An Equivalent Control Based Observer for Biomass in a Batch Process

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Abstract—In this paper a sliding-mode observer for a batch bioprocess, the δ-endotoxins production of *bacillus thuringiensis* (BT), is presented. The proposed observer is based on the equivalent control method and a class of second-order sliding mode operators. The use of these operators in the observer design allows the fixed-time convergence of the measured variables, while the unmeasured variables converge exponentially. This structure allows to estimate the biomass in the δ-endotoxins production of *BT*, even, under noisy measurement conditions. Simulations show the feasibility of the proposed observer. Convergence proofs are also presented.

I. INTRODUCTION

State estimation is a topic with a great interest in real applications such as automatic control, process monitoring and fault detection. Several results has been obtained using structures such as Kalman filters, Luenberger observers among others. In these type of estimation techniques the mainly idea is either, to obtain the estimation state by reducing the effect of noise or to design the observer to reach a specific performance in the error of the estimation [1].

Additionally, controllers or observers based on sliding mode (SM) are obtained by means of a non-smooth terms depending on the output error, into the controlling or observing system [2]. The SM approaches have been widely used for the problems of dynamic systems control and observation due to their characteristics of finite time convergence, robustness to uncertainties and insensitivity to external bounded disturbances [3], [4]. In this sense, by using that non-smooth function of the error to drive the sliding mode observer, the observer trajectories become insensitive to many forms of noise. Hence, some sliding mode observers have attractive properties similar to those of the Kalman filter (i.e. noise resilience) but with simpler implementation [5].

Taking advantage of the noise resilience feature, in this paper a SM observer design for the process of δ-endotoxins production is considered. The observer structure is based on the observer presented in [2] and it is an improved version of a previous work presented in [6]. The observer inputs are proposed applying the generalized super twisting algorithm (GSTA) [7], a fixed-time stable extension of the well known super twisting algorithm [8]. The proposed observer allows a filtered reconstruction of the biomass (vegetative cells and sporulated cells) in the reactor.

In the following, the Section II presents some mathematical preliminaries in order to introduce the basics of the GSTA and the fixed time stability. The Section III presents the mathematical model δ-endotoxins production with BT. The estimation structure is presented in Section IV. The Section V presents simulation results of the proposed observer. Finally, the conclusions of this paper are exposed in the Section VI.

II. MATHEMATICAL PRELIMINARIES

A lot of processes can be modeled by the dynamic system

$$\dot{\xi} = f(t, \xi) \tag{1}$$

where $\xi \in \mathbb{R}^n$ and $f : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^n$. If the function f is discontinuous (or non-smooth), the equation (1) is understood in Filippov sense [9]. The following definition is necessary in order to design controllers and observers.

Definition 2.1 (Globally fixed-time attraction [10]): Let a non-empty set $M \subset \mathbb{R}^n$. It is said to be globally fixed-time attractive for the system (1) if any solution $\xi(t, \xi_0)$ of (1) reaches M in some finite time moment $t = T(\xi_0)$ and the settling-time function $T : \mathbb{R}^n \to \mathbb{R}_+ \cup \{0\}$ is bounded by some positive number $T_{\max}$, i.e. $T(\xi_0) \leq T_{\max}$ for $\xi_0 \in \mathbb{R}^n$.

With the definition of a globally fixed-time attractive set, it is presented the GSTA [7], a fixed-time stable extension of the super twisting algorithm [8], as follows:

$$
\begin{aligned}
\dot{\xi}_1 &= -\lambda_1 \phi_1(\xi_1) + \xi_2 \\
\dot{\xi}_2 &= -\lambda_2 \phi_2(\xi_1) + \Delta,
\end{aligned} \tag{2}
$$

