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REAL-TIME LEAK ISOLATION BASED ON STATE ESTIMATION WITH FITTING LOSS COEFFICIENT CALIBRATION IN A PLASTIC PIPELINE*

Adrian Navarro, Ofelia Begovich, Juan Sánchez, and Gildas Besancon

ABSTRACT

This paper presents a leak isolation methodology using a fitting loss coefficient calibration. Two stages are considered for this purpose: First, the equivalent straight length (ESL) is fixed by an model-base observer designed as an extended Kalman filter. Once the leak is detected, the previous observer is stopped and the second system, based on an algebraic observer, is started with the ESL value fixed by the previous observer. Finally, the estimated leak position is recovered in original coordinates since the observer deal with ESL coordinates. In order to tackle the friction variations problem, the so-called Swamee-Jain equation is embedded explicitly, instead of a constant parameter as in other studies. The approach assumes only flow and pressure sensors at the ends of the duct. Experimental results with data obtained from a plastic pipeline prototype are presented to assess the method efficiency.

Key Words: Leak detection and isolation, Kalman filter, algebraic observer, dynamic model.

I. INTRODUCTION

Many analytical methods (fault model and fault sensitive approaches) have shown to be an efficient tool to isolate a leak in a pipeline, as can be seen in [2,3]. However, due to several phenomena present in a real pipeline, reliable and efficient mode-based leak detection and isolation (LDI) algorithms are still being researched.

One example of this phenomena is that, in plastic pipes, a flow regime that ensures a constant friction factor is difficult to achieve, since the friction value is sensitive to small flow rate variations. So, with leak occurrence, the flow changes, and so does the friction.

Several papers in the literature do not take into consideration these phenomena due to the fact that in traditional pipelines (iron, concrete, *etc.*), the friction value is quasi constant. However, nowadays the plastic pipes have become more and more popular, and the friction variation are more significant, so the analysis should include the friction variations.

For instance, Jian in [4] tackles the leak isolation problem using the partial derivative equation model and the gradient method to locate one leak. The paper achieves a satisfying location precision of the leak but that work only perform a statistic treatment of the noise. Besides, the authors do not include friction variation in this study then robustness and precision are compromised.

Benkherouf [2] and Aamo [5] design a nonlinear observer in order to locate one leak. Although the authors present good results, they fit the friction factor as a constant which leads to a low performance in plastic pipelines.

Liu [6] proposes an adaptive particle filter algorithm to locate one leak. The authors, although only presenting simulation results, show that the accuracy and speed in the leak location is increased. This work does not consider the influence of the friction variation.

In [7], the author extends the leak detection and isolation problem to two leaks using a nonlinear observer. The algorithm presented shows a good performance in the estimation of two leaks, but the fact that the work does not consider the friction variation could present lack of exactness in plastic pipeline implementation.

To take into account the friction factor variations, in [8] the friction is estimated into a Kalman Filter. In [9] and [10], where a fault sensitive approach (the model of the leak is explicitly characterized into the general model) is used, a gradient algorithm is implemented

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A. Navarro is with ITESO, Universidad Jesuita de Guadalajara, Periférico Sur Manuel Gómez Morín 8585, Tlaquepaque, Jalisco, México C.P. 45604 (corresponding author, e-mail: adriannavarro@iteso.mx).
O. Begovich and Juan Diego Sánchez-Torres are with the Automatic Control Laboratory, CINVESTAV-IPN Guadalajara, Av. del Bosque 1145 Col. El Bajío C.P. 45019, México (e-mails: obegovi@gdl.cinvestav.mx and dsanchez@gdl.cinvestav.mx).
G. Besançon is with Control Systems Department, Gipsa-lab, Ense3 Grenoble INP, BP46, 38402, Saint-Martin d'Hères, France, and with Institut Universitaire de France (e-mail: Gildas.Besancon@Grenoble-inp.fr).
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* A previous conference version of this paper is presented in [1].

