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Polynomial-Based Surrogate Modeling of RF and Microwave Circuits in Frequency Domain Exploiting the Multinomial Theorem

José L. Chávez-Hurtado and José E. Rayas-Sánchez

Abstract—A general formulation to develop EM-based polynomial surrogate models in frequency domain utilizing the multinomial theorem is presented in this paper. Our approach is especially suitable when the number of learning samples is very limited and no physics-based coarse model is available. We compare our methodology against other four surrogate modeling techniques: response surface modeling, support vector machines, generalized regression neural networks, and Kriging. Results confirm that our modeling approach has the best performance among these techniques when using a very small amount of learning base points on relatively small modeling regions. We illustrate our technique by developing a surrogate model for an SIW interconnect with transitions to microstrip lines, a dual band T-slot PIFA handset antenna, and a high-speed package interconnect. Examples are simulated on a commercially available 3D FEM simulator.

Index Terms—EM-based design, FEM, multinomial theorem, package interconnect, PIFA antenna, polynomial surrogates, SIW interconnect, surrogate modeling.

I. INTRODUCTION

Typically, direct optimization of microwave structures using full-wave electromagnetic (EM) simulators is computationally too expensive. A single EM simulation may take several hours, especially when using detailed physical models with high resolution discretization (fine EM model). Surrogate models can be exploited to accelerate the direct optimization process of high-frequency structures [1], [2], which additionally can be useful to hide the intellectual property of the EM design. In order to be useful for design optimization, surrogate models should be computationally cheap, smooth, and sufficiently accurate in the region of interest for the design parameters.

We can identify two types of surrogate models: physical and functional surrogates. A physical surrogate is usually implemented by a quasi-static approximation or an equivalent circuit; it can also be implemented in the same EM simulator used for the original structure under design, but using a coarse discretization [3] and removing some details of the original structure to speed up the simulation time [4]. However, these simplified and coarsely discretized full-wave EM models may exhibit numerical noise, discontinuous behavior, and non-negligible simulation time with respect to the corresponding original fine model [5].

On the other hand, functional surrogates, also called metamodels, are generated by using a set of learning base points from the original fine EM model. These learning base points are usually selected within a specified modeling region and can be allocated using design of experiments (DoE) techniques [3]. Functional surrogate approaches treat the EM model as a black box, with the goal to approximate the relationship between inputs and outputs of the structure under study [6].

Different approaches can be used to develop functional surrogate models for EM-based design. Among the most popular are: response surface methodology (RSM), artificial neural networks (ANN), support vector machines (SVM) and Kriging.

RSM uses a second-order polynomial to generate a surrogate model that represents the relationship between model inputs and outputs [7]-[9]. The number of unknowns in the second-order approximation defines the required number of function evaluations [7]. Some RSM techniques, such as the D-optimal DoE, require twice that number of function evaluations [10].

ANN can be seen as non-linear data modeling tools capable of representing complex relationships between model inputs and outputs [11], [12]. A multilayer ANN can approximate any input-output deterministic relationship by using a suitable amount of data and the correct number of hidden neurons [13]. Generally, the learning error hypersurface contains many local minima, making global optimization algorithms more desirable to train the ANN [14]. Additionally, the number of learning base points needed to approximate a function grows exponentially with the ratio between dimensionality and the degree of smoothness [13]. Generalized regression neural network (GRNN) is a special type of ANN that does not require an iterative training procedure [15]. The number of neurons in the hidden layer of a GRNN is equal to the number of learning samples. Some advantages of GRNN are fast learning and convergence to the optimal regression surface as the number of samples become very large, and no need of a minimum set of learning base points to train the neural network [16], [17]. Abundant examples of ANN-based surrogate modeling for microwave structures have been published, e.g., [13], [14], [18], [19].

SVM modeling solves a constrained quadratic optimization
problem, allowing to find a global optimum for the model parameters. The optimization problem is feasible due to the utilization of a kernel function, including linear, polynomial, and radial basis functions. The radial basis function is the most employed kernel since it creates a nonlinear map, taking the samples into a higher dimensional space with less numerical difficulties to find an optimum solution [20], [21]. SVM models are trained by using the structural risk minimization principle, instead of the empirical risk minimization principle used by ANN models. The structural risk minimization principle allows SVM models to have a good trade-off between model complexity and generalization capability [22]. Since SVM models are based on small sample statistical learning theory, an optimum solution can be found by using a limited number of samples [21]. Examples of SVM modeling for microwave structures can be found in [23]-[26].