where, for $\xi \in \mathbb{R}$, $\phi_1(\xi) = \lfloor \xi \rceil^{\frac{1}{2}} + \theta \lfloor \xi \rceil^{\frac{3}{2}}$ and $\phi_2(\xi) = \phi_1(\xi)\frac{\partial \phi_1(\xi)}{\partial \xi} = \frac{1}{2}\lfloor \xi \rceil^0 + 2\theta\xi + \frac{3}{2}\theta^2\lfloor \xi \rceil^2$, with the parameter $\theta \geq 0$, the function $\lfloor \xi \rceil^\alpha = |\xi|^\alpha \text{sign}(\xi)$ is defined for $\alpha \geq 0$, where $\text{sign}(\xi) = 1$ for $\xi > 0$, $\text{sign}(\xi) = -1$ for $\xi < 0$ and $\text{sign}(0) \in \{-1, 1\}$; and λ_1, $\lambda_2 > 0$.

The fixed-time stability of the system (2) in spite of a persistent and bounded disturbance Δ and an estimation of its settling time are presented in [7], both based on Lyapunov stability analysis.

III. BATCH PROCESS MODEL

A model of the δ-endotoxins production of BT, which is proposed on [11], [12] is used here. The model equations are:

$$
\begin{aligned}
\dot{s}_p &= -\left(\frac{\mu}{y_{x/s}} + m_s\right) x_v \\
\dot{o}_d &= K_3 Q_A \left(o_d^* - o_d\right) - K_1 \left(\mu - k_e(t)\right) x_v - K_2 \left(x_v + x_s\right) \\
\dot{x}_v &= \left(\mu - k_s - k_e(t)\right) x_v \\
\dot{x}_s &= k_s x_v,
\end{aligned}
\tag{3}
$$

where s_p is the substrate concentration, o_d is the dissolved oxygen concentration, x_v is the vegetative cells concentration, x_s is the sporulated cells concentration, μ is the specific growth rate, $y_{x/s}$ is the growth yield, m_s is the maintenance constant, Q_A is the airflow that enters the bioreactor, o_d^* is the oxygen saturation concentration, K_1 is the oxygen consumption dimensionless constant by growth, K_2 is the oxygen consumption constant for maintenance, K_3 is the ventilation constant, k_s is the spore formation kinetics and $k_e(t)$ is the specific cell death rate. For the observer design purposes, it is assumed that continuous measurements of the outputs s_p and o_d are available.

Defining $x_1 = s_p$, $x_2 = o_d$, $x_3 = x_v$, $x_4 = x_s$, the model (3) can be written as:

$$
\begin{aligned}
\dot{x}_1 &= b_1(x_1, x_2)x_3 \\
\dot{x}_2 &= f_2(x_2) + b_{21}(x_1, x_2)x_3 + b_{22}x_4 \\
\dot{x}_3 &= b_3(x_1, x_2)x_3 \\
\dot{x}_4 &= b_4(x_1, x_2)x_3
\end{aligned}
\tag{4}
$$

with

$$
\begin{aligned}
b_1(x_1, x_2) &= -\left(\frac{\mu}{y_{x/s}} + m_s\right) \\
f_2(x_2) &= K_3 Q_A \left(o_d^* - o_d\right) \\
b_{21}(x_1, x_2) &= -K_1 \left(\mu - k_e(t)\right) - K_2 \\
b_{22} &= -K_2 \\
b_3(x_1, x_2) &= \mu - k_s - k_e(t) \\
b_4(x_1, x_2) &= k_s
\end{aligned}
$$

The nominal parameters and the constitutive equations details for the system (4) were given in reference [6]. Additionally, an observability analysis was performed in the same reference, concluding that since the function $b_1(x_1, x_2)$ is positive, then the system (4) with the measurements of the outputs s_p and o_d is observable.

