The Aggressive Space Mapping (ASM) Algorithm: Over Two Decades of Development and Engineering Applications

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This article presents a historical account and technical reassessment of the most widely used space mapping (SM) approach to efficient design optimization: the Aggressive Space Mapping (ASM) algorithm. It starts from the invention of the space mapping concept, and briefly makes an overview of the most fundamental SM optimization methods developed until now, in which ASM is framed. It reviews over two decades of ASM evolution, in terms of not only the theoretical contributions directly incorporated into the ASM algorithm, but also in terms of its most significant engineering applications publically documented. Clearly, ASM is not the most powerful and advanced space mapping design optimization approach invented until now. However, the historical evidence proves it is the most widely adopted SM optimization method, both in academia and industry. In the author's opinion, two main characteristics have made of ASM the most popular SM optimization technique: 1) it is simple, and 2) it is very efficient (when it works, it works extremely well). For those reasons, this article also revisits the ASM algorithm, emphasizing key steps for its successful implementation, as well as typical scenarios where ASM may fail. Finally, some future directions regarding ASM are ventured.

I. Overview on SM-based Optimization Methods

Prof. John Bandler invented the space mapping (SM) technique in 1994 [1]. A great description about how it was originated, along with intriguing analogies to human cognition and a qualitative illustration of the multiple faces of SM, was realized in 2013 by its inventor [2]. Excellent technical reviews on general SM methods for modeling and design optimization are found in [3] and [4], made in 2004 and 2008, respectively. A specific review on SM-based optimization exploiting artificial neural networks, made in 2004, is in [5]. Making an up-to-date review on general SM technologies seems to be now pertinent.

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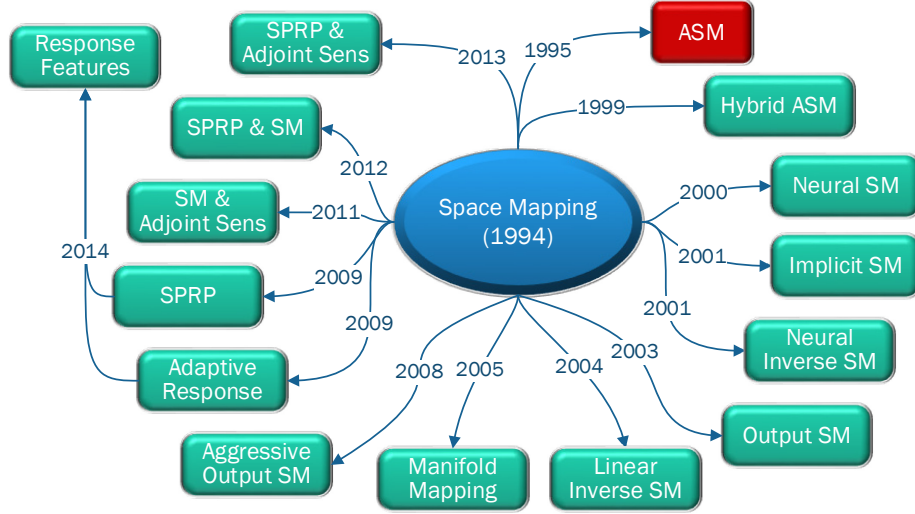
Fig. 1 Fundamental design optimization methods emerged from the SM concept: aggressive SM [6,7]; hybrid ASM [8,9]; neural SM [10-12]; implicit SM [13-17]; neural inverse SM [18,19]; output SM [20,21]; linear inverse SM [22-24]; manifold mapping [25-28]; aggressive output SM [29,30]; adaptive response correction (ARC) [31,32]; shape-preserving response prediction (SPRP) [33,34]; SM with adjoint sensitivities [35-37]; SPRP exploiting SM [38]; SPRP using adjoint sensitivities [39,40]; response features [41-44] (emerged from ARC and SPRP).

In spite of that, the present article does not aim at making a general review on SM. It is focused on the aggressive space mapping (ASM) approach to design optimization. Nevertheless, to place into proper context the ASM algorithm, Fig. 1 briefly illustrates, in a schematic manner, the most fundamental design optimization algorithms emerged from the SM concept.

SM optimization methods belong to the general class of surrogate-based optimization algorithms [45]. They are specialized on the efficient optimization of computationally expensive objective functions.

ASM emerged in 1995, almost two decades ago. Since then, many other design optimization algorithms have been proposed, as seen in Fig. 1. They aim at making SM optimization more general, more robust, and more efficient. Excepting perhaps implicit space mapping [16], most of them have a significantly higher complexity than ASM, making them more difficult to exploit by non-optimization experts. Any quick (or diligent) search in IEEE Xplore and other recognized digital libraries will confirm that the number of applications using more sophisticated SM design optimization methods is significantly smaller than that one using ASM.

