P}_t = \hat{P}_{t-1} + Q,$$

where $\hat{P}_{t-1}$ is the covariance of $(\bar{x} - \hat{x})$.

The correction state equations.

$$k_t = \frac{\bar{P}_t}{\bar{P}_t + R},$$

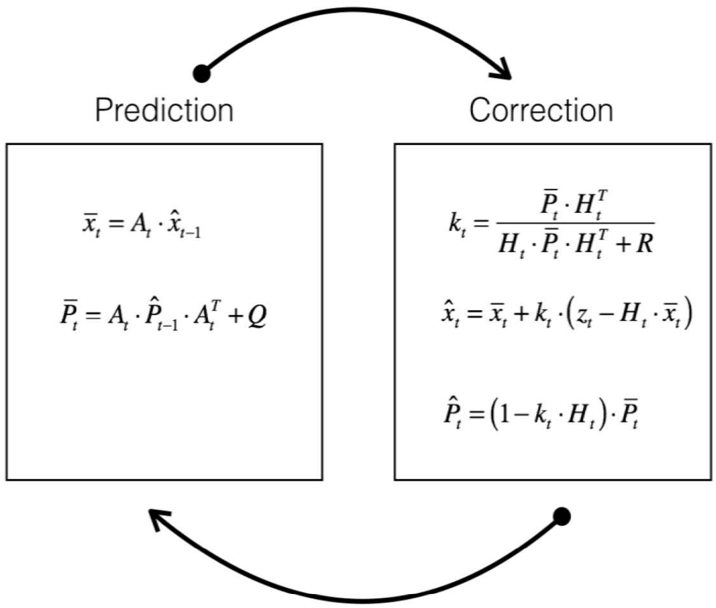
k_t is the Kalman gain which influences over the difference between the measurement minus the measurement domain $(Z_t - \bar{x}_t)$ or correction term.

$$\hat{\boldsymbol{x}}_{\boldsymbol{t}} = \bar{\boldsymbol{x}}_{\boldsymbol{t}} + \boldsymbol{k}_{\boldsymbol{t}} \cdot (\boldsymbol{Z}_{\boldsymbol{t}} - \bar{\boldsymbol{x}}_{\boldsymbol{t}}),$$

$$\hat{P}_t = (1 - k_t) \cdot \bar{P}_t.$$

In this study, first-order differential equations are combined. Differential equations relate an independent variable with its derivatives. For instance, equations relating time, speed, acceleration and distance, when looking for a vehicle position where Z_t is a vector of measurement (Faragher, 2012). In case these kind of equations exist, then it will be a Jacobian matrix or state transition matrix that maps the state vector parameters on the prediction state (A_t) and another one on the correction state (H_t); having as a result the algorithm in Figure 5. Now R is the measurement domain noise and Q the system noise covariance matrices.

Figure 5
Kalman filter states



3.3 *Two steps regression, Fama & MacBeth* (1973)

The procedure divides the sample into two steps, the first step consists of temporal series regressions where α and β factors are estimated, the same quantity of observations is taken into consideration based on what the data window indicates. For example, if the data window corresponds to 52 weeks, then 52 weeks will be considered for the first step and the following 52 weeks for the second step. The second step consists of cross-sectional regressions using estimated values based on the factors calculated in first step.

4. Results

Three types of procedures for estimating the CAPM model were performed.

1. Coefficients' estimation by means of ordinary least squares (OLS).

2. Closing prices are modified by the Kalman's filter before performing the coefficients' estimation using the first procedure.
3. The two steps regression from Fama & MacBeth's is performed.

For each procedure, four types of yields are used (one week, one month, six months and one year). Table 1 exhibits that the procedures with annual yields show the highest average R-square. Similarly, for the annual yields, in the longer-term window (5 years) the highest R-square can be appreciated, when following procedure 1 and 2. These findings match with researches revised, where it is suggested the usage of a 5-year window in order to calculate the CAPM. In the other hand, those studies do not agree with monthly yield usage, but the annual yield usage. In Mexico´s case, when having annual yields, the volatility in performance is greatly reduced. Therefore, this information reinforces the model. Table 1 shows the relation between the yield and the model strengthening; the higher the yield, the greater the model reinforcement.

It is important to note the procedure type influences the outcome. In this case procedure 1 shows to offer the best R-square.

Table 1
Results of the different procedures

	Window (weeks)	*CAPM*		*CAPM-Kalman filter*		*CAPM-FAMA-MacBeth*	
		Average R-square	*Standard deviation*	*Average R-square*	*Standard deviation*	*Average R-square*	*Standard deviation*
Weekly returns	52	0.283	0.175	0.043	0.049	0.289	0.176
	156	0.298	0.159	0.035	0.033	0.303	0.162
	260	0.322	0.152	0.036	0.030	0.298	0.130
Monthly returns	52	0.315	0.199	0.220	0.157	0.315	0.200
	156	0.328	0.176	0.232	0.136	0.317	0.177
	260	0.356	0.158	0.252	0.124	0.290	0.137
Biannual returns	52	0.428	0.296	0.416	0.293	0.423	0.299
	156	0.415	0.271	0.406	0.270	0.380	0.279
	260	0.451	0.243	0.442	0.242	0.304	0.219
Annual returns	52	0.441	0.304	0.422	0.307	0.446	0.308
	156	0.457	0.301	0.451	0.299	0.423	0.300
	260	0.480	0.261	0.471	0.259	0.314	0.234

Note: returns means risk premium.

5. Conclusion

Three types of procedures were compared when looking to increase the R-square using the CAPM model for calculation. It may be noted that using a 5-year window and average annual yields it is possible to obtain an R-square of 0.48 by OLS, which are findings in accordance with other available studies. The usage of the Kalman's filter provides similar results, since in average you can have a 0.47 R-square. The best result employing the two-step regression of Fama and MacBeth is using annual yields and a one year window, which is 0.446. Although there is a significant difference between using less than a month yields or less than one-year window, a robust model cannot be achieved. The suggestion for future research is the study of the general additive model (GAM), maximum likelihood (ML), generalized method of moments, generalized least squares (GLS) usage, as a regression method in combination with particle systems (Dablemont et al., 2007; Shanken & Zhou, 2007).

Glossary of Terms

Financial Asset is a financial instrument which derives an assigned value. Stocks, bonds, bank

deposits, etc. are all examples of financial assets. This value assigned is different to real assets.

Performance is an indicator that evaluates certain variables which are relevant to who evaluates certain phenomenon. Omega ratio, Sharpe ratio and Jensen's alpha are examples of performance indicators.

Risk Premium is the return over the risk-free rate of return that an investment expects to obtain as a result of an exchange associated to certain additional risk

Return describes the yield earned during a certain period of time in the past. Return and yield are terms frequently used to describe the performance of and investment.

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