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A Fixed-Time Second Order Sliding Mode Observer for a Class of Nonlinear Systems

Juan Diego Sánchez-Torres and Alexander G. Loukianov

***Abstract*— This paper presents a second order fixed time sliding mode observer based on an extension of the super-twisting algorithm. This observer can be applied to a class of nonlinear system with a block-wise representation. The block structure provides a straightforward form to the application of the proposed second order sliding mode algorithm, yielding to finite-time convergence with a settling time independent to the system initial conditions. Finally, as numerical simulation example, the case of a linear induction motor is studied, exposing the efficiency and feasibility of the proposal.**

I. Introduction

The sliding mode (SM) algorithms are applied with the idea to drive the dynamics of a system to a sliding manifold that is an integral manifold with finite reaching time [1]. Generally, this approach exhibits very interesting and desirable features such as the work with reduced observation error dynamics, the possibility to decompose the design problem into two sub problems of the reduced order, the robustness of the closed-loop system in presence of parameter variations and external disturbances and, finite-time stability [2]–[4].

Considering the observation error as a sliding variable, the SM algorithms can be considered as an effective solution to the problem of observers design for nonlinear systems [5], specially when finite-time convergence of the observed states to the real ones is required. An important class of SM observers use the equivalent control method [6] to obtain information of the system by means of continuous equivalent values of the discontinuous observer inputs in SM motion [7]. With this idea, several designs have been proposed as the cascade observers [8], step-by-step observers [9], a SM observer where the estimation of unknown inputs problem has been considered [10], fixed time designs [11], among others.

Another class of SM observers are based on the second order SM feature of the super-twisting algorithm (STA) [12]. Those attractive characteristics of the STA algorithm have been exploited and extended for fixed time convergent methods [13]–[15], adaptive controllers [16]–[21], multivariable structures [22], most of them based on the stability studies presented in [23]–[29]. For the case of observers, a design for mechanical systems is presented in [30] being extended to electrical drives [31], more general forms as in [32] and systems with noisy measurements [33], [34].

All these methods present high performance. However, most of them are presented in scalar form. And, the multivariable structure introduced in [22] converges in finite time but not fixed.

Under that consideration, this paper is aimed to present a SM observer for a class of nonlinear systems based on a fixed time STA with fixed time convergence. This design allows the problem to be solved without the individual selection of each stabilizing input, instead a multivariable function, based on the unit control [2], [35], is used. On the other hand, the fixed time stability [13], [36] ensures the existence of a finite time independent to the initial conditions in which the system converges. Thus, the proposed approach have very attractive features as: fixed time convergence to the observed variables and a fixed parameters number (six for this case), regardless of the state dimension.

The linear induction motor is considered as case study. The effectiveness the proposed observer is demonstrated by means of numerical simulation, showing a good performance of this proposal.

This paper is organized as follows: Section II introduces a multivariable fixed time stable STA . Section III describes the proposed observers. The simulations are presented in Section IV. Finally, in Section V the conclusions are given.

II. Preliminary Result

Let the vectors $x_1, x_2 \in \mathbb{R}^n$. Now, consider the system

$$\begin{aligned} \dot{x}_1 &= -k_1 \frac{x_1}{\|x_1\|^{1/2}} - k_2 x_1 - k_3 x_1 \|x_1\|^{1/2} + x_2 + \Delta_1 \\ \dot{x}_2 &= -k_4 \frac{x_1}{\|x_1\|} - k_5 x_1 - k_6 x_1 \|x_1\|^{1/2} + \Delta_2 \end{aligned} \tag{1}$$

where $k_1, \ldots, k_6 > 0$, and the disturbances are regarded as $\|\Delta_1\| \leq \delta_1 \|x_1\|$ and $\|\Delta_2\| \leq \delta_2$ with $\delta_1, \delta_2 > 0$.

With the Lyapunov function

$$V = 2k_3 \|x_1\| + k_4 \|x_1\|^2 + \frac{1}{2} \|x_2\|^2 + \nu^T \nu \tag{2}$$

where $\nu = k_1 \frac{x_1}{\|x_1\|^{1/2}} + k_2 x_1 + k_3 x_1 \|x_1\|^{1/2} - x_2$, it is possible to show there exists constants $\gamma_1 = \gamma_1(\theta), \gamma_2 = \gamma_2(\theta) > 0$, $\theta = (k_1, k_2, k_3, k_3, k_4, k_5, k_6, \delta_1, \delta_2)$, such that

$$\dot{V} \leq -\gamma_1 V^{1/2} - \gamma_2 V^{3/2}. \tag{3}$$

Therefore, from (2) – (3), the system (1) is globally fixed time stable [36].