Kriging methodology exploits the best linear unbiased estimator (BLUE) of the output value for a given input to choose the weighting factors that minimize the prediction variance [27]. Implementation of RSM and ANN models usually employ classical DoE, where extreme scenarios are commonly simulated, such as the corners of the corresponding modeling region. However, Kriging methodology is based on space-filling experiments, usually implemented by using the Latin hypercube design (LHD) [27]. Kriging models aim at covering the whole experimental area, consequently, they can be seen as a global metamodels [28]. If there are not enough function evaluations of the EM model, the Kriging estimated correlation function tends to be noisy and the predictions become inaccurate [27]. Different Kriging techniques have been developed, such as simple Kriging, ordinary Kriging, universal Kriging, Taylor Kriging, and dynamic Kriging [29]. Examples of Kriging surrogate modeling of microwave structures can be found in [30], [31].

The surrogate modeling methodology proposed in this work exploits the multinomial theorem to represent the relationship between model inputs and outputs by using polynomial functions at each simulated frequency point. Corresponding surrogate model weighting factors are calculated in closed form by using frugal learning base point distributions (e.g., star or box distribution), achieving a global minimum in the least squares sense. In contrast to RSM, the order of the polynomial function is not fixed and can be increased until generalization performance deteriorates.

A similar approach to develop polynomial surrogate models is proposed in [32], however, our new approach differs in three aspects: the surrogate model formulation, the calculation of weighting factors, and the surrogate order determination. For the surrogate model formulation, [32] implements the Nth order surrogate model by using an element-wise power operator, which creates some redundant terms, while our new formulation exploits the multinomial theorem, allowing us to expand a polynomial raised to an arbitrary power including all cross terms and no redundant terms. For the weighting factors calculation, the approach in [32] calculates simultaneously all weighting factors available for each surrogate model order. In contrast, the proposed approach in this work automatically calculates the weighting factors by assuming that lower-order surrogates are fixed or by calculating all weighting factors simultaneously, and the selection between both manners is based on the conditional number of the system matrix. Finally, the order of the surrogate model can be different for each simulated frequency point, while in [32] and [33] the same surrogate model order is used for all simulated frequency points.

The present article expands our work in [34] by: a) presenting the general formulation for the Nth-order surrogate model; b) comparing and discussing the number of weighting factors required by the proposed new polynomial-based surrogate modeling (PSM) methodology versus the polynomial formulation presented in [32]; c) developing two additional surrogate modeling examples: a single-layer substrate integrated waveguide (SIW) interconnect with transitions to microstrip lines and a high-speed package interconnect; d) comparing at some testing base points the actual EM responses of the original fine model and the corresponding PSM model responses; and e) presenting and discussing the performance of our PSM proposal when varying the size of the region of interest, as well as some potential applications.

The organization of this paper is as follows: Section II describes the general PSM formulation. Section III presents the PSM training, including the selection of the polynomials order. Section IV shows the performance of our proposal for three surrogate modeling examples, comparing it versus other approaches. Section V elaborates on the modeling region size and potential applications. Finally, Section VI concludes this work.

II. GENERAL PSM FORMULATION

Let \( \mathbf{R}_t \in \mathbb{R}^p \) denote a fine model response sampled at \( q \) frequency points. We assume that \( \mathbf{R}_t \) only depends on the design variables \( \mathbf{x} \in \mathbb{R}^n \). We treat the fine model as a multidimensional vector function, \( \mathbf{R}(\mathbf{x}) : \mathbf{x} \rightarrow \mathbb{R}^q \) whose domain is \( \mathbf{x}_f \subseteq \mathbb{R}^n \). We aim to develop a surrogate model \( \mathbf{R}(\mathbf{x}) : \mathbf{x} \rightarrow \mathbb{R}^q \) that approximates \( \mathbf{R}(\mathbf{x}) \) in a region of interest \( \mathbf{x}_f \subseteq \mathbf{x}_f \), around the reference design \( \mathbf{x}(0) \), where \( \Delta \mathbf{x} \in \mathbb{R}^n \) is the distance from a given design to that reference. \( \Delta \mathbf{x} = \mathbf{x} - \mathbf{x}(0) \).