to estimate the friction. A simple new idea was proposed in [3] where the value of friction in the model is replaced by an expression directly function of the flow, given by the so-called Swamee-Jain equation [11]. In this way, a direct estimate of the friction can be obtained based on a measured signal (flow rate) avoiding problems associated with identification or estimation such as persistent excitation, local minimum, convergence rate, *etc.* All these works present a good leak position estimation by taking a non-constant friction factor, but another drawback exists. The necessity of using a model representing a straight pipeline (which is difficult to satisfy when a designer is concerned with a real non-straight pipeline). To tackle this problem, an equivalent straight pipeline model must be obtained from the non-straight pipeline. For this purpose, a designer should make a virtual substitution of each fitting (elbow, joint, tee, *etc.*) by an equivalent straight segment of pipe presenting the same head loss as the fitting in question. In other words, the equivalent straight length (ESL) of the fitting must be obtained. The previous task involves the use of the so-called *loss coefficient* (see chapter 5 in [11]) of a particular fitting (FLC). This parameter can be found in the pipe manufacturer datasheet, which is normally denoted as K. However, in such a specification, the K value is usually larger than its real one to provide a security margin in the design of a pipeline system. As a consequence, if this value is used to calculate the ESL of a fitting (needed to design a model-based leak isolation system), its uncertainty can lead to a bad leak isolation. Then, it is critical to know with a good accuracy the FLC values to isolate a leak position in a more efficient way. Since, an accurate value of K is difficult to obtain via data-sheet, it is necessary to draw upon other methods to achieve adequate leak isolation.

In light of this background, the necessity to improve the algorithms for one leak detection in isolation in plastic pipelines continue to be a case worthy of study. Moreover, in a real case, the concurrence of two or more leaks is unlikely.

For all these reasons, the present paper focuses on the detection and isolation of one leak in a non-straight plastic pipeline. For the purpose aforementioned, this work proposes an LDI system to isolate one leak made of two stages. First, a nonlinear state observer is proposed to get a good estimation of the FLC value. In order to do that, a finite-dimensional model resulting from the classical infinite-dimensional model description of water dynamics in pipelines (Water Hammer equations [11]) is derived. Then, the model is augmented with a state variable related to the FLC and their own relations. In the second stage (when the leak is detected), the previous observer is frozen and a second observer is started. For the design of the second observer, the method shown in [1] is used. Here the state variables are flows, pressure head at the leak point and, in this approach, the model is augmented with the leak position and a parameter related to the leak intensity.

In both cases, “Swamee-Jain expression” is used instead of considering a constant parameter for the friction in the model. In the FLC approach the resulting continuous-time nonlinear model is discretized in time by using Heun’s method. Then, an Extended Kalman Filter is designed in order to estimate the “K” value [12].

For the isolation purpose, the paper explores the use of a leak position estimator based on an algebraic observer, taking advantage of its non-asymptotic convergence, robustness to uncertainties and capacity to deal with measurement noise without any assumption on its statistical properties [13].

Finally, the leak position in the original coordinates is computed (remember that the leak isolation task is done in equivalent length coordinates).

In order to assess its performance, the proposed LDI system is tested with measurements taken from the pipeline prototype described in [10].

The paper continues as follows: Section II provides the mathematical model and Section III describes the proposed model-based detection approach. Section IV then present some successful experimental results. Finally Section V concludes the paper.

II. PIPELINE MATHEMATICAL PRELIMINARIES

This section presents the two partial differential equations modelling the water dynamics in a pipeline. Then, the finite dimensional model to design the LDI system is described, as well as the notion of equivalent straight length.

2.1 Governing equations

Assuming the fluid to be slightly compressible and the duct walls slightly deformable; the convective changes in velocity to be negligible; the cross section area of the pipe and the fluid density to be constant, then the dynamics of the pipeline fluid can be described by the following partial differential equations [11].

Momentum equation.

$$\frac{\partial Q(z,t)}{\partial t} + Ag\frac{\partial H(z,t)}{\partial z} + \mu Q(z,t)\,|Q(z,t)| = 0 \qquad (1)$$

Continuity equation.

$$\frac{\partial H(z,t)}{\partial t} + \frac{b^2}{Ag}\frac{\partial Q(z,t)}{\partial z} = 0 \tag{2}$$

where Q is the flow rate (m^3/s), H the pressure head (m), z the length coordinate (m), t the time coordinate (s), g the gravity acceleration (m/s^2), A the cross-section area (m^2), b the speed of the pressure wave in the fluid (m/s), $\mu = \frac{\tau}{2DA}$, D the diameter (m) and τ the friction factor.