IV. OBSERVATION SCHEME

A. Observer Structure

From the system model (4), inspired in the equivalent control observer structure [2] (to take advantage of its filtering capabilities), and using the GSTA algorithm (2) the following observer is proposed:

$$
\begin{aligned}
\dot{\hat{x}}_1 &= b_1(\hat{x}_1, \hat{x}_2)\hat{x}_3 + \lambda_{11}\phi_1(\tilde{x}_1) + v_1 \\
\dot{v}_1 &= \lambda_{12}\phi_2(\tilde{x}_1) \\
\dot{\hat{x}}_2 &= f_2(\hat{x}_2) + b_{21}(\hat{x}_1, \hat{x}_2)\hat{x}_3 + b_{22}\hat{x}_4 + \\
&\quad \lambda_{12}\phi_1(\tilde{x}_2) + v_2 \\
\dot{v}_2 &= \lambda_{22}\phi_2(\tilde{x}_2) \\
\dot{\hat{x}}_3 &= b_3(\hat{x}_1, \hat{x}_2)\hat{x}_3 + \lambda_3 \left[\lambda_{11}\phi_1(\tilde{x}_1) + v_1\right] \\
\dot{\hat{x}}_4 &= b_4(\hat{x}_1, \hat{x}_2)\hat{x}_3 + \lambda_4 \left[\lambda_{12}\phi_1(\tilde{x}_2) + v_2\right],
\end{aligned}
\tag{5}
$$

where $\hat{x}_1$, $\hat{x}_2$, $\hat{x}_3$ and $\hat{x}_4$ are the estimates of x_1, x_2, x_3 and x_4, respectively; $\tilde{x}_1 = x_1 - \hat{x}_1$ and $\tilde{x}_2 = x_2 - \hat{x}_2$ are the error variables; the observer input injections $\phi_1(\cdot)$ and $\phi_2(\cdot)$ are of the form presented in (2), and λ_{11}, λ_{12}, λ_{21}, λ_{22}, λ_3, λ_4 are the observer gains.

B. Convergence Analysis

Defining the additional error variables $\tilde{x}_3 = x_3 - \hat{x}_3$ $\tilde{x}_4 = x_4 - \hat{x}_4$, it follows

$$
\begin{aligned}
\dot{\tilde{x}}_1 &= b_1(x_1, x_2)x_3 - b_1(\hat{x}_1, \hat{x}_2)\hat{x}_3 - \lambda_{11}\phi_1(\tilde{x}_1) - v_1 \\
\dot{v}_1 &= \lambda_{12}\phi_2(\tilde{x}_1) \\
\dot{\tilde{x}}_2 &= f_2(x_2) - f_2(\hat{x}_2) + b_{21}(x_1, x_2)x_3 - b_{21}(\hat{x}_1, \hat{x}_2)\hat{x}_3 \\
&\quad + b_{22}\tilde{x}_4 - \lambda_{12}\phi_1(\tilde{x}_2) - v_2 \\
\dot{v}_2 &= \lambda_{22}\phi_2(\tilde{x}_2) \\
\dot{\tilde{x}}_3 &= b_3(x_1, x_2)x_3 - b_3(\hat{x}_1, \hat{x}_2)\hat{x}_3 \\
&\quad - \lambda_3 \left[\lambda_{11}\phi_1(\tilde{x}_1) + v_1\right] \\
\dot{\tilde{x}}_4 &= b_4(x_1, x_2)x_3 - b_4(\hat{x}_1, \hat{x}_2)\hat{x}_3 \\
&\quad - \lambda_4 \left[\lambda_{12}\phi_1(\tilde{x}_2) + v_2\right],
\end{aligned}
\tag{6}
$$

Define now the perturbation variables Δ_1 and Δ_2 as

$$
\begin{aligned}
\Delta_1 &= b_1(x_1, x_2)x_3 - b_1(\hat{x}_1, \hat{x}_2)\hat{x}_3 \tag{7} \\
\Delta_2 &= f_2(x_2) - f_2(\hat{x}_2) + b_{21}(x_1, x_2)x_3 \\
&\quad - b_{21}(\hat{x}_1, \hat{x}_2)\hat{x}_3 + b_{22}\tilde{x}_4. \tag{8}
\end{aligned}
$$

and the auxiliary variables

$$
\begin{aligned}
q_1 &= \Delta_1 - v_1 \tag{9} \\
q_2 &= \Delta_2 - v_2. \tag{10}
\end{aligned}
$$