II. Evolution of ASM Theory and Applications

The most significant theoretical contributions to aggressive space mapping, as well as the
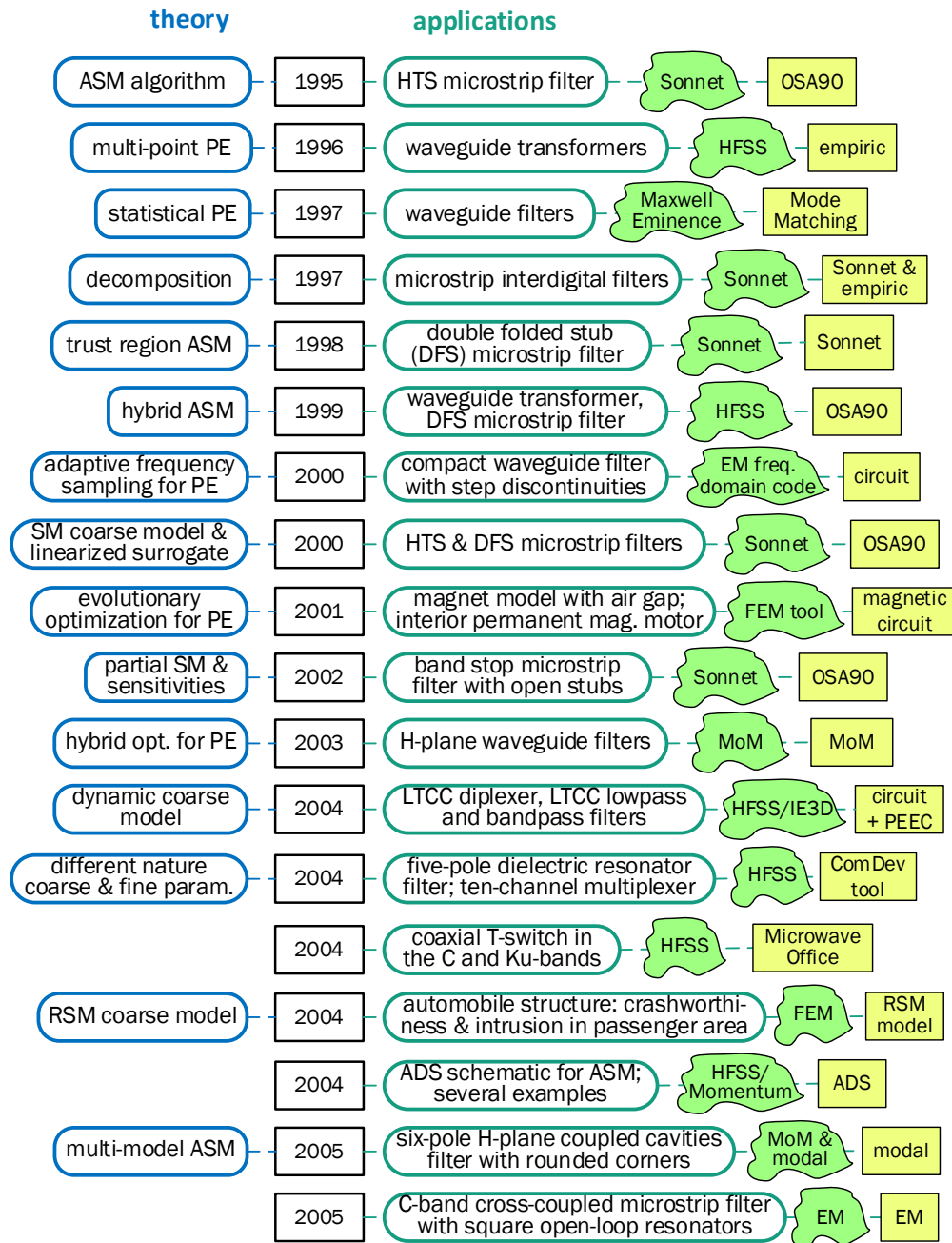
Fig. 2   First decade of evolution of Aggressive Space Mapping (ASM): key theoretical elements contributed to ASM and main applications to design optimization using ASM (indicating fine and coarse models employed).

main publically documented applications of ASM in engineering fields are highlighted in Figs. 2 and 3, indicating also the fine and coarse models utilized on each application case.
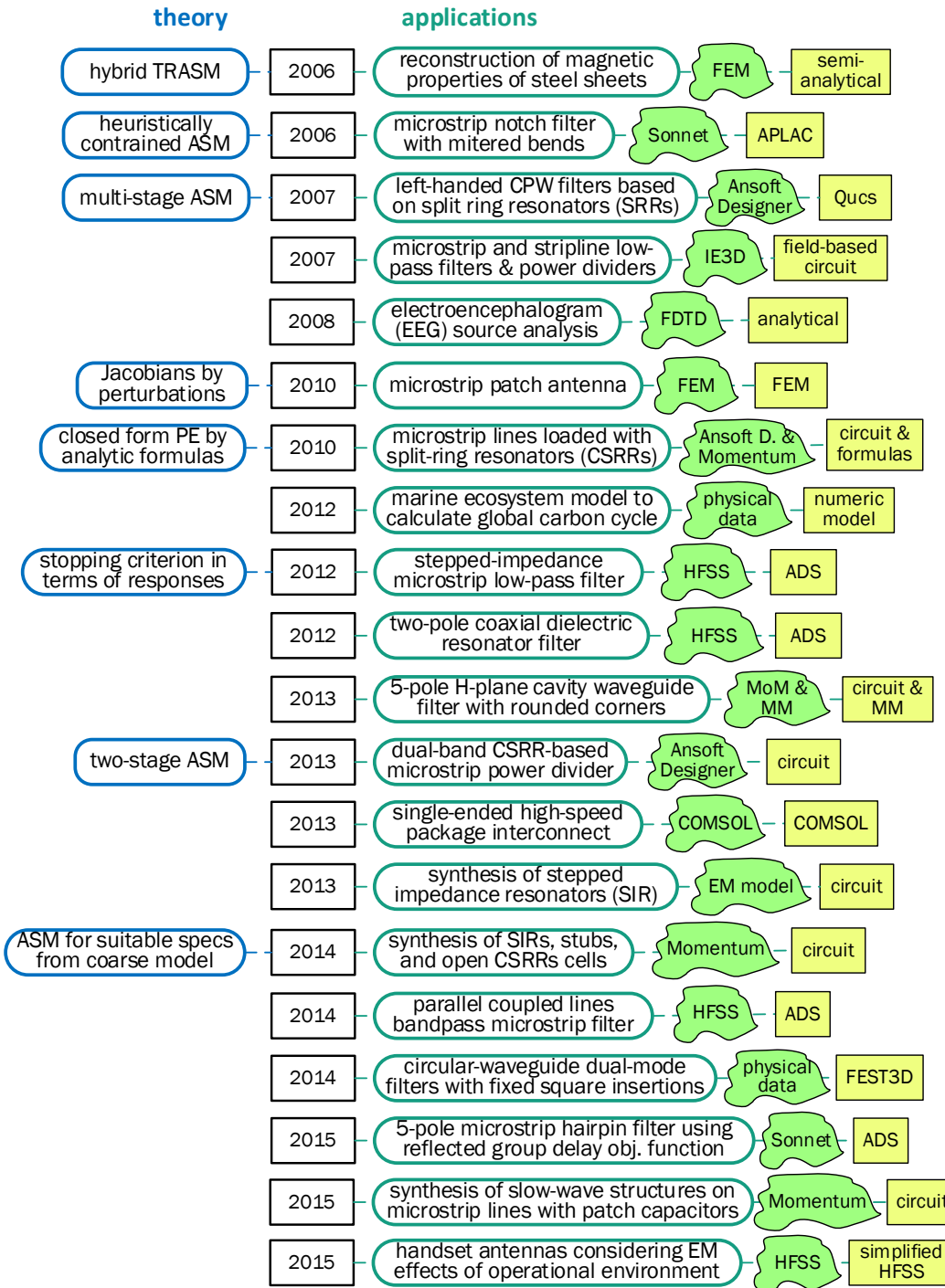
Aggressive Space Mapping (ASM) – Second Decade