A. Nth-order Surrogate Model

A general expression for an Nth-order surrogate model at the \( k \)-th simulated frequency point can be written as

\[
R^{(N)}_k(\mathbf{x}) = R^{(N-1)}_k(\mathbf{x}) + w^{(N)}_k \mathbf{q}^{(N)}(\Delta \mathbf{x})
\]

where \( w^{(N)}_k \in \mathbb{R}^{(N + n - 1)/(n - 1)/N(n-1)/(N-1)} \) has the weighting factors and \( \mathbf{q}^{(N)}(\Delta \mathbf{x}) \in \mathbb{R}^{(N + n - 1)/(n - 1)/N(n-1)/(N-1)} \) the multinomial terms for \( \Delta \mathbf{x} \).

The scalar elements of \( \mathbf{q}^{(N)} \) are given by

\[
q^{(N)}(\Delta \mathbf{x}) = \Delta x_{j_1} \Delta x_{j_2} \cdots \Delta x_{j_n}
\]

for \( j_1 = 1 \), \( j_2 = \beta_1 \), \( \beta_2 = \beta_1 \), \( \ldots \), \( j_N = \beta_{N-1} \).

The zero-order model response is equal to \( \mathbf{R}_t(\mathbf{x}(0)) \) at all frequency points, as described in [34]. The first-order surrogate model is formulated by including weighting factors \( w^{(1)}_k \) and multinomial terms \( \mathbf{q}^{(1)} \). The second-order surrogate model is formulated by incorporating the second-order terms \( w^{(2)}_k \) and \( \mathbf{q}^{(2)} \), etc. See [34] for more details.
Fig. 1. Comparison in terms of total number of weighting factors for both polynomial surrogate formulations: a) old formulation [32] (solid line) and b) new formulation (dashed line).

B. Comparing Polynomial Surrogate Model Structures

The total number of weighting factors, \( N_w \), used by the \( N \)-th order polynomial surrogate model employed in [32] is

\[
N_w = (N-1)n^2 + n
\]

(3).

On the other hand, our new polynomial surrogate model uses

\[
N_w = n + n(n+1) + \frac{n(n+1)(n+2)}{2!} + \frac{n(n+1)(n+2)(n+3)}{3!} + \ldots + \frac{n(n+1)(n+2)(n+3)\ldots(n+n-1)!}{N(n-1)!}
\]

(4).

A comparison between both formulations in terms of the total number of weighting factors is realized in Fig. 1. Both formulations use the same number of weighting factors in the following cases: a) when \( N = 1 \), for any \( n \); b) when \( n = 1 \), for any \( N \); c) when \( N = 3 \), for \( n = 5 \); and d) when \( N = 4 \), for \( n = 2 \). The new formulation uses a smaller number of weighting factors in two cases: a) when \( N = 2 \), for any \( n \); and b) when \( N = 3 \), for \( n \leq 4 \). Moreover, since our new formulation does not incorporate redundant terms, the generalization performance is expected to be better than in [32], at least in all the previously mentioned cases, as it was experimentally confirmed in [33].

III. PSM Training

“Training” the surrogate models formulated in the previous section is done by calculating in closed form the corresponding weighting factors. We define a region of interest \( X_r \) delimited by a vector \( \tau \in \mathbb{R}^n \) containing the relative deviations for each design variable with respect to \( x^{(0)} \). The corresponding weighting factors are calculated in closed form by using \( L \) learning base points within \( X_r \), denoted as \( x^{(1)} \), \( x^{(2)} \), …, \( x^{(L)} \). The generalization performance is measured by using \( T \) testing base points within the same region. Weighting factors can be calculated in two different forms: a) by reusing lower-order weights and calculating only current-order weights, or b) by calculating all weights simultaneously for each surrogate model order.