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Juna Diego and Alexander G. Loukianov are with the Automatic Control Lab., CINVESTAV-IPN Gdl, Av. del Bosque 1145, CP 45019, México, {dsanchez, louk}@gdl.cinvestav.mx

III. Fixed-Time Second Order Sliding Mode Observer

A. Observer Design

Consider the system written in the following block-wise form:

$$\begin{aligned} \dot{x}_1 &= B_1(x_1)x_2 + f_1(x_1,u) \\ \dot{x}_2 &= B_2(x_1)x_2 + f_2(x_1) \\ y &= x_1 \end{aligned} \tag{4}$$

where $x = [x_1 \quad x_2]^T$ and the vectors $x_1, x_2 \in \mathbb{R}^n$. The matrix $B_1(x_1)$ is considered to be invertible.

Based on the system (4), the following observer is proposed in order to provide a uniform finite estimation of the state x:

$$\begin{aligned} \dot{\hat{x}}_1 &= B_1(x_1)\hat{x}_2 + f_1(x_1,u) + \phi_1(\tilde{x}_1) \\ \dot{\hat{x}}_2 &= B_2(x_1)\hat{x}_2 + f_2(x_1) + B_1^{-1}(x_1)\phi_2(\tilde{x}_1) \end{aligned} \tag{5}$$

where $\hat{x}_1$ and $\hat{x}_2$ are the estimates of x_1 and x_2, respectively and, the observer errors are given by $\tilde{x}_1 = \hat{x}_1 - x_1$ and $\tilde{x}_2 = \hat{x}_2 - x_2$. The observer inputs $\phi_1(\tilde{x}_1)$, and $\phi_2(\tilde{x}_1)$ are defined as

$$\begin{aligned} \phi_1(\tilde{x}_1) &= k_1 \frac{\tilde{x}_1}{\|\tilde{x}_1\|^{1/2}} + k_2\tilde{x}_1 + k_3\tilde{x}_1 \|\tilde{x}_1\|^{1/2} \\ \phi_2(\tilde{x}_1) &= k_4 \frac{\tilde{x}_1}{\|\tilde{x}_1\|} + k_5\tilde{x}_1 + k_6\tilde{x}_1 \|\tilde{x}_1\|^{1/2} \end{aligned} \tag{6}$$

where $k_1, \ldots, k_6 > 0$.

B. Convergence Analysis

To analyze the observer convergence, consider the dynamics of the errors $\tilde{x}_1$ and $\tilde{x}_2$. From (4) and (5) it follows

$$\begin{aligned} \dot{\tilde{x}}_1 &= B_1(x_1)\tilde{x}_2 - \phi_1(\tilde{x}_1) \\ \dot{\tilde{x}}_2 &= B_2(x_1)\tilde{x}_2 - B_1^{-1}(x_1)\phi_2(\tilde{x}_1). \end{aligned} \tag{7}$$

Defining $q = B_1(x_1)\tilde{x}_2$, the system (7) is transformed to

$$\begin{aligned} \dot{\tilde{x}}_1 &= \phi_1(\tilde{x}_1) + q \\ \dot{q} &= \phi_2(\tilde{x}_1) + B(x_1)q \end{aligned} \tag{8}$$

where $B(x_1) = \left[\dot{B}_1(x_1) + B_1(x_1)B_2(x_1)\right] B_1^{-1}(x_1)$.

Considering $\|B(x_1)q\| < \delta$ where $\delta > 0$, with a suitable choice of the gains $k_1, \ldots, k_6$, it follows from (2) – (3) that the system (8) is globally fixed time stable. Therefore, the observer variables converges to the real ones in fixed time.

IV. Numerical Simulation Results

This section shows numerical simulations results of the proposed observer for a linear induction motor. The measured variables are the velocity and the currents. The observed variables are the flux and the load torque, introduced as step form. This is motivated, due the difficulty of the flux and torque direct measurement [37].

The model for the induction can be described by equations for the stator current and rotor fluxes in stationary reference frame $\alpha\beta$ as follows:

$$\begin{aligned} \frac{d\Theta}{dt} &= d_1\left(\lambda_{\alpha r} i_{\beta s} - \lambda_{\beta r} i_{\alpha s}\right) - d_2\Gamma - d_3\Theta \\ \frac{d\lambda_{\alpha r}}{dt} &= -\eta_1\lambda_{\alpha r} + \eta_2\Theta\lambda_{\beta r} + \eta_3 i_{\alpha s} \\ \frac{d\lambda_{\beta r}}{dt} &= -\eta_1\lambda_{\beta r} - \eta_2\Theta\lambda_{\alpha r} + \eta_3 i_{\beta s} \\ \frac{di_{\alpha s}}{dt} &= -\eta_4 i_{\alpha s} + \eta_5\lambda_{\alpha r} - \eta_6\Theta\lambda_{\beta r} + \eta_7 v_{\alpha s} \\ \frac{di_{\beta s}}{dt} &= -\eta_8 i_{\beta s} + \eta_9\lambda_{\beta r} + \eta_{10}\Theta\lambda_{\alpha r} + \eta_{11} v_{\beta s} \end{aligned} \tag{9}$$

where $\lambda_{\alpha r}$ and $\lambda_{\beta r}$ are the rotor magnetic-flux-linkage components, respectively; $i_{\alpha s}$ and $i_{\beta s}$ are the stator current components, respectively, $v_{\alpha s}$ and $v_{\beta s}$ are the voltage of α and β axes in the stator, respectively.