theory   **applications**

theory	year	applications	fine model	coarse model
hybrid TRASM	2006	reconstruction of magnetic properties of steel sheets	FEM	semi-analytical
heuristically contrained ASM	2006	microstrip notch filter with mitered bends	Sonnet	APLAC
multi-stage ASM	2007	left-handed CPW filters based on split ring resonators (SRRs)	Ansoft Designer	Qucs
	2007	microstrip and stripline low-pass filters & power dividers	IE3D	field-based circuit
	2008	electroencephalogram (EEG) source analysis	FDTD	analytical
Jacobians by perturbations	2010	microstrip patch antenna	FEM	FEM
closed form PE by analytic formulas	2010	microstrip lines loaded with split-ring resonators (CSRRs)	Ansoft D. & Momentum	circuit & formulas
	2012	marine ecosystem model to calculate global carbon cycle	physical data	numeric model
stopping criterion in terms of responses	2012	stepped-impedance microstrip low-pass filter	HFSS	ADS
	2012	two-pole coaxial dielectric resonator filter	HFSS	ADS
	2013	5-pole H-plane cavity waveguide filter with rounded corners	MoM & MM	circuit & MM
two-stage ASM	2013	dual-band CSRR-based microstrip power divider	Ansoft Designer	circuit
	2013	single-ended high-speed package interconnect	COMSOL	COMSOL
	2013	synthesis of stepped impedance resonators (SIR)	EM model	circuit
ASM for suitable specs from coarse model	2014	synthesis of SIRs, stubs, and open CSRRs cells	Momentum	circuit
	2014	parallel coupled lines bandpass microstrip filter	HFSS	ADS
	2014	circular-waveguide dual-mode filters with fixed square insertions	physical data	FEST3D
	2015	5-pole microstrip hairpin filter using reflected group delay obj. function	Sonnet	ADS
	2015	synthesis of slow-wave structures on microstrip lines with patch capacitors	Momentum	circuit
	2015	handset antennas considering EM effects of operational environment	HFSS	simplified HFSS

Fig. 3   Second decade of evolution of Aggressive Space Mapping (ASM): key theoretical elements contributed to ASM and main applications to design optimization using ASM (indicating fine and coarse models used).

A more detailed description of these advances are summarized in Tables I and II, where