A. Weighting Factors Calculation Reusing Weights

Considering that the lower-order weights are already calculated and fixed, the fine model and the \( N \)-th order surrogate model are matched at the \( j \)-th learning base point,

\[
R_0(x^{(j)}) = R_0^{(N-1)}(x^{(j)}) + w_k^{(N)}q^{(N)}(\Delta x^{(j)})
\]

(5), for \( j = 1, \ldots, L \). The \( N \)-th order weights are then calculated by solving for \( w_k^{(N)} \) the following system of linear equations

\[
Q^{(N)}w^{(N)} = \Delta R_k^{(N)} \quad \text{for} \quad k = 1, \ldots, p
\]

(6), with \( Q^{(N)} \in \mathbb{R}^{(N-n-1)!((n-1)!)} \) and \( \Delta R_k^{(N)} \in \mathbb{R}^T \) defined as

\[
Q^{(N)} = \begin{bmatrix}
q^{(N)}(\Delta x^{(1)})^T \\
q^{(N)}(\Delta x^{(2)})^T \\
\vdots \\
q^{(N)}(\Delta x^{(L)})^T
\end{bmatrix}
\]

(7),

\[
\Delta R_k^{(N)} = \begin{bmatrix}
R_{lk}(x^{(1)}) - R_{lk}^{(N-1)}(x^{(1)}) \\
R_{lk}(x^{(L)}) - R_{lk}^{(N-1)}(x^{(L)})
\end{bmatrix}
\]

(8).

B. Calculating All Weighting Factors Simultaneously for Each Surrogate Model Order

Consider now that all the weighting factors are simultaneously calculated at each surrogate model order. The corresponding responses of the fine model and the \( N \)-th order surrogate model are matched at the \( j \)-th learning base point,

\[
R_0(x^{(j)}) = R_0^{(N-1)}(x^{(j)}) + w_k^{(N)}q^{(N)}(\Delta x^{(j)}) + \ldots + w_k^{(N)}q^{(N)}(\Delta x^{(j)})
\]

(9), for \( j = 1, \ldots, L \). Weighting factors are then calculated by solving for \( W^{(N)} \) the following system of linear equations

\[
Q_0^{(N)}W^{(N)} = \Delta R_k^{(N)} \quad \text{for} \quad k = 1, \ldots, p
\]

(10), with \( Q_0^{(N)}W^{(N)} \in \mathbb{R}^{(N-n-1)!((n-1)!)} \) and \( C \) defined as

\[
Q_0^{(N)} = \begin{bmatrix}
Q^{(1)} \\
Q^{(2)} \\
\vdots \\
Q^{(L)}
\end{bmatrix},
\]

(11),

\[
W^{(N)} = \begin{bmatrix}
w_k^{(1)} \\
w_k^{(2)} \\
\vdots \\
w_k^{(N)}
\end{bmatrix},
\]

(12),

\[
C = \sum_{1}^{N}(N+n-1)!/(n-1)!(N-n)!)
\]

(13).

C. Selecting Weighting Factors Calculation

The selection between the both previously defined forms to calculate the weighting terms is automatically realized in our surrogate modeling algorithm by comparing the condition number of matrices \( Q^{(N)} \) in (6) and \( Q_0^{(N)} \) in (10). If the condition number of matrix \( Q^{(N)} \) is smaller than the condition number of matrix \( Q_0^{(N)} \), weighting factors are calculated by solving (6), otherwise, by solving (10). This test is inexpensive, since no fine model evaluations are implied. In summary, we solve the best conditioned system of linear equations between (6) and (10), which only depends on the input learning base points and the order of the current polynomial surrogate.

D. Setting the Order of the Polynomial Function

As mentioned before, a polynomial function is developed at each simulated frequency point. The order of that polynomial is increased until generalization error deteriorates. This allows us to use a different polynomial order at each simulated frequency point, depending on the behavior, within the region of interest, of the EM response of the structure under study.

IV. EXAMPLES

For developing the surrogate models in all the following
examples, we employ a small amount of learning base points by using star and box distributions. The resultant number of base points are $2\pi$ and $2^n$, respectively. For reliable assessment of the generalization performance we use $K^n$ testing base points uniformly distributed, where $K$ corresponds to the number of points for each design parameter.

For all the examples, we calculate the maximum absolute error in all the learning base points at each simulated frequency point, denoted as $\epsilon_l$, as well as the maximum absolute error in all the testing base points at each simulated frequency point, denoted as $\epsilon_t$. Since both errors, $\epsilon_l$ and $\epsilon_t$, are calculated from S-parameters, they are dimensionless. To denote the largest maximum absolute error in the complete frequency sweep we use $\epsilon_{l_{\text{max}}}$ and $\epsilon_{t_{\text{max}}}$, for the learning and testing sets, respectively.