Friction model. In several works, the friction factor has been deemed to be a constant value. However, in smooth pipes (pipes with a relative roughness usually less than 1×10^{-3}), the complete turbulence zone is difficult to reach (*i.e.* the zone where friction factor is almost constant). For this reason, when the LDI problem is considered in plastic pipes, it is preferable to obtain a more realistic value for the friction by using either some formula to calculate it or some algorithm to estimate its value. In the present work the friction factor is calculated by using the well-known Swamee-Jain equation, where the friction factor is function of the Reynolds number (which is in turn function of the flow) [11]:

$$\tau(Q) = \frac{0.25}{\left[\log_{10}\left(\frac{\varepsilon}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2} \tag{3}$$

where ε (m) is the roughness height and Re the Reynolds number given by:

$$\mathrm{Re} = \frac{QD}{\nu A}$$

and ν is the kinematic viscosity (m^2/s).

Leak model. Furthermore, one leak arbitrarily located at point z_1 (see Fig. 1) in a pipeline can be modeled as follows [11]:

$$Q_L = \lambda\sqrt{H_L} \tag{4}$$

where the constant λ is a function of the orifice area and the discharge coefficient; Q_L is the flow through the leak and H_L is the head pressure at the leak point [11].

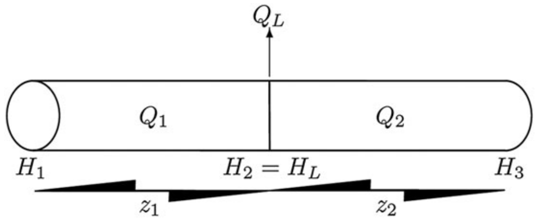

Fig. 1. Discretization of the pipeline with a leak Q_L

This leak produces a discontinuity in the system. Furthermore, due to the law of mass conservation, Q_L must satisfy the next relation:

$$Q_b = Q_a + Q_L \tag{5}$$

where Q_b and Q_a are the flows in an infinitesimal length before and after of the leak, respectively.

2.2 Spatial discretization of the modeling equations

In order to obtain a finite dimensional model from (1) and (2), the partial differential equations are discretized with respect to the spatial variable z, as in [7,14], by using the following relationships:

$$\frac{\partial H}{\partial z} \approx \frac{H_{j+1} - H_j}{z_j} \tag{6}$$

$$\frac{\partial Q}{\partial z} \approx \frac{Q_j - Q_{j-1}}{z_{j-1}} \tag{7}$$

where index j is related to the discretized variable at position z_j.

Assuming only two partitions in the pipeline as shown in Fig. 1, z_j ($j = 1, 2$) becomes the distance from the beginning of the pipe to the point of the leak and from the point of the leak to the end of the pipe, respectively. Notice that $z_2 = L - z_1$ where L is the total length of the pipeline and the leak position is assumed to be different from 0 and L in this description. Applying approximations (6) and (7) to equations (1) and (2) together with (4) and (5), we get

$$\begin{bmatrix} \dot{Q}_1 \\ \dot{H}_2 \\ \dot{Q}_2 \end{bmatrix} = \begin{bmatrix} -\frac{Ag}{z_1}(H_2 - u_1) - \mu Q_1 |Q_1| \\ -\frac{b^2}{Agz_1}(Q_2 - Q_1 + \lambda\sqrt{H_2}) \\ -\frac{Ag}{L-z_1}(u_2 - H_2) - \mu Q_2 |Q_2| \end{bmatrix} \tag{8}$$

Here, the input vector is $u = [H_1 \; H_3]^T = [u_1 \; u_2]^T$, and the output vector is $y = [Q_1 \; Q_2]^T$. Even if it is very simplified, this model can be useful for actual leak isolation, as will be seen from the results hereafter.