TABLE I

Historical Account of ASM – First Decade					
year	ref.	theoretical contributions to ASM	ASM applications	fine model	coarse model
1995	[6,7]	ASM algorithm invented. Huber norm for parameter extraction (PE). Linear frequency mapping with exact penalty functions for severe model misalignment.	High temperature superconductive (HTS) microstrip filter.	Sonnet	equivalent circuit in OSA90
1996	[46]	Multi-point PE to increase uniqueness of PE solution and to improve ASM convergence.	Waveguide transformers.	HFSS	empirical model
1997	[47,48]	Statistical approach to parameter extraction involving penalty concepts for PE uniqueness and consistency.	H-plane resonator waveguide filters with rounded corners.	Maxwell Eminence (Ansoft)	Mode-matching / equiv. circuit
1997	[49]	Structural decomposition to build accurate coarse model combining EM models with a coarse grid and empirical models for noncritical substructures.	Microstrip interdigital filter.	Sonnet	Sonnet with coarse grid / equiv. circuit
1998	[50,51]	Trust region ASM with multi-point PE. Non-conventional quasi-Newton step with an empirical parameter to ensure next candidate is within trust region.	Microstrip double-folded stub (DFS) filter.	Sonnet	Sonnet with coarse grid
1999	[8,9]	Trust region ASM combined with direct optimization. Lemma to calculate the fine model response Jacobian as a function of the coarse model response Jacobian and the Broyden matrix.	Waveguide transformer; microstrip double-folded stub filter.	HFSS	equivalent circuit in OSA90
2000	[52,53]	New surrogate: combination of a mapped coarse model with a linearized fine model. Next iterate accepted if improves objective function, otherwise surrogate is enhanced by additional fine data.	HTS filter; two-section impedance transformer; double-folded stub filter.	Sonnet	equivalent circuit in OSA90
2000	[54]	Sampling algorithm to minimize the number and automate the selection of fine model frequency points for ASM.	Low-pass compact rectangular waveguide filter with capacitive step discontinuities.	frequency domain EM code	equivalent circuit
2001	[55]	Evolutionary optimization method to perform PE (to extract global optimum at each ASM iteration).	Magnet model with air gap; interior permanent magnet motor.	FEM tool	magnetic equivalent circuit (MEC)
2002	[56,57]	Partial space mapping exploiting fine model exact sensitivities in PE and mapping update.	Bandstop microstrip filter with open stubs; two-section impedance transformer.	Sonnet	equivalent circuit in OSA90
2003	[58,59]	Geometrical segmentation to decrease design variables. Combination of optimization methods for coarse model optimization and for PE.	Tunable H-plane waveguide filters with tuning posts operating at 11 and 13 GHz.	Method of Moments (MoM)	MoM with small number of modes
2004	[60,61]	Dynamic coarse model: a combination of an evolutionary equivalent-circuit model and quasi-static EM PEEC model (highly accurate coarse model).	LTCC frequency-selective passive modules: LTCC diplexer; low-pass and band-pass LTCC filters.	HFSS and IE3D	equivalent-circuit and PEEC model
2004	[62]	Coarse and fine design parameters of different nature (coupling coefficients *vs* geometrical dimensions). Broyden matrix initialized by finite differences.	Double-terminated five-pole dielectric resonator filter; ten-channel manifold-coupled output multiplexer.	Ansoft HFSS	ComDev internal circuit analysis tool
2004	[63]	-	Four port electromechanical coaxial T-switch in the C and Ku-bands.	HFSS	Microwave Office
2004	[64]	Response surface methodologies (RSM) to develop coarse models for ASM design optimization.	Automobile structure optimized (crashworthiness and intrusion in the passenger compartment).	large industrial FEM model	RSM model
2004	[21,65]	-	ADS schematic for ASM: microstrip transformer; H-plane waveguide filter; interdigital microstrip filter.	HFSS and Momentum	equivalent circuit in ADS
2005	[66]	ASM with multiple models of increasing accuracy: coarse model is the fastest, while the finest model is used in the last iteration (gradual mapping to avoid divergence).	Six-pole H-plane coupled cavities filter with rounded corners in the coupling windows due to die casting fabrication.	MoM combined with modal techniques	modal techniques
2005	[67]	-	C-band cross-coupled bandpass microstrip filter using square open-loop resonators.	full-wave EM model	coarsely discretized EM model

the corresponding references are also provided. A number of interesting observations can be inferred from these two tables:

TABLE II

year	ref.	theoretical contributions to ASM	ASM applications	fine model	coarse model
		Historical Account of ASM – Second Decade			
2006	[68]	Trust region ASM combined with direct optimization; similar to [8],[9].	Reconstruction of magnetic properties of steel sheets by needle probe	FEM model	semi-analytical model
2006	[69,70]	Heuristically constrained ASM: if next candidate falls outside predefined limits, step size is decreased in same quasi-Newton direction (using empirical shrinking factor).	Microstrip notch filter with mitered bends.	Sonnet	equivalent circuit in APLAC
2007	[71]	Multi-stage ASM to address manufacturing limitations in metamaterial structure. Heuristic constraints used as in [70].	Left-handed coplanar waveguide filters based on split ring resonators (SRRs).	EM simulator Ansoft Designer	equivalent circuit in Qucs
2007	[72-74]	-	Microstrip and stripline low-pass filters; microstrip and stripline power dividers.	IE3D	field-based equivalent circuit
2008	[75]	-	Localization of electric current sources within the brain from electroencephalograms (EEG)	FDTD head model	analytical head model
2010	[76]	Approximation of the Jacobian matrix by perturbations, combined with Broyden update.	Microstrip patch antenna.	fine-mesh FEM model	coarse-mesh FEM model
2010	[77-80]	PE in closed form using analytical design formulas. Coarse and fine design parameters of different nature (lumped elements *vs* geometrical dimensions).	Resonant-type metamaterial transmission lines: microstrip lines loaded with comp. split-ring resonators (CSRRs).	Ansoft Designer, Agilent Momentum	lumped circuit and analytical formulas
2012	[81]	-	Marine ecosystem model to calculate global carbon cycle (oceanic CO_2 uptake).	physical data	coarse time discretization num. model
2012	[82]	Additional stopping criterion for ASM: error between target response and fine model response (already used in [12]).	Stepped-impedance microstrip low-pass filter.	Ansoft HFSS	equivalent circuit in ADS
2012	[83]	-	Two-pole coaxial dielectric resonator filter.	Ansoft HFSS	equivalent circuit in ADS
2013	[84]	-	5-pole H-plane direct-coupled-cavity waveguide bandpass filter with rounded corners for space applications in the C band.	Method of Moments & Mode Matching	equivalent circuit & Mode Matching
2013	[85,86]	Two-stage ASM: 1) pre-optimization to determine a convergence region for implementable equivalent circuits; and 2) conventional ASM.	Stopband microstrip filters by cascading CSRR-loaded line unit cells; dual-band CSRR-based power divider.	EM simulator Ansoft Designer	equivalent circuit
2013	[87]	-	Fabricated prototype of a single-ended high-speed package interconnect.	COMSOL	simplified and coarsely meshed COMSOL
2013	[88]	-	Synthesis of stepped impedance resonators (SIR); fabricated 3rd-order elliptic microstrip low pass filter.	full-wave EM model	equivalent circuit
2014	[89-91]	Two-stage ASM: 1) pre-optimization to determine suitable design specs from lumped circuit; 2) conventional ASM. PE in closed form using design formulas in terms of characteristic responses.	Synthesis of SIRs, shunt stubs, and open CSRRs individual cells; wideband bandpass filters by cascading individual cells (negligible EM interaction).	Agilent Momentum	equivalent circuit
2014	[92]	-	Parallel coupled lines bandpass microstrip filter.	HFSS	equivalent circuit in ADS
2014	[93,94]	-	Circular-waveguide dual-mode filters with fixed square insertions (avoiding tuning screws).	physical data (VNA)	FEST3D
2015	[95]	-	Five-pole microstrip hairpin filter using a reflected group delay objective function.	Sonnet	equivalent circuit in ADS
2015	[96]	-	Synthesis of slow-wave structures based on microstrip lines with patch capacitors.	Agilent Momentum	equivalent circuit
2015	[97]	-	Handset antennas considering EM effects of mobile phone components and human head.	HFSS	Simplified HFSS ignoring environment