For the three-phase linear induction motor in $\alpha\beta$ frame, the voltages are presented of the form

$$v_{\alpha s} = v_s \sin(\omega t) \tag{10}$$

$$v_{\beta s} = -v_s \sin(\omega t). \tag{11}$$

Thus, for this case, the parameters are: $\eta_1 = \frac{R_r}{L_r}$, $\eta_2 = n_p\left(\frac{\pi}{\tau}\right)$, $\eta_3 = \frac{R_r L_m}{L_r}$, $\eta_4 = \frac{R_s}{\left(\frac{{L_s}^2 L_r - L_s {L_m}^2}{L_s L_r}\right)} + \frac{1-\left(\frac{L_s L_r - {L_m}^2}{L_s L_r}\right)}{\left(\frac{L_s L_r - {L_m}^2}{L_s L_r}\right)\frac{R_r}{L_r}}$, $\eta_5 = \frac{L_m R_r}{\left(\frac{L_s L_r - {L_m}^2}{L_s L_r}\right) L_s {L_r}^2}$, $\eta_6 = n_p\left(\frac{\pi}{\tau}\right)\frac{L_m}{\left(\frac{L_s L_r - {L_m}^2}{L_s L_r}\right) L_s L_r}$, $\eta_7 = \frac{1}{\left(\frac{L_s L_r - {L_m}^2}{L_s L_r}\right) L_s}$, $\eta_8 = \eta_4$, $\eta_9 = \eta_5$, $\eta_{10} = \eta_6$, $\eta_{11} = \eta_7$, $d_1 = \frac{3 n_p \pi L_m}{2 L_r \tau M}$, $d_2 = \frac{1}{M}$, $d_3 = \frac{D}{M}$ where R_s and L_s are the resistance and inductance of the stator, respectively. τ is the pole pitch, M is the total mass of the moving element, D is viscous friction, $\Theta = v$ is the linear velocity and $\Gamma = F_L$ is the external force.

For the observer design, the availability of continuous measurements of motor speed and currents is assumed. In addition the mechanic load Γ is considered as an unknown and slowly-varying perturbation to be estimated, that is $\dot{\Gamma} = 0$. Thus, the system (9) can be and, the blocks are $x_1 = [\Theta \quad i_{\alpha s} \quad i_{\beta s}]^T$ and, $x_2 = [\lambda_{\alpha r} \quad \lambda_{\beta r} \quad \Gamma]^T$, with $u = [v_{\alpha s} \quad v_{\beta s}]^T$. Here $B_1(x_1) = \begin{bmatrix} d_1 i_{\beta s} & d_1 i_{\alpha s} & -d_2 \\ \eta_5 & -\eta_6\Theta & 0 \\ \eta_{10}\Theta & \eta_9 & 0 \end{bmatrix}$, $f_1(x_1,u) = \begin{bmatrix} -d_3\Theta \\ \eta_7 v_{\alpha s} - \eta_4 i_{\alpha s} \\ \eta_{10}\Theta \end{bmatrix}$, $B_2(x_1) = \begin{bmatrix} -\eta_1 & \eta_2\Theta & 0 \\ -\eta_2\Theta & -\eta_1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and, $f_2(x_1) = \begin{bmatrix} \eta_3 i_{\alpha s} \\ \eta_3 i_{\beta s} \\ 0 \end{bmatrix}$.

$B_1^{-1}(x_1)$ is the inverse of the matrix $B_1(x_1)$ and is given by $B_1^{-1}(x_1) = \begin{bmatrix} 0 & \frac{\eta_9}{\eta_6\eta_{10}\Theta^2 + \eta_5\eta_9} & \frac{\eta_6\Theta}{\eta_6\eta_{10}\Theta^2 + \eta_5\eta_9} \\ 0 & -\frac{\eta_{10}\Theta}{\eta_6\eta_{10}\Theta^2 + \eta_5\eta_9} & \frac{\eta_5}{\eta_6\eta_{10}\Theta^2 + \eta_5\eta_9} \\ -\frac{1}{d_2} & \frac{d_1(\eta_9 i_{\beta s} + \eta_{10} i_{\alpha s}\Theta)}{d_2(\eta_6\eta_{10}\Theta^2 + \eta_5\eta_9)} & -\frac{d_1(\eta_5 i_{\alpha s} - \eta_6 i_{\beta s}\Theta)}{d_2(\eta_6\eta_{10}\Theta^2 + \eta_5\eta_9)} \end{bmatrix}$.

