# GPS BASED DESIGN OF THE LOCAL CLOCK CONTROL SYSTEM BASED ON THE OPTIMALLY UNBIASED MOVING AVERAGE FILTER

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Abstract

In this paper we made the simulation steering of the local clock time errors with simple moving average (MA), optimally unbiased moving average (OMA), the two and three-state Kalman filters. The references signal (precise time) was suministred by GPS. In this task we have two important activities, estimating and the error control, so the principal parameter in this study is the root mean square error (RMSE) of steering. When steering the GPS-based time error in the local clock with four filters, we found out that, of the filter with the same time constant, the optimally unbiased MA filter desmostred the steering error between the two and three state Kalman filter.

## INTRODUCTION

To be outside of synchronization is be outside of control, without precise time reference in applications that requiered of synchronization, so we are undefeat in problems related with the time. It is well known that timeekeping solve two important tasks: estimating the time error of a local clock with reference time signal (GPS) and elimination with the synchronization loop. In our particular case we take the synchronization loop as an optimal stochastical error control. Optimal lineal stochastic estimation theory, named Kalman filtering theory [1], solves both tasks straightforwardly. In this task we have three important variables for simulation, **N** is the points numbers in the average for FIR filters (simple MA and OMA filter), ***m*** *is the points numbers taken for the lineal regression* (LR), so LR is used for estimate the first values of time error, frequency and frequency aging of the local clock, and **K3** is the control coefficient for to obtain the least RMSE of steering.

To determine the tradeoff, we examine each filter for time error model identified by the finite polynomial [2]

$$x_n = x_0 + y_0 \Delta n + \frac{D}{2} \Delta^2 n^2 + w_{xn} , \tag{1}$$

where $n = 0,1,\ldots$; $\Delta = t_n - t_{n-1}$ *is the sample time;* $t_n$ *is the discrete time;* $x_0$ *is the initial time* error; $y_0$ is the initial fractional frequency offset of a local clock from the reference frequency; $D$ is the lineal fractional frequency drift rate; and $w_{xn}(t)$ is the random time error deviation component. For the simulation steering we have an important formula for determine the time constant

$$\mathbf{Tr} = \frac{\mathbf{T(N-1)}}{3600} , \tag{2}$$

where **T** is the digitalization time in seconds; 3600 equal in seconds per hour.

## DEVELOPMENT

The signal for simulation steering is showed in figure 1, so it has the reference signal with respect of the local clock, the observation measured with respect to GPS timing and additive Gaussian noise with mean zero.

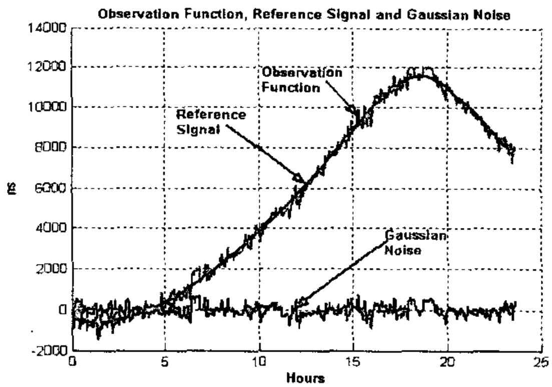


Figure 1 – Observation function, reference signal and Gaussian noise.

As a first step, we analyze the simple MA filter, so this filter produces the lowest noise for stationary processes, but in non-stationary processes is not good, the estimate equation of time error for simple MA filter is

$$\hat{x}'_n = \frac{1}{N}\sum_{i=0}^{N-1}\xi_{n-i}\,, \tag{3}$$

where $\xi_{n-i}$ is the observation signal.

In (3) the constant coefficient $\frac{1}{N}$ produce inherently the bias.

For the steering task, the simple MA algorithm is give it in controlled estimated (4) and controlled observation (5)

$$\lambda ma_n = \frac{1}{N}\sum_{i=0}^{N-1}\xi ma_{n-i}\,, \tag{4}$$

$$\xi ma_{n+1} = \xi G_{n+1} - \mathbf{K3}\lambda ma_n\,. \tag{5}$$

The optimally unbiased MA filter for estimate the time error is defined by

$$\hat{x}_n = \sum_{i=0}^{N-1}\frac{2(2N-1)-6i}{N(N+1)}\xi_{n-i}\,, \tag{6}$$

in (7) we have the weightig function as

$$W_i(N) = \begin{cases} \dfrac{2(2N-1)-6i}{N(N+1)}, & 0 \le i \le N-1 \\ 0, & otherwise \end{cases} \tag{7}$$

this evidently is nonzero on the averaging interval only [3]. In case of steering algorithm with optimally unbiased MA filter, we have the next equations

$$\lambda omae_n = \sum_{i=0}^{N-1} \frac{2(2N-1)-6i}{N(N+1)} \xi oma_{n-i}\,, \tag{8}$$

$$\xi oma_{n+1} = \xi G_{n+1} - \mathbf{K3}\lambda omae_n\,. \tag{9}$$

For two and three state Kalman filter, we have the next equations, in (10) it's the general form for observation signal, where $\mathbf{H}_n$ is the measurement matrix, $\mathbf{u}_n$ is a control vector, $\mathbf{n}_{\lambda n}$ is a Gaussian noise vector

$$\xi_n = \mathbf{H}_n \boldsymbol{\lambda}_n + \mathbf{u}_n + \mathbf{n}_{0n}\,, \tag{10}$$

$$\boldsymbol{\lambda}_n = \mathbf{A}_{n-1}\boldsymbol{\lambda}_{n-1} + \mathbf{n}_{\lambda n}\,, \tag{11}$$

$$\mathbf{H}_n = \begin{bmatrix} 1 & 0 \end{bmatrix},\ \mathbf{H}_n = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}. \tag{12}$$