- Diversity of engineering disciplines. ASM has been applied not only to electromagnetics-based design optimization of RF and microwave circuits, as originally intended, but also to several other areas, including magnetic circuits, mechanical engineering, materials design, medical instrumentation, environmental sciences, etc.

- Diversity of CAD tools. Models of the optimized structures have been implemented using a variety of numerical simulators, including commercially available CAD tools and internal tools. Physical data obtained from direct measurements have also been incorporated as "fine models".

- Diversity of contributors. A very significant number of theoretical contributions and applications have been made from research groups outside the originator group at McMaster University, especially for the second decade of evolution.

- Stable production of applications. A quite steady generation of engineering applications of ASM over time, spanning over two decades. No signs of a proximate end to ASM applications is observed.

III. THE BEAUTY OF ASM: SIMPLICITY

ASM efficiently finds an approximation of the optimal design of a computationally expensive model, termed as fine model. It does that by exploiting a fast but inaccurate surrogate of the original fine model, named as coarse model. It starts from having a coarse model optimal design whose coarse model response satisfies the design specifications and provides a target or desired response for the fine model. ASM aims at finding a solution that makes the fine model response close enough, from an engineering perspective, to the desired response. ASM is naturally rooted in the engineering design practice.

Finding the actual fine model optimal design x_f^* could be realized by minimizing with respect to the fine model design parameters x_f a suitable objective function U that encodes the design specifications in terms of the fine model response R_f, by solving

$$x_f^* = \arg \min_{x_f} U(R_f(x_f))$$

However, the above optimization problem is unfeasible in most practical cases given the high computational cost implied by each evaluation of the fine model response. ASM does not aim at finding x_f^*. Instead, ASM aims at finding a solution x_f^{SM}, called space mapped solution, that
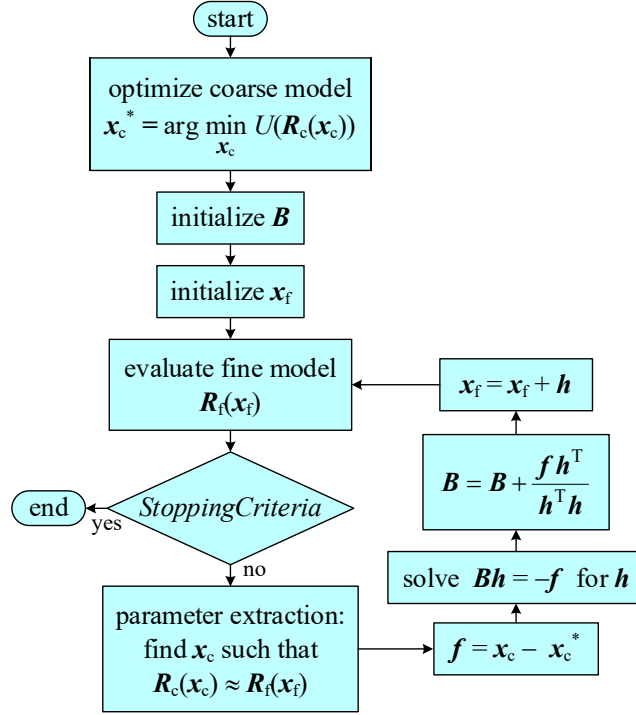
Fig. 4   Flow diagram of the Aggressive Space Mapping (ASM) algorithm [6],[7].

makes the fine model response close enough to the optimal coarse model response, $R_f(x_f^{SM}) \approx R_c^*$.

A. ASM Flow Diagram

A flow diagram for the ASM algorithm is shown in Fig. 4. It starts by finding the optimal coarse model design x_c^* that yields the target response, $R_c(x_c^*) = R_c^*$. This is typically done by directly optimizing the coarse model using classical optimization methods, as in the case of Fig. 4. However, analytical procedures can also be applied to find x_c^* using classical engineering design methods on idealized (coarse) models, e.g., classical filter synthesis procedures.