For three-phase linear induction motor the parameter are presented as [38]:

Three-phase linear			
H.P.	4	V_s	180 (V)
f	60 (Hz)	n_p	2
R_s	5.3685 (Ω)	R_r	3.5315 (Ω)
L_s	0.02846(H)	L_r	0.02846 (H)
L_m	0.02419 (H)	M	2.78 (kg)
D	36.0455 (Kg/s)	τ	0.027 (m)
$I_{\max}$	14.2 (A)		
μ_1	1	μ_2	1
m_{11}	640	m_{12}	640
m_{13}	45	m_{21}	64000
m_{22}	64000	m_{23}	20

The observer gains are chosen as $k_1 = 5$, $k_2 = 10$, $k_3 = 2$, $k_4 = 10$, $k_5 = 5$ and $k_6 = 1$. The simulation results are shown in the following figures:

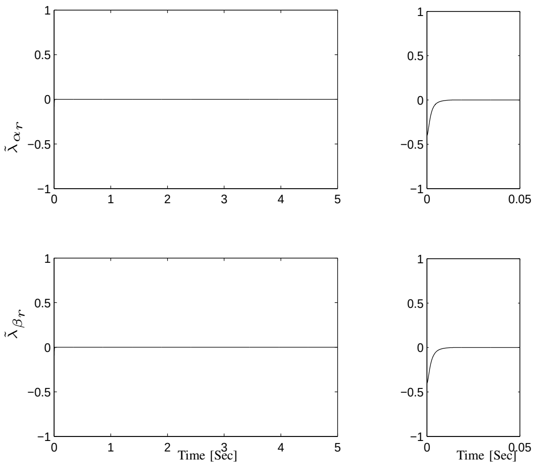

Fig. 1. Error of rotor flux $\tilde{\lambda}_{\alpha_r}$ and $\tilde{\lambda}_{\beta_r}$ of TLIM.

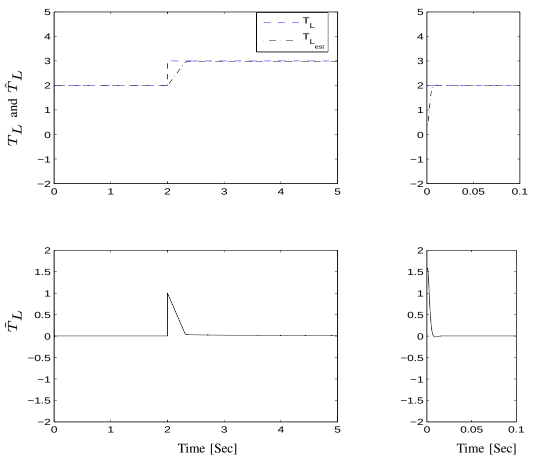

Fig. 2. Load torque estimated $\hat{T}_L$ and error of load torque $\tilde{T}_L$ of TLIM.

In Figure 1 the time evolution of the rotor flux $\tilde{\lambda}_{\alpha_r}$ and $\tilde{\lambda}_{\beta_r}$ errors of induction motors are shown, while Fig. 2 presents the time evolutions of the estimated load torque $\hat{T}_L$ and the load estimation error $\tilde{T}_L$ of induction motors cases.

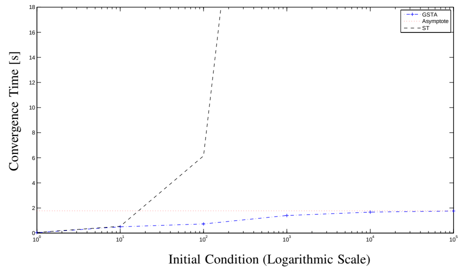

Fig. 3. Convergence time of both observers by growing initial condition norm.

The Figure 3 presents a comparison of the proposed observer with another which uses the mutivariable STA [22], that means fixing $k_3 = 0$ and $k_6 = 0$ in (6). Here is highlighted that the convergence time for the multivariable STA grows unboundedly with the norm of the initial condition, while the convergence time of the proposed observer is asymptotically bounded by a constant for growing initial condition's norm.

V. CONCLUSIONS

In this work a fixed time convergent observer was proposed. The scheme was applied to the model on the stationary frame $\alpha\beta$ for induction motors. The flux and load torque were estimated, all of them are shown to give appreciable results in order of convergence time to estimate the rotor flux and the load torque.

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