2.2.1 Time discretization

Notice finally that, in order to design an extended Kalman filter, it can be useful to perform a temporal discretization of model (8). In this work, the well-known Heun‘s Method is used. In this method, the solution for the initial value problem

$$\dot{x}(t) = \xi(x(t), u(t)), \; x(t_0) = x_0$$

is given by [15]

$$x^{i+1} = x^i + \frac{\Delta t}{2}\left[\xi\left(x^i, u^i\right) + \xi\left(x^i + \Delta t\xi\left(u^i, x^i\right), u^{i+1}\right)\right] \tag{9}$$

where Δt is the time step.

2.3 Equivalent Straight Length

As explained before, even if the pipe is not straight, an "Equivalent Straight Length" (ESL) can be obtained by considering loses due to each "non straight" (fitting) element. The head loss (h_L) produced by a fitting is calculated as [16]

$$h_L = K\frac{Q^2}{2gA}$$

where K is the fitting loss coefficient (FLC) for the particular fitting in question.

In accordance with the Darcy-Weisbach formula [16], K satisfies the relation

$$K = \tau\frac{l_e}{D} \tag{10}$$

where l_e is the ESL of the fitting. In this way, by obtaining the ESL of each fitting, a non-straight pipeline can be transformed to an equivalent straight pipeline whose total ESL (L_e) can be calculated as

$$L_e = L_r + \frac{D\sum_{j=1}^{n} K_j}{\tau(Q)} \tag{11}$$

where L_r is the pipeline physical length $[m]$ measured between the sensors placed at the ends of the pipeline, K_j is the FLC for the jth fitting and n the total number of the pipeline fittings. Then, (11) is commonly used to calculate the total ESL of a pipeline [17]. On the other hand, L_e must be such that it satisfies the following leak-free steady state relation (Darcy-Weisbach equation):

$$H_{in} - H_{out} = \frac{\tau}{2DA^2g}Q^2L_e \tag{12}$$

where H_{in} and H_{out} denote the pressure head at inlet and outlet of the pipeline, Q is the flow through the pipeline, and L_e is the total length for a straight pipeline. However, if the ESL is not well calculated, due to uncertainty in the K_j values, then the right hand of (12) could not match the left hand of this one (remember that H_{in} and H_{out} are measured online). Therefore, the K_j values are important to obtain a good modelling of a pipeline.

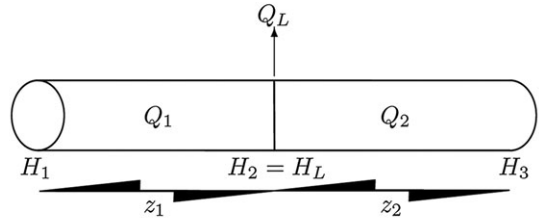

Fig. 2. LDI scheme.

III. OBSERVER SCHEMES

As has been mentioned, the LDI scheme proposed in this work comprises two main stages: First, an observer which estimates the ESL of the pipe through the knowledge of fitting loss coefficient is designed. For this purpose, the pipeline model state equation (8) is augmented with a factor related with the FLC of each fitting. When the leak is detected, the first observer is stopped and the ESL is fixed. Then, a second observer that estimates the values related to the leak is started. At this point, the state of the Eq. 8 is now augmented with the leak location and the λ parameter. Fig. 2 depicts a scheme for this approach.

3.1 Fitting loss coefficients observer

As can be seen in equation (11), the ESL could be computed directly with the knowledge of each K_j. These values are given by the manufacturer data-sheet but, most of the time, these values are larger than the real ones. This work proposes an observer that estimates the K_j values. In order to do that, one could try to find an augmented model of (8) including as extra states each K_j. Unfortunately, due to the limited number of measurements, this model would be unobservable. However, a model involving only one kind of K, which keeps relation with the other K_js , leads to an observable model. This shall be outlined in this paper.

By selecting the lowest value of the FLC in the data-sheet (denoted by K), the relation (11) can be expressed as:

$$L_e = L_r + \frac{D\sum_{j=1}^{n} K_j}{f(Q)} = L_r + \frac{DK\sum_{j=1}^{n} c_j}{f(Q)} \tag{13}$$

where c_j is the proportional factor of K with respect to each other fitting. The c_j values can be determined based on the manufacturer data-sheet. For example, for the pipeline used in this work, the FLC of an elbow is approximately eight times the FLC of a coupling [18].