The corresponding polynomial-based surrogate model performance for each example is compared with other four functional surrogate modeling techniques: RSM, SVM, Kriging and GRNN. SVM, Kriging and GRNN are implemented using the corresponding Matlab Toolboxes with the default settings.

In all the following examples, we perform the EM simulations using a conventional laptop (core i5 CPU with 8 GB RAM).

A. SIW Interconnect with Transitions to Microstrip Lines

Consider the SIW interconnect with transitions to microstrip lines proposed in [32], whose geometry is shown in Fig. 2. The purpose of the tapered microstrip transitions is to perform field conversion and impedance matching of the two dissimilar guiding structures [35].

1) Design Parameters

The SIW is designed on a substrate with a relative permittivity $\varepsilon_r = 3.6$ and a thickness $H = 0.4064$ mm. The SIW interconnect has a length $L_{\text{SIW}} = 4W$ and an external width $W = 9.6446$ mm. According to the required cutoff frequency for the dominant mode $f_{10} = 10$ GHz, the internal width of the SIW interconnect is $W_{\text{SIW}} = 8.6845$ mm [32]. Vias have a diameter $d = 0.4801$ mm and are separated from their neighboring via by a center-to-center spacing $s = 2d$. The microstrip lines have a width $W_p = 0.8672$ mm and a length $L_p = 1.5W$. The transition uses $L_{\text{tap}} = 3W_{\text{SIW}}$ and $W_{\text{tap}} = (W_{\text{SIW}} + W_p)/2$.

Our model, implemented in COMSOL, neglects dielectric and metallic losses. Lateral and upper walls of the simulation bounding box are configured as scattering boundary condition. We follow a meshing scheme by zones as proposed in [36] and the bounding box dimensions proposed in [37]. We use a fine resolution discretization. Each frequency sweep takes around 1 h 20 min.

2) Surrogate Model Implementation

We develop a polynomial surrogate model for $|S_{21}|$ as a function of $x = [W_{\text{tap}}, L_{\text{tap}}]^T$ (see Fig. 2), using as a reference design $x^{(0)} = [(W_{\text{SIW}} + W_p)/2, 3W_{\text{SIW}}]^T$, over a region defined by $\tau = [5\% \ 5\%]^T$. EM response of the SIW interconnect at $x^{(0)}$ is shown in Fig. 3. To measure the generalization performance of the resultant polynomial surrogate we use a uniform distribution with $K = 4$ (resulting 16 testing base points).

The maximum absolute testing errors as well as the order of the polynomial implemented at each simulated frequency point, for both learning base point distributions, are shown in Fig. 4. Notice that there are some frequency points where the polynomial reaches the $10^{th}$ order. This is due to the high nonlinearity of the EM responses, within the modeling region, at those frequency points.

EM responses of the surrogate model at some testing base
points, with polynomial surrogates trained with both learning base point distributions, are shown in Fig. 5. We can see there is a very good match between the fine and the surrogate model responses at the testing base points.

3) Performance Comparison

Using the same learning and testing base point distributions, the corresponding maximum absolute testing errors at each simulated frequency point, for the five implemented surrogate models, are shown in Fig. 6. Numerically, Table I shows the largest maximum absolute learning and testing errors for each implemented surrogate model. These results show that the proposed polynomial-based surrogate model achieves the best generalization performance with both learning base point distributions.

B. Dual-Band Planar Inverted F Handset Antenna with Slotted Ground

For the second example, consider the T-slot planar inverted F handset (PIFA) antenna proposed in [38]. Its geometry is shown in Fig. 7. The bandwidth is increased by removing two portions of the metallization at the ground plane. The antenna design is intended to operate at the following bands: GSM900 (880-960 MHz), GSM1900 (1850-1990 MHz), UMTS2100 and WCDMA2100 (1920-2170 MHz).

1) Design Parameters

The PIFA is implemented on a substrate with a relative permittivity \(\varepsilon_r = 2.2\), a dielectric loss tangent \(\tan(\delta) = 0.009\) and a thickness \(H = 3.962\) mm. The design parameters values are defined as \(W_1 = 3.83\) mm, \(W_2 = 8.85\) mm, \(W_3 = 11\) mm, \(W_4 = 1.54\) mm, \(L_1 = 8.10\) mm, \(L_2 = 20.34\) mm, \(L_3 = 24\) mm, \(Y_1 = 14\) mm, \(X_1 = 19.16\) mm, \(Y_2 = 18.9\) mm and, \(X_2 = 1\) mm.