In (13) we can see the vector states of local clock like time error $x_n$; frequency offset $y_n$, and frequency aging $\alpha_n$

$$\boldsymbol{\lambda}_n = \begin{bmatrix} x_n \\ y_n \end{bmatrix},\quad \boldsymbol{\lambda}_n = \begin{bmatrix} x_n \\ y_n \\ \alpha_n \end{bmatrix}. \tag{13}$$

So, in (14) Kalman algorithm required a transition matrix related the sample time and in (15) are the covariance matrix

$$\mathbf{A}_n = \begin{bmatrix} 1 & \Delta \\ 0 & 1 \end{bmatrix},\ \mathbf{A}_n = \begin{bmatrix} 1 & \Delta & \Delta^2/2 \\ 0 & 1 & \Delta \\ 0 & 0 & 1 \end{bmatrix}, \tag{14}$$

$$\boldsymbol{\Psi}_{\lambda n} = \frac{N_{\alpha n}}{2}\Delta \begin{bmatrix} \Delta^2/3 & \Delta/2 \\ \Delta/2 & 1 \end{bmatrix},\ \boldsymbol{\Psi}_n = E\{n_{\lambda n} n_{\lambda n}^T\} = \begin{bmatrix} \Psi_{11n} & \Psi_{12n} & \Psi_{13n} \\ \Psi_{21n} & \Psi_{22n} & \Psi_{23n} \\ \Psi_{31n} & \Psi_{23n} & \Psi_{33n} \end{bmatrix}. \tag{15}$$

After of this, the algorithm for estimated (16) and steering (17)-(18) by means of the Kalman filter are:

$$\hat{\boldsymbol{\lambda}}_n = \mathbf{A}_{n-1}\hat{\boldsymbol{\lambda}}_{n-1} + \mathbf{K}_n\left(\xi_n - \mathbf{u}_n - \mathbf{H}_n\mathbf{A}_{n-1}\hat{\boldsymbol{\lambda}}_{n-1}\right), \tag{16}$$

$$\lambda e_n = \mathbf{A}_{n-1}\lambda e_{n-1} + \mathbf{K}_n\left(\xi_n - \mathbf{H}_n\mathbf{A}_{n-1}\lambda e_{n-1}\right), \tag{17}$$

$$\xi_{n+1} = \xi G_{n+1} - \mathbf{K3}\lambda e_n\,, \tag{18}$$

where $\mathbf{K}_n$ is the Kalman gain, $\xi G_{n+1}$ is the observation signal from PRN file.

We determined the statiscal steering error by means a general error function, this general error function is similar for the four filters, so its structure was determinated by the time interval $M = 0 \cdots ni - N - 1$,

$$\lambda fc_{M+1} = \lambda_{M+N+1} - \mathbf{K3}\lambda oma_{M+N}\,, \tag{19}$$

where $ni$ is the length of the observation file, in the right-part of (19) we show only the time error controlled for optimally unbiased MA filter, but we can replace easily it for any time error controlled like simple MA, two or three state Kalman filter.

Now, the bias $\Delta\hat{x} = E[\lambda fc]$, the variance $\sigma_{\varepsilon}^2 = E\left[(\lambda fc - \Delta\hat{x})^2\right]$, root mean square deviation (RMSD) $\sigma = \sqrt{\sigma_{\varepsilon}^2}$, root mean square error (RMSE) $\varepsilon_{RMS} = \sqrt{E[\lambda fc^2]} = \sqrt{\Delta\hat{x}^2 + \sigma_{\varepsilon}^2}$ and maximal error $\varepsilon_{global} = 0.5(\varepsilon_{RMS} + \lambda fc_{\max})$ of steering were calculated from (19).

## SIMULATION AND CONCLUSIONS

As the principal goal was the steering of the time error in the local clock, in figure 2 we have all steering error for N average points (order filter), the simulation was evaluated in Matlab and Mathcad Software. As we can see, the worst filter was the simple MA and this was because of the process is non-stationary, the best filter is the two state Kalman filter and the optimally unbiased MA filter is situated between two and three state Kalman filter; when filter order increases, transition time increases too, therefore a filter with N close to 80 or 100 average points is recommended for steering error of the local clock. The FIR filter did not require any a *priori* knowledge about the GPS-based time error process in the opposite with Kalman filters [4].

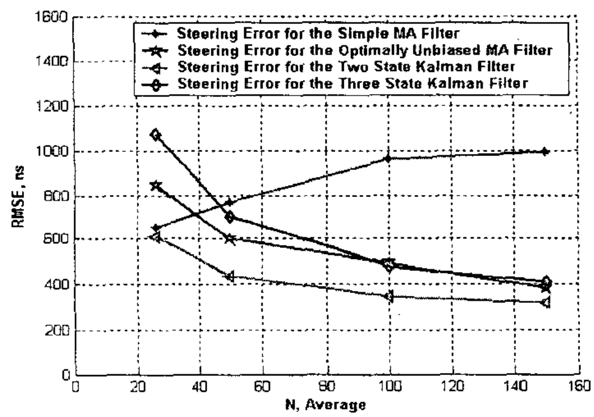


Figure 2 – Least RMSE of steering with all filters.

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