Then, with the new variables (7)-(10), the error system (6) is transformed into:

$$
\begin{aligned}
\dot{\tilde{x}}_1 &= q_1 - \lambda_{11}\phi_1(\tilde{x}_1) \\
\dot{q}_1 &= -\lambda_{12}\phi_2(\tilde{x}_1) + \dot{\Delta}_1 \\
\dot{\tilde{x}}_2 &= q_2 - \lambda_{12}\phi_1(\tilde{x}_2) \\
\dot{q}_2 &= -\lambda_{22}\phi_2(\tilde{x}_2) + \dot{\Delta}_2 \\
\dot{\tilde{x}}_3 &= b_3(x_1, x_2)x_3 - b_3(\hat{x}_1, \hat{x}_2)\hat{x}_3 \\
&\quad - \lambda_3 \left[\lambda_{11}\phi_1(\tilde{x}_1) + v_1\right] \\
\dot{\tilde{x}}_4 &= b_4(x_1, x_2)x_3 - b_4(\hat{x}_1, \hat{x}_2)\hat{x}_3 \\
&\quad - \lambda_4 \left[\lambda_{12}\phi_1(\tilde{x}_2) + v_2\right],
\end{aligned}
\tag{11}
$$

where the disturbances Δ_1 and Δ_2 are assumed to be unknown but with bounded dynamics. Therefore, $\left|\dot{\Delta}_1\right| < \delta_1$ and $\left|\dot{\Delta}_2\right| < \delta_2$, and δ_1, δ_2 are known positive constants.

If the gains λ_{11}, λ_{12}, λ_{21} and λ_{22} are chosen such that $0 < \lambda_{11} \leq 2\sqrt{\delta_1}$, $0 < \lambda_{21} \leq 2\sqrt{\delta_2}$, $\lambda_{12} > \frac{\lambda_{11}^2}{4} + \frac{4\delta_1^2}{\lambda_{11}^2}$ and $\lambda_{22} > \frac{\lambda_{21}^2}{4} + \frac{4\delta_1^2}{\lambda_{21}^2}$; or $\lambda_{11} > 2\sqrt{\delta_1}$, $\lambda_{21} > 2\sqrt{\delta_2}$, $\lambda_{12} > 2\delta_1$ and $\lambda_{22} > 2\delta_2$, then a sliding mode appears in the system (11) on the manifold $(\tilde{x}_1, \tilde{x}_2, q_1, q_2) = (0, 0, 0, 0)$ in a fixed-time $t_q > 0$ [7].

Once the dynamics (11) is constrained to the manifold $(\tilde{x}_1, \tilde{x}_2, q_1, q_2) = (0, 0, 0, 0)$ the following two equivalent control signals are given as [3]:

$$\{\lambda_{11}\phi_1(\tilde{x}_1) + v_1\}_{\text{eq}} = b_1(x_1, x_2)\tilde{x}_3$$
$$\{\lambda_{12}\phi_1(\tilde{x}_2) + v_2\}_{\text{eq}} = b_{21}(x_1, x_2)\tilde{x}_3 + b_{22}\tilde{x}_4$$

and, therefore, the sliding mode dynamics is

$$\begin{aligned} \dot{\tilde{x}}_3 &= b_3(x_1, x_2)\tilde{x}_3 - \lambda_3 b_1(x_1, x_2)\tilde{x}_3 \\ \dot{\tilde{x}}_4 &= b_4(x_1, x_2)\tilde{x}_3 - \lambda_4 b_{21}(x_1, x_2)\tilde{x}_3 - \lambda_4 b_{22}\tilde{x}_4. \end{aligned} \quad (12)$$

The equation (12) is an affine system with vanishing disturbances. Since, the function $b_1(x_1, x_2)$ is positive, the gains λ_3 and λ_4 can be selected large enough such the the system is asymptotically stable [13].