The structure is implemented in COMSOL by using the simulation bounding box dimensions, boundary conditions, and the meshing scheme proposed in [38]. All metals are defined as perfect electric conductors. We use a relatively coarse resolution discretization, such that each frequency sweep takes around 2 min 20 s.

2) Surrogate Model Implementation

To develop the surrogate model, we define as design variables the parameters \(x = [W_1 \ W_2 \ L_1 \ L_2]^T\), using as a reference design \(x^{(0)} = [3.83 \ 8.85 \ 8.10 \ 20.34]^T\) (mm), over a region defined by \(x = \pm [5\% \ 5\% \ 5\% \ 5\%]^T\). The EM response of the antenna at \(x^{(0)}\) is illustrated in Fig. 8. To

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**TABLE I**

<table>
<thead>
<tr>
<th>SIW INTERCONNECT SURROGATE MODELS PERFORMANCE FOR</th>
<th>star distribution</th>
<th>box distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>model</td>
<td>(\epsilon_{T\max})</td>
<td>(\epsilon_{L\max})</td>
</tr>
<tr>
<td>RSM</td>
<td>0.16921</td>
<td>1.93e-14</td>
</tr>
<tr>
<td>PSM</td>
<td>0.11067</td>
<td>0.08567</td>
</tr>
<tr>
<td>SVM</td>
<td>0.16160</td>
<td>0.14324</td>
</tr>
<tr>
<td>Kriging</td>
<td>0.19523</td>
<td>0.12441</td>
</tr>
<tr>
<td>GRNN</td>
<td>0.20462</td>
<td>0</td>
</tr>
</tbody>
</table>

---
measure the generalization performance of the resultant surrogate model we use a uniform distribution of testing base points with \( K = 3 \) (resulting 81 testing base points).

For both learning base point distributions, the maximum absolute testing errors as well as the order of the polynomial implemented at each simulated frequency points are shown in Fig. 9. Notice that, for the star distribution, a polynomial order higher than 2 is not required, while for the box distribution, a polynomial order higher than 4 is not required.

EM responses of the surrogate model at some testing base points, with polynomial surrogates trained with both learning base point distributions, are shown in Fig. 10. It is seen that there is an excellent match between the fine and the surrogate model responses at the testing base points.

3) Performance Comparison

Using the same learning and testing base point distributions, the corresponding maximum absolute testing errors at each simulated frequency point, for the five implemented surrogate models, are shown in Fig. 11. Numerically, Table II shows the largest maximum absolute learning and testing errors for the
surrogate models. Again, these results indicate that the proposed polynomial-based surrogate model achieves the best generalization performance.

C. Package Via-Stripline Interconnect

Finally, as a third example, consider the package via-stripline-via interconnect shown in Fig. 12 (similar to that one in [39]). The interconnect starts with a coaxial port in contact with a via pad, whose via connects the top and middle layers. In the middle layer there is a stripline. At the end of the stripline there is another via that goes up to the top layer and connects to the other coaxial port.

1) Design Parameters

The interconnection is implemented on an FR4 substrate with a relative dielectric permittivity $\varepsilon_r = 3.34$, a loss tangent $\tan \delta = 0.018$, and thickness $H_1 = H_2 = 22.5 \mu$m. The length and width of the stripline are $l_{tr} = 3000 \mu$m and $W_{tr} = 21 \mu$m, respectively. Vias, pads and antipads radius are $r_{v1} = r_{v2} = 26 \mu$m, $r_{p1} = r_{p1} = r_{p2} = 54 \mu$m, $r_{a1} = r_{a2} = 104 \mu$m, $r_{a1} = r_{a2} = 220 \mu$m. Distances from the interconnect to lateral and front walls are $x_{gap} = y_{gap} = 280 \mu$m, respectively. Lossless metals with thickness $t = 15 \mu$m are used (thick metals).

The structure is implemented in COMSOL by using the simulation bounding box dimensions, boundary conditions, coaxial ports, and the meshing scheme proposed in [39]. We use a relatively coarse resolution discretization, such that each frequency sweep takes approximately 3 min.