Using Eq. 8 with the pipeline divided in two equal parts (*i.e.* $z1 = \frac{L_e}{2}$), by substituting the left hand of (13), and then incorporating the K value as an additional state, a leak free dynamic model (since in the first stage the pipe is not leaking) of the pipeline in a state space representation is given by the following nonlinear equations:

$$\begin{bmatrix} \dot{Q}_1 \\ \\ \dot{H}_2 \\ \\ \dot{Q}_2 \\ \\ \dot{K} \end{bmatrix} = \begin{bmatrix} -\frac{gA}{0.5L(K)}(H_2 - u_1) \\ -\mu^* \tau\left(Q_1\right) Q_1 \left|Q_1\right| \\ -\frac{b^2}{0.5gAL(K)}(Q_2 - Q_1) \\ -\frac{gA}{0.5L(K)}(u_2 - H_2) \\ -\mu^* \tau\left(Q_2\right) Q_2 \left|Q_2\right| \\ 0 \end{bmatrix} \tag{14}$$

where $\mu^* = \frac{1}{2DA}$ and $L(K)$ is as follows:

$$L(K) = L_r + \frac{DK \sum_{j=1}^{n} c_j}{\tau(Q)} \tag{15}$$

and the input and output vector remain the same (*i.e.* $u = [H_1\ H_3]^T = [u_1\ u_2]^T$ and $y = [Q_1\ Q_2]^T$).

In equation (14) the dynamics of $\dot{K}$ is equal to zero since K is not a function of time.

Now, applying equation (9) to equation (14) yields a difference equations that can be written in compact form as (see [1] for details)

$$\begin{aligned} x^{i+1} &= \xi(x^i, u^{i+1}, u^i) \\ y^i &= \mathbf{H}x^i \end{aligned} \tag{16}$$

where

$$x^i = [Q_1^i\ H_2^i\ Q_2^i\ K^i]^T$$

$$\mathbf{H} = \begin{bmatrix} 1\,0\,0\,0 \\ 0\,0\,1\,0 \end{bmatrix}$$

where the symbol T stand for matrix transpose.

3.1.1 Extended Kalman filter application

In order to minimize the estimated error covariance a Kalman Filter is implemented to the model (equation (16)) in the following form [12]:

$$\hat{x}^i = \hat{x}^{i^-} + \mathbf{K}^i(y^i - \mathbf{H}\hat{x}^{i^-})$$

where $\hat{x}^{i^-}$ is the *a priori* estimate of x^i and $\mathbf{K}^i$, the Kalman gain for the observer.

$\mathtt{R}$ and $\mathtt{Q}$ are known as the covariance matrices of the measure and process noises, respectively. Notice that:

$$\hat{x}^i = \left[\hat{Q}_1^i\ \hat{H}_2^i\ \hat{Q}_2^i\ \hat{K}_1^i\right]$$

$$P_{1,2}^{0^-} = \left(P_{1,2}^{0^-}\right)^T > 0, \mathtt{R} = \mathtt{R}^T > 0 \text{ and } \mathtt{Q} = \mathtt{Q}^T > 0.$$

3.2 Leak isolation scheme

Pipeline equations (8) together with the two new estates related with the leak (z_1 and λ) yields

$$\begin{bmatrix} \dot{Q}_1 \\ \dot{H}_2 \\ \dot{Q}_2 \\ \dot{z}_1 \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} -\frac{Ag}{z_1}(H_2 - u_1) - \mu Q_1 \left|Q_1\right| \\ -\frac{b^2}{Agz_1}(Q_2 - Q_1 + \lambda\sqrt{H_2}) \\ -\frac{Ag}{L-z_1}(u_2 - H_2) - \mu Q_2 \left|Q_2\right| \\ 0 \\ 0 \end{bmatrix} \tag{17}$$

As before, the input and output vector remain the same (*i.e.* $u = [H_1\ H_3]^T = [u_1\ u_2]^T