V. SIMULATION RESULTS

The observer was applied to the model of δ-endotoxins production of bacillus thuringiensis (BT) and probed by mean of simulation. All simulations presented here were conducted using the Euler integration method, with a fundamental step size of $1 \times 10^{-5}[h]$. The model parameters are shown in reference [6]. The parameters were taken according to the range to $20\,[\text{g} \cdot \text{L}^{-1}] < s_{p,\text{max}} < 32\,[\text{g} \cdot \text{L}^{-1}]$. The value $s_{p,\text{max}}$ corresponds to the initial condition of s_p since $\dot{s}_p \leq 0$.

The initial conditions for the model were selected as: $x_1(0) = 32\,[\text{g} \cdot \text{L}^{-1}]$, $x_2(0) = 0.74 \times 10^{-2}\,[\text{L} \cdot \text{h}^{-1}]$, $x_3(0) = 0.645\,[\text{g} \cdot \text{L}^{-1}]$ and $x_4(0) = 1 \times 10^{-5}\,[\text{g} \cdot \text{L}^{-1}]$; and for the observer: $\hat{x}_1(0) = 32.64\,[\text{g} \cdot \text{L}^{-1}]$, $\hat{x}_2(0) = 0.7252 \times 10^{-2}\,[\text{L} \cdot \text{h}^{-1}]$, $\hat{x}_3(0) = 1.29\,[\text{g} \cdot \text{L}^{-1}]$, $\hat{x}_4(0) = 0.5\,[\text{g} \cdot \text{L}^{-1}]$, $v_1(0) = 0$ and $v_2(0) = 0$. The observer gains were adjusted to $\lambda_{11} = 10$, $\lambda_{12} = 5$, $\lambda_{21} = 4.4721$, $\lambda_{22} = 5$, $\lambda_3 = -10$ and $\lambda_4 = -6.8587 \times 10^3$. Finally, for the functions $\phi_1(\cdot)$ and $\phi_2(\cdot)$ it is taken $\theta = 1$.

This section is divided into two parts. In the first part, noiseless measurements of the substrate concentration $s_p = x_1$ and the dissolved oxygen concentration $o_d = x_2$ were assumed; in the second part instead, these measurements were assumed to be corrupted by a normally distributed random signal.

A. Noiseless Measurements

In this subsection, there is assumed no noise in the measurements. Figs. 1-4 show the comparison between the actual and estimated variables corresponding to substrate concentration $s_p = x_1$, dissolved oxygen concentration $o_d = x_2$, vegetative cell concentration $x_v = x_3$ and sporulated

cells concentration $x_s = x_4$, respectively, for noiseless measurements.
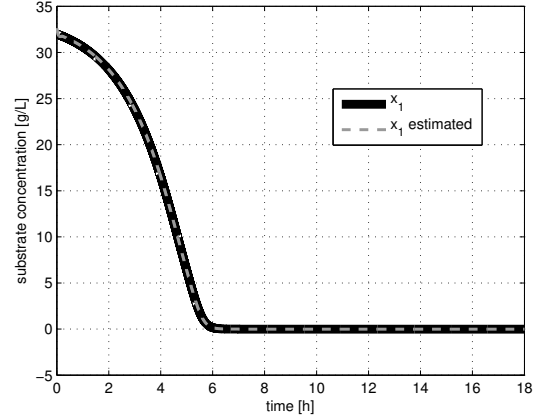

