THREE-DIMENSIONAL OPTIMAL KALMAN ALGORITHM FOR GPS-BASED POSITIONING ESTIMATION OF THE STATIONARY OBJECT

I. E. Villalón-Turrubiates, O. G. Ibarra-Manzano, Y. S. Shmaliy,
J. A. Andrade-Lucio

1Universidad de Guanajuato.
Facultad de Ingeniería Mecánica, Electrónica y Electrónica.
Maestría en Ingeniería Eléctrica, Opción: Instrumentación y Sistemas Digitales.
Prolongación Tampico No. 912, Tel. 01 464 6480911 ext. 119, Col. Bellavista.
36730 Salamanca, Gto., México.

INTRODUCTION

Navigation is defined as the science of getting a craft or person from one place to another [1]. Some navigation aids are very complex and transmit electronic signals, that are referred to as radionavigation aids. Various types of radionavigation aids exist, which can be either ground-based or space-based. The Global Positioning System (GPS) was created in the early 1960s by the National Aeronautics and Space Administration (NASA), developing satellite systems for positioning determination. GPS is now being used to provide positioning and timing information for a number of applications where it is essential that the accuracy and reliability of the GPS information can be assured [2]. The susceptibility of the GPS signals to interference is of concern to the GPS user community. Because of the low receiver power of the GPS signals, outages can easily occur due to unintentional interference.

Theoretically, the Kalman filter is an estimator [3] for what is called the \textit{linear-quadratic problem}, which is the problem of estimating the instantaneous “state” of a linear dynamic system perturbed by white noise. The resulting estimator is statistically optimal with respect to any quadratic function of estimation error. R. E. Kalman’s paper describing a recursive solution of the discrete-data linear filtering problem was published in 1960 [4].

This project presents the design and development of a multidimensional Kalman filter with the purpose to estimate the tri-dimensional position of a stationary object based on GPS measurements. Because this is not the only filtering algorithm available, a comparison with other four types of filters (one-dimensional optimal Kalman algorithm, quasi-optimal stationary Kalman algorithm, simple moving average algorithm and optimally unbiased moving average algorithm) is also developed.

METHODOLOGY

Consider a discrete observation signal \( \xi_v \) that is a linear combination of a discrete signal \( \lambda_v \) and a white noise \( n_m \). The observation and state equations are [5]:

\[
\xi_v = H \lambda_v + u_v + n_m, \tag{1}
\]

\[
\lambda_v = A_{v-1} \lambda_{v-1} + n_{\lambda}, \tag{2}
\]

where \( v = 0,1,2,\ldots \) corresponds to discrete-time \( t_v \) and measuring time interval \( \Delta = t_v - t_{v-1} \), \( \xi_v \) is \( m \)-dimensional observation vector formed by the reference short-term noisy GPS signals, \( \lambda_v \) is \( n \)-dimensional three-dimensional position state vector (latitude, longitude, altitude), \( H \), is \( m \times n \) dimensional measurement matrix, \( u_v \) is \( m \)-dimensional vector that contains the control signals (which for the filtering task is taken as zero), \( A_v \) is \( n \times n \) dimensional state transition matrix,

\( A_v = e^{A_{\Delta}} = I + A_{\Delta} + \frac{1}{2!} A_{\Delta}^2 + \cdots \),

\( \Delta = t_v - t_{v-1} \),

\( n_{\lambda} \) is \( n \)-dimensional measurement noise vector.

The observation and state equations are expressed in a matrix form as:

\[
\begin{align*}
\xi_v &= H \lambda_v + u_v + n_m, \\
\lambda_v &= A_{v-1} \lambda_{v-1} + n_{\lambda},
\end{align*}
\]

The optimal filtering process involves the solution of the following Riccati equation:

\[
P_v = A_{v-1} P_{v-1} A_{v-1}^T + Q_v - A_{v-1} P_{v-1} A_{v-1}^T R_v^{-1},
\]

where \( P_v \) is the covariance matrix of the estimation error, \( Q_v \) is the covariance matrix of the measurement noise, \( R_v \) is the covariance matrix of the process noise.
matrix, \( n_y \) and \( n_x \), are jointly independent vector white Gaussian noises with mean-zero and covariance matrices \( V_y \) and \( \Psi_y \) which are of \( m \times m \) and \( n \times n \) dimensions, respectively. As usually, we will deal with single observations and estimate some states. It means that if \( m < n \) then one may use the following algorithm of lineal Kalman filtering based on (1) and (2):

\[
R_y = A_{y-1}R_{y-1}A_{y-1}^T + \Psi_y (3)
\]

\[
K_y = R_y H_y (H_y R_y H_y^T + V_y)^{-1} (4)
\]

\[
\hat{\lambda}_y = \lambda_{y-1} + K_y (\xi_y - u_y - H_y \lambda_{y-1}) (5)
\]

\[
R_y = (I - K_y H_y) R_y (6)
\]

where \( \lambda_y \) is a vector of three-dimensional positioning estimates, I is unit matrix, and \( R_y \) is the error covariance matrix. Solution (3) is justified for a common case and may not be simplified, as a rule. Errors between estimates and original signals will be determined using:

- Maximal Value of Error (maximal value of \( \xi_y \)).
- Bias of Error (Bias of \( \xi_y \)).
- Root Mean Square Deviation (RMSD, \( \sigma_y \)).
- Root Mean Square Error (RMSE, \( \sqrt{\sum e^2 + \sigma_y^2} \)).
- Maximal Error (maximal value of \( e \)).