2) Surrogate Model Implementation

For developing the surrogate model of the interconnect we use as input parameters the pads radius with $r_{p1} = r_{p1} = r_{p2}$. Then, $x = r_{1p}$. We use as a reference design $x^{(0)} = 54$ (\mu m) over a region defined by $\tau = \pm 5\%$. The EM response of the interconnect at $x^{(0)}$ is shown in Fig. 13.

To measure the generalization performance of the resultant polynomial surrogate model we use a uniform distribution with $K = 10$. Since this is a one-design variable example, star and box distribution yields to the same 2 learning base points, corresponding to the limits of the modeling region.

The maximum absolute testing errors as well as the order of the polynomial implemented at each simulated frequency point are shown in Fig. 14. Notice that a polynomial order higher than 2 is not required.
The EM responses of the surrogate model at some testing base points are shown in Fig. 15. As in the previous examples, a very good match is obtained between the fine and the surrogate model responses at the testing base points.

3) Performance Comparison

Performance comparison for the five implemented surrogate models is illustrated in Fig. 16. Numerically, Table III shows the largest maximum absolute learning and testing errors for each implemented surrogate model. Once again, the results confirm that the polynomial surrogate model achieves the best generalization performance.

V. DISCUSSION ON THE MODELING REGION SIZE AND POTENTIAL APPLICATIONS

In all the previous examples (five different cases in total), we consider a relatively small modeling region size, using symmetric deviations of 5% for each design variable. The corresponding results for the surrogate models show that the proposed PSM exhibits the best performance, according to the generalization error, in all the five cases. We also implemented these examples using larger regions, with deviations of ±10% and ±15% for each design variable. The corresponding numerical results are in Tables IV to IX. It is seen that when using deviations of ±10%, the proposed PSM has the best performance in 4 out of the 5 cases, while when using deviations of ±15%, PSM shows the best performance in 3 out of the 5 cases. In those cases where PSM does not have the best performance, it still shows a very competitive behavior with respect to the surrogate modeling technique with best performance, which turned out different for each case. We have observed this same general trend when using even larger regions, finding that, when using a large modeling region size combined with a small number of learning base points, none of these surrogate modeling approaches consistently exhibits the best performance.

Our PSM technique can be useful for statistical analysis and yield estimations of RF and microwave circuits. A reliable Monte Carlo yield prediction requires a very high number of EM fine simulations to cover the entire statistics of possible outcomes [40]. To reduce this high computational cost, a PSM could be developed around the nominal design considering typical parameter tolerances due to the manufacturing process variability, which are usually small. Then, the statistical analysis could be realized directly on the PSM surrogate model, which is computationally cheap to develop, extremely fast to evaluate, and accurate for a small design region.

Our PSM methodology could also be used for design optimization of RF and microwave circuits. At each iteration, a
TABLE IV
SIW INTERCONNECT SURROGATE MODELS PERFORMANCE FOR $|S_{11}|$

<table>
<thead>
<tr>
<th>model</th>
<th>star distribution</th>
<th>box distribution</th>
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<tr>
<td></td>
<td>$\epsilon_{\text{Tmax}}$</td>
<td>$\epsilon_{\text{Lmax}}$</td>
</tr>
<tr>
<td>RSM</td>
<td>0.24860</td>
<td>0.59749</td>
</tr>
<tr>
<td>PSM</td>
<td>0.18947</td>
<td>0.09072</td>
</tr>
<tr>
<td>SVM</td>
<td>0.40140</td>
<td>0.28565</td>
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<tr>
<td>Kriging</td>
<td>0.41565</td>
<td>0.26696</td>
</tr>
<tr>
<td>GRNN</td>
<td>0.32309</td>
<td>0.0</td>
</tr>
</tbody>
</table>

TABLE V
PIFA ANTENNA SURROGATE MODELS PERFORMANCE FOR $|S_{11}|$

<table>
<thead>
<tr>
<th>model</th>
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<tr>
<td></td>
<td>$\epsilon_{\text{Tmax}}$</td>
<td>$\epsilon_{\text{Lmax}}$</td>
</tr>
<tr>
<td>RSM</td>
<td>0.31910</td>
<td>0.62052</td>
</tr>
<tr>
<td>PSM</td>
<td>0.22117</td>
<td>0.11758</td>
</tr>
<tr>
<td>SVM</td>
<td>0.40407</td>
<td>0.14461</td>
</tr>
<tr>
<td>Kriging</td>
<td>0.28935</td>
<td>0.01368</td>
</tr>
<tr>
<td>GRNN</td>
<td>0.38304</td>
<td>0.14887</td>
</tr>
</tbody>
</table>