After obtaining x_c^*, the initial guess of the Broyden matrix B is defined, and the fine model design parameters x_f are initialized. The Broyden matrix B linearly approximates the relationship between both parameters spaces, x_c and x_f, as further explained in the following sections. Having the initial x_f, the corresponding fine model response R_f is calculated at that point. Next, the stopping criteria are tested; if fulfilled, the algorithm ends, otherwise it continues by performing parameter extraction (PE), which consist of finding the coarse model design that makes the coarse model response as close as possible to the current fine model response.

The difference f between the extracted parameters and the optimal coarse model design is then calculated. Next, a linear system is solved to calculate the step h. The Broyden matrix is

updated and the next iterate is calculated, at which a new fine model evaluation is realized and the algorithm proceeds.

B. Initializing the Broyden Matrix and the Fine Model Design Parameters

When the design parameters in both models, x_c and x_f, have the same nature (for instance, both containing the same geometrical dimensions), B should be initialized by the identity matrix, which has the reasonable and implicit assumption that the coarse model is not too deviated from the fine model. However, if x_c and x_f have different nature (for instance, x_c containing lumped circuit element values while x_f containing geometrical dimensions), then B can be initialized by estimating the Jacobian of x_c with respect to x_f by finite differences [62,79].

Similarly, if both x_c and x_f have the same nature, the fine model design parameters are initialized with the optimal coarse model solution, $x_f = x_c^*$, otherwise, they are initialized as $x_f = B^{-1} x_c^*$.

C. Parameter Extraction (PE)

The PE process is the weakest part of ASM. It is usually formulated as an optimization sub-problem that aims at minimizing the differences between coarse and fine model responses at the i-th iteration (local alignment), by solving

$$x_c^{(i)} = \arg \min_{x_c} \left\| R_f(x_f^{(i)}) - R_c(x_c) \right\|$$

This optimization sub-problem may present multiple local minima, some of them yielding a good match (several coarse model designs able to approximate with acceptable accuracy the current fine model response). Non-uniqueness of the PE solution may lead to oscillations or even divergence in the ASM algorithm [48,49]. Several successful strategies have been proposed to overcome this difficulty [3].

When the coarse model consists of an equivalent circuit model, or some physics-based analytical approximation, or some metamodel (response surface model, polynomial model, neural network model, etc.), solving the PE optimization sub-problem is computationally very inexpensive. However, if the coarse model consists of a coarsely discretized full-wave EM model, its computational cost starts being non-negligible, and may exhibit numerical noise and discontinuous behavior [98].

Another interesting approach to perform PE consists of completely avoiding the above

optimization sub-problem, and its inherent difficulties, by following a synthesis approach, i.e., by finding in closed form the coarse model (physics-based) parameter values that synthesize the current fine model response, as in [79,90].

D. The Root of ASM: Finding Roots

The parameter extraction problem described above can be interpreted as a multidimensional vector function $\boldsymbol{P}$ representing the mapping between both design parameter spaces, $\boldsymbol{x}_c^{(i)} = \boldsymbol{P}(\boldsymbol{x}_f^{(i)})$. If the current extracted parameters $\boldsymbol{x}_c^{(i)}$ correspond approximately to $\boldsymbol{x}_c^*$, then the current fine model response approximates the desired response, $\boldsymbol{R}_f(\boldsymbol{x}_f^{(i)}) \approx \boldsymbol{R}_c^*$. From here, it is seen that the ASM algorithm (see Fig. 4) iteratively finds a solution to the following system of nonlinear equations:

$$\boldsymbol{f}(\boldsymbol{x}_f) = \boldsymbol{P}(\boldsymbol{x}_f) - \boldsymbol{x}_c^*$$

since any root $\boldsymbol{x}_f^{SM}$ of the above system of equations $\boldsymbol{f}(\boldsymbol{x}_f)$ implies that $\boldsymbol{R}_f(\boldsymbol{x}_f^{SM}) \cong \boldsymbol{R}_c^*$ (either $\boldsymbol{R}_f(\boldsymbol{x}_f^{SM}) = \boldsymbol{R}_c^*$ or $\boldsymbol{R}_f(\boldsymbol{x}_f^{SM}) \approx \boldsymbol{R}_c^*$; the latter case due to a possible residual in matching the responses during PE). Therefore, ASM is essentially equivalent to the classical Broyden's method for solving systems of nonlinear equations [99], also known as the "method of secants". ASM makes a linear approximation of $\boldsymbol{f}(\boldsymbol{x}_f)$ at each iteration. It iteratively approximates the Jacobian of the mapping function $\boldsymbol{P}$ by matrix $\boldsymbol{B}$ using the Broyden's rank one updating formula (see Fig. 4), where each evaluation of the system $\boldsymbol{f}(\boldsymbol{x}_f)$ implies at least one fine model simulation, and the next iterate is predicted from a quasi-Newton step. It has been shown [100] that ASM is not expected to yield the exact fine model optimum $\boldsymbol{x}_f^*$, but a space mapped solution $\boldsymbol{x}_f^{SM}$ whose accuracy is usually sufficient from a practical engineering perspective.