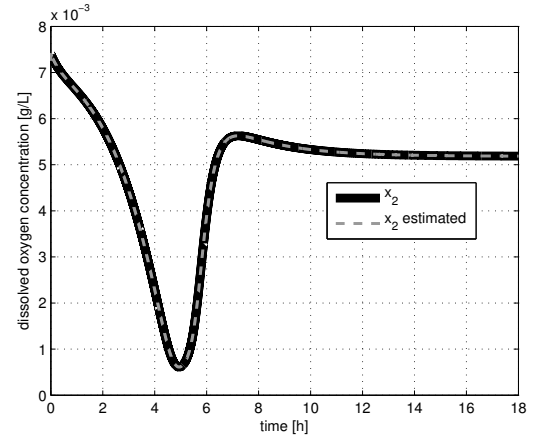Figure 1. Substrate concentration s_p (actual and estimated).

Figure 2. Dissolved oxygen concentration o_d (actual and estimated).
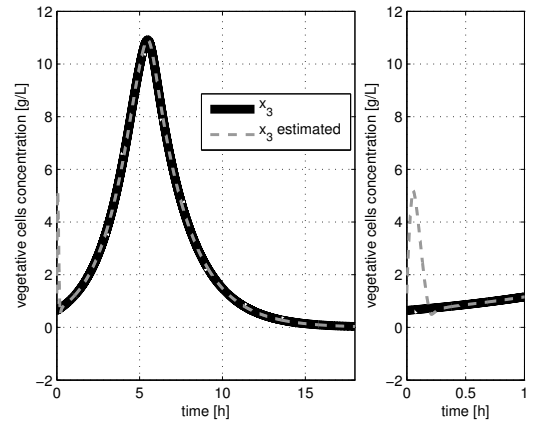

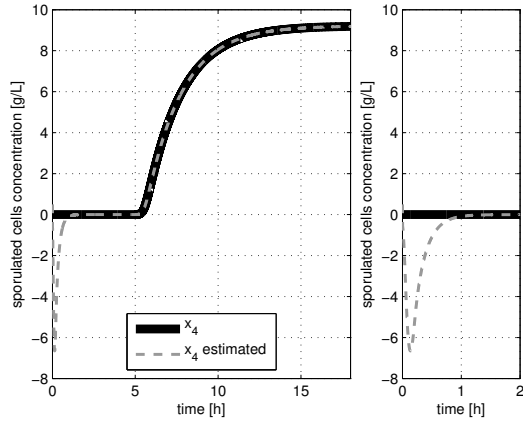Figure 3. Vegetative cells concentration x_v (actual and estimated).

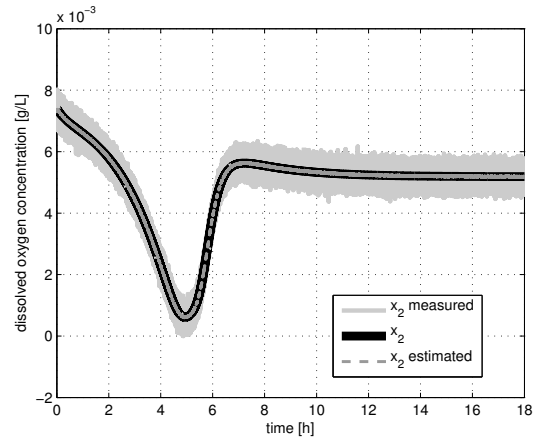Figure 4. Sporulated cells concentration x_s (actual and estimated).

Figure 6. Dissolved oxygen concentration o_d (measured, actual and estimated). Noisy measurements.