TABLE VI
PACKAGE INTERCONNECT SURROGATE MODEL PERFORMANCE FOR $|S_{11}|$

<table>
<thead>
<tr>
<th>model</th>
<th>$\epsilon_{\text{Tmax}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>RSM</td>
<td>0.01062</td>
</tr>
<tr>
<td>PSM</td>
<td>0.01062</td>
</tr>
<tr>
<td>SVM</td>
<td>0.02667</td>
</tr>
<tr>
<td>Kriging</td>
<td>0.02592</td>
</tr>
<tr>
<td>GRNN</td>
<td>0.01655</td>
</tr>
</tbody>
</table>

TABLE VII
SIW INTERCONNECT SURROGATE MODELS PERFORMANCE FOR $|S_{21}|$

<table>
<thead>
<tr>
<th>model</th>
<th>star distribution</th>
<th>box distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\epsilon_{\text{Tmax}}$</td>
<td>$\epsilon_{\text{Lmax}}$</td>
</tr>
<tr>
<td>RSM</td>
<td>0.33525</td>
<td>0.36726</td>
</tr>
<tr>
<td>PSM</td>
<td>0.22262</td>
<td>0.16685</td>
</tr>
<tr>
<td>SVM</td>
<td>0.47614</td>
<td>0.34309</td>
</tr>
<tr>
<td>Kriging</td>
<td>0.49633</td>
<td>0.31750</td>
</tr>
<tr>
<td>GRNN</td>
<td>0.57570</td>
<td>0.0</td>
</tr>
</tbody>
</table>

TABLE VIII
PIFA ANTENNA SURROGATE MODELS PERFORMANCE FOR $|S_{21}|$

<table>
<thead>
<tr>
<th>model</th>
<th>star distribution</th>
<th>box distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\epsilon_{\text{Tmax}}$</td>
<td>$\epsilon_{\text{Lmax}}$</td>
</tr>
<tr>
<td>RSM</td>
<td>0.49730</td>
<td>0.76730</td>
</tr>
<tr>
<td>PSM</td>
<td>0.35426</td>
<td>0.17725</td>
</tr>
<tr>
<td>SVM</td>
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<td>0.22893</td>
</tr>
<tr>
<td>Kriging</td>
<td>0.40881</td>
<td>0.02462</td>
</tr>
<tr>
<td>GRNN</td>
<td>0.51803</td>
<td>0.23650</td>
</tr>
</tbody>
</table>

PSM model with a reduced number of learning base points around the current iterate can be obtained on a relatively small region; the optimal solution of the current PSM model would yield the next iterate, around which a new PSM model can be obtained, etc. This approach should require a smaller number of EM fine model simulations than that one required by direct EM optimization.

A third potential application for the proposed PSM methodology consists of using it to build mathematical maps between coarse and fine model design variables, for input space mapping approaches [41]. Since the relationship between fine and coarse model variables is usually not too complex [42], the use of low-order polynomial surrogates is appropriate to represent it. This application is currently being developed by the authors.

VI. CONCLUSIONS

We presented a general formulation for a polynomial surrogate modeling (PSM) approach exploiting the multinomial theorem. Some features of our proposal are: a) the polynomial is expanded using the multinomial theorem, which includes all cross terms and avoids redundant terms; b) globally optimal weighting factors are calculated in closed form using two different approaches, automatically selecting one of them according to the best conditioned system; c) a minimum number of learning base points is not required for developing the surrogate model; and d) the order of the polynomial can be different at each simulated frequency point, according to the local generalization performance.

Our PSM methodology was compared with four surrogate modeling techniques: response surface methodology, support vector machines, generalized regression neural networks and Kriging; by modeling three microwave structures. Our proposal showed the best performance when the size of the region of interest is small, and a very good performance for larger regions, in spite of its simplicity of formulation and implementation. Our PSM methodology proves to be an excellent candidate for developing EM surrogate models when the amount of base points is very limited and the size of the region of interest is relatively small.

REFERENCES
Design of an ultrawide band k, RNN neural network in...
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