A typical evolution of ASM from the perspective of the system of nonlinear equations associated to the mapping function is illustrated in Fig. 5, where the Broyden matrix is initialized with the identity (a one-dimension design optimization problem is considered for simplicity). In this illustration, it is assumed that the initial design is very bad (or a very deviated coarse model), implying a very large value of $\| \boldsymbol{f}(\boldsymbol{x}_f^{(0)}) \|$. In spite of that, ASM converges very quickly to a space mapped solution $\boldsymbol{x}_f^{SM}$.

Plots in Fig. 5 also provide some insight regarding the famous efficiency of ASM, by which many highly complex problems are frequently solved in just a few fine model evaluations, regardless of the number of optimization variables, even in cases were the initial fine model
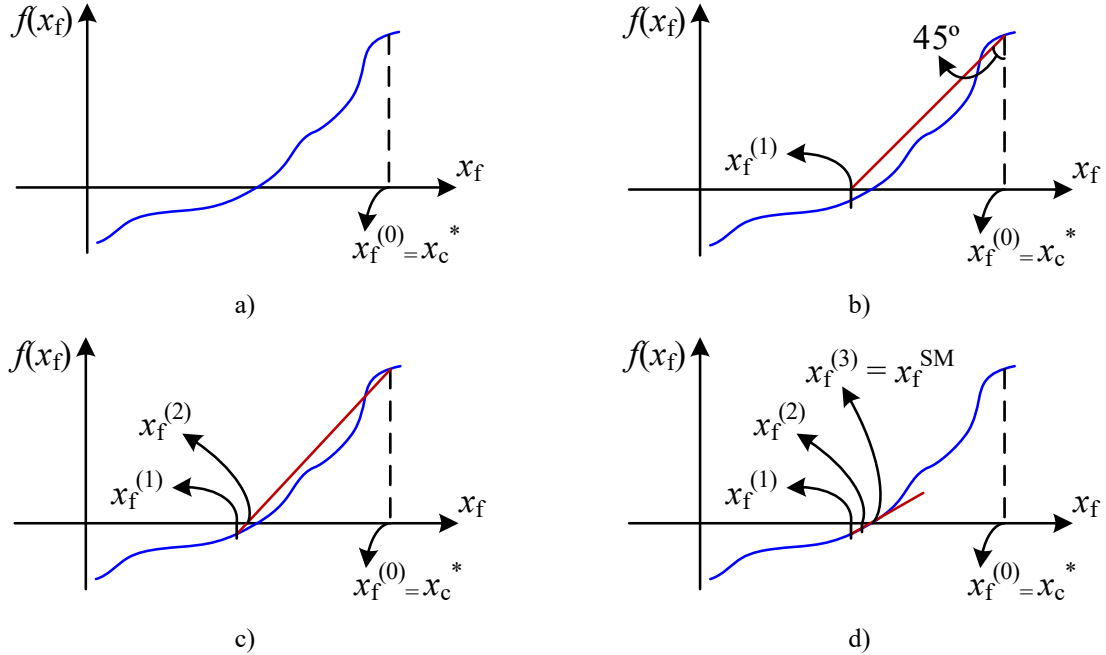
Fig. 5   Typical evolution of the aggressive space mapping (assuming a one-dimension design optimization problem and a very bad initial design): a) initial fine model response calculated, first extracted parameters are very different to x_c^*; b) Broyden matrix is initialized with the identity and first iterate is predicted; c) Broyden matrix is updated with formula and next iterate calculated; d) Broyden matrix is updated and next iterate is practically a root (extracted parameters are practically equal to x_c^*).

response $R_f(x_c^*)$ is very much deviated from the target response R_c^*. As seen in Fig. 5, the efficiency of ASM depends on the degree of nonlinearity of $f(x_f)$, which in turns depends on the degree of nonlinearity of the mapping P between both model parameter spaces. If the mapping is relatively linear (maybe with a large offset), ASM solves the design problem in a few iterations, regardless of the problem dimensionality, even when the initial fine model response is significantly deviated from the target, as in [97].

It is clear then that, assuming the parameter extraction process is correctly implemented, ASM can face the four scenarios depicted in Fig. 6 (again, a one-dimensional design problem is assumed to simplify matters):

a) A unique and exact solution exists for the SM problem. ASM finds a fine model design whose response matches, either exactly or approximately, the desired response. This scenario may occur in practice, usually when the desired response is only approximated by the fine model response at the space mapped solution.