B. Noisy Measurements

In this subsection, the measurements of substrate concentration $s_p = x_1$ and dissolved oxygen concentration $o_d = x_2$ were assumed to be corrupted by a normally distributed random signals with zero mean, and variances of $\left(\frac{2}{3}\right)^2$ and $\left(\frac{5 \times 10^{-4}}{3}\right)^2$, respectively, which correspond to concentration sensors with an accuracy of $\pm 2[g/L]$ and $\pm 5 \times 10^{-4}[g/L]$, respectively. Figs. 5-8 show the comparison between the actual and estimated variables corresponding to substrate concentration $s_p = x_1$, dissolved oxygen concentration $o_d = x_2$, vegetative cell concentration $x_v = x_3$ and sporulated cells concentration $x_s = x_4$, respectively, for noisy measurements.
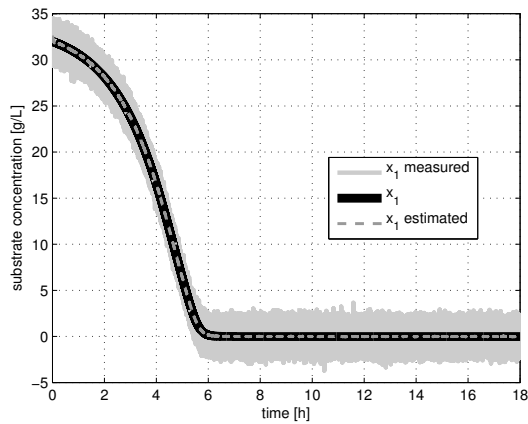

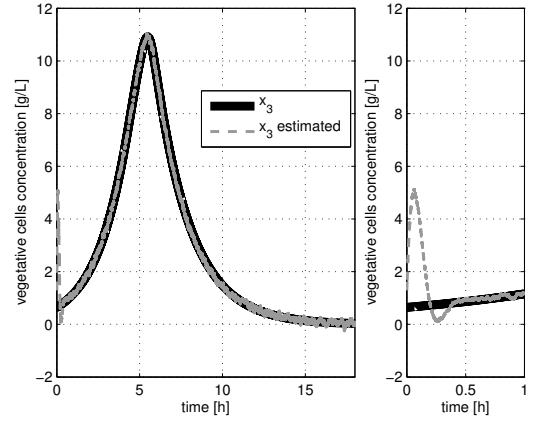Figure 7. Vegetative cells concentration x_v (actual and estimated). Noisy measurements.

Figure 5. Substrate concentration s_p (measured, actual and estimated). Noisy measurements.
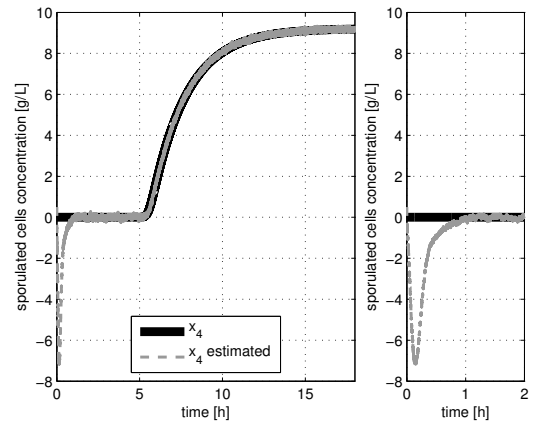

Figure 8. Sporulated cells concentration x_s (actual and estimated). Noisy measurements.

Based on the presented figures, it can be observed a good performance of the proposed scheme in both noiseless and

noisy measurement conditions. A correct and fast estimation of the state using the equivalent control based observer is achieved. Furthermore, under noisy measurement conditions, the estimations of the measured variables are much closer to their actual values than their measurements (Figs. 5 and 6). This performance is very important in real applications because, for instance, the variations in the estimated variables will generate errors in a control system, or false alarms in a fault detection system.

VI. CONCLUSION

An observer to estimate the biomass in a batch bioprocess was presented in this paper. The proposed observer structure was based on a class of second order sliding mode algorithms and the equivalent control method. This structure allowed a robust estimation of the biomass, even, under noisy measurement conditions. Convergence proofs were given and and numerical simulations showed the feasibility of the proposed observer.

Future work would be focus on the optimal selection of the observers gains, because this selection still depends on the designer expertise.

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