RESULTS

Now, we will compare the 3-Dimensional Kalman estimates with the One-dimensional Optimal Kalman algorithm [6], the Quasi-Optimal stationary Kalman algorithm [6], the Simple Moving Average (MA) algorithm [6] and the Optimally Unbiased Moving Average (OMA) algorithm [6]. For this case, we use the same conditions and the information from the Observation \( \xi_y \). Tables 1, 2 and 3 show the statistical data obtained from the errors for the three signals, respectively. Also, Figures 1, 2 and 3 show the comparison between the different filters for the three signals, respectively.

<table>
<thead>
<tr>
<th>Table 1 – Statistical values obtained from the different algorithms for the signal x</th>
<th>Estimate's mean</th>
<th>Bias of Error</th>
<th>RMSD</th>
<th>RMSE</th>
<th>Maximal Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-D Kalman</td>
<td>4054.923</td>
<td>31.358</td>
<td>349.359</td>
<td>350.764</td>
<td>1460.309</td>
</tr>
<tr>
<td>1-D Kalman</td>
<td>4054.320</td>
<td>31.433</td>
<td>335.451</td>
<td>336.920</td>
<td>1365.408</td>
</tr>
<tr>
<td>1-D Stationary</td>
<td>4055.245</td>
<td>31.638</td>
<td>304.002</td>
<td>305.644</td>
<td>1153.318</td>
</tr>
<tr>
<td>Simple MA</td>
<td>3818.170</td>
<td>29.475</td>
<td>241.340</td>
<td>243.133</td>
<td>498.533</td>
</tr>
<tr>
<td>Optimal MA</td>
<td>3824.408</td>
<td>36.128</td>
<td>303.026</td>
<td>305.172</td>
<td>712.668</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2 – Statistical values obtained from the different algorithms for the signal y</th>
<th>Estimate's mean</th>
<th>Bias of Error</th>
<th>RMSD</th>
<th>RMSE</th>
<th>Maximal Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-D Kalman</td>
<td>3008.725</td>
<td>71.779</td>
<td>285.029</td>
<td>293.929</td>
<td>1152.731</td>
</tr>
<tr>
<td>1-D Kalman</td>
<td>3009.500</td>
<td>71.753</td>
<td>274.267</td>
<td>283.497</td>
<td>1081.556</td>
</tr>
</tbody>
</table>
Table 3 – Statistical values obtained from the different algorithms for the signal z.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Estimate's mean</th>
<th>Bias of Error</th>
<th>RMSD</th>
<th>RMSE</th>
<th>Maximal Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-D Kalman</td>
<td>5017.712</td>
<td>-14.174</td>
<td>469.401</td>
<td>469.615</td>
<td>1767.886</td>
</tr>
<tr>
<td>1-D Kalman</td>
<td>5021.790</td>
<td>-14.298</td>
<td>451.979</td>
<td>452.206</td>
<td>1649.259</td>
</tr>
<tr>
<td>1-D Stationary</td>
<td>5021.343</td>
<td>-14.648</td>
<td>432.605</td>
<td>432.865</td>
<td>1384.148</td>
</tr>
<tr>
<td>Simple MA</td>
<td>4710.869</td>
<td>-32.100</td>
<td>355.820</td>
<td>357.265</td>
<td>974.333</td>
</tr>
<tr>
<td>Optimal MA</td>
<td>4713.880</td>
<td>-29.027</td>
<td>436.138</td>
<td>437.103</td>
<td>833.336</td>
</tr>
</tbody>
</table>

Figure 1 – Estimates for the Signal x.
Figure 2 – Estimates for the Signal y.
Figure 3 – Estimates for the Signal z.
CONCLUSIONS

The comparison between the three different Kalman algorithms showed that the general behavior of the estimates are similar, only with a very minimal difference that is not possible to be seen on the Figures, only with the statistical data. For the comparison with the simple MA and the Optimally Unbiased MA estimates, it was possible to see that the filters can provide accurate estimates of the signals, but their problems are the transients. If we want to obtain reduced transients, the bias are going to increment, on the other hand, to obtain a reduced bias, the transients will increase. For the stationary object, this algorithms, specially the simple MA, can provide a good result in a very easy way (the filter is very simple).

ACKNOWLEDGEMENTS

This work was supported by CONACyT under the project J38818-A and the project “Apoyo a Actividades Académicas de los Programas de Postgrado de Excelencia de CONACyT”.

REFERENCES