b) Several exact solutions exists. This is another theoretically possible but infrequent practical scenario. It implies that several fine model designs are able to match the optimal
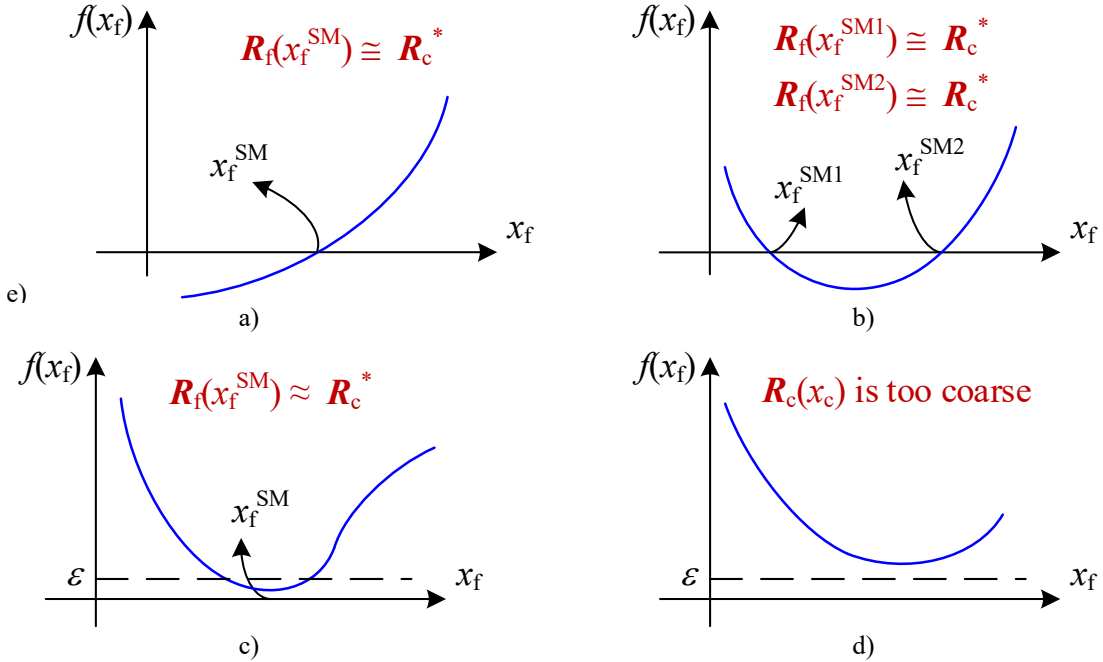
Fig. 6 Aggressive space mapping scenarios (assuming a one-dimension design optimization problem): a) a unique and exact space mapped solution exists; b) several exact SM solutions exists; c) an acceptable SM solution exists; and d) no acceptable SM solution exists due to a too inaccurate coarse model. The desired response is denoted as R_c^*, which is equal to the coarse model response at the optimal coarse model design, $R_c(x_c^*)$. Symbol '$\cong$' denotes equal or approximately equal.

coarse model response. ASM would find only one of those space mapped solutions, depending on the starting point.

c) An acceptable solution exists. This is the most common scenario found in practice for successful ASM design optimizations. Of course, a closely related scenario can happen when several acceptable solutions exists (less likely to happen).

d) There is no acceptable space mapped solution. In this scenario ASM fails. It occurs when the coarse model is too inaccurate with respect to the fine model.

At the end of a successful ASM algorithm, we have not only a fine model design whose corresponding response approximates the desired response, $R_f(x_f^{SM}) \approx R_c^*$, but also a fast linear input-mapped coarse model that makes a good approximation of the fine model around the space mapped solution, $R_c(P(x_f)) \approx R_f(x_f)$ for x_f around x_f^{SM}. This final linear mapping is given by $P(x_f) = Bx_f + c$, with $c = x_c^* - Bx_f^{SM}$ and B as the final Broyden matrix (see Fig. 4).

E. Stopping Criteria

Since ASM aims at finding the roots of $f(x_f)$, the most natural and widely used stopping criterion is when the maximum absolute error in the solution of the system of nonlinear equations

is small enough. However, it has been found in practice that by incorporating other criteria the ASM performance can be significantly enhanced [12,82]. Appropriate additional stopping criteria include: when the maximum relative error in the fine model response with respect to the target response is small enough; when the relative change in the fine model design parameters is small enough; or when a maximum number of iterations is reached. In summary, the above four criteria to finalize ASM at the i-th iteration can be implemented as follows,

$$\left\| \boldsymbol{f}(\boldsymbol{x}_{\mathrm{f}}^{(i)}) \right\|_{\infty} < \varepsilon_1 \ \lor \ \dots$$

$$\left\| \boldsymbol{R}_{\mathrm{f}}(\boldsymbol{x}_{\mathrm{f}}^{(i)}) - \boldsymbol{R}_{\mathrm{c}}(\boldsymbol{x}_{\mathrm{c}}^{*}) \right\|_{\infty} \leq \varepsilon_2 (\varepsilon_2 + \left\| \boldsymbol{R}_{\mathrm{c}}(\boldsymbol{x}_{\mathrm{c}}^{*}) \right\|_{\infty}) \ \lor \ \dots$$

$$\left\| \boldsymbol{x}_{\mathrm{f}}^{(i+1)} - \boldsymbol{x}_{\mathrm{f}}^{(i)} \right\|_{2} \leq \varepsilon_3 (\varepsilon_3 + \left\| \boldsymbol{x}_{\mathrm{f}}^{(i)} \right\|_{2}) \ \lor \ \dots$$

$$i > i_{\max}$$

where ε_1, ε_2, and ε_3 are arbitrary small positive scalars. Since ASM is normally very efficient, a suitable maximum number of iterations, $i_{\max}$, to stop ASM is $3n$ or $4n$, where n is the total number of design variables. Exceeding that amount of iterations is typically a sign of anomalous ASM behavior, most probably caused by an inadequate parameter extraction process, or by a too coarse model.

IV. FINAL REMARKS AND ASM FUTURE DIRECTIONS

In the context of all the SM-based design optimization approaches overviewed in the first section of the present article, and considering the described essential characteristics of aggressive space mapping, a more technical name for this SM design technique would be Broyden-based input space mapping algorithm.

Studying in more detail the most recent applications of ASM, listed in Table II, it seems that there is a trend towards the development of fully automated CAD tools, based on ASM, for efficient and accurate synthesis and design optimization algorithms dedicated to particular structures in specific technologies. This trend might lead to the future incorporation into industrial CAD tools of ASM-based built-in design functions.

As time goes by, perhaps some of the most recent and advanced SM-like optimization approaches, indicated in Fig. 1, will prove to be as popular as the ASM algorithm has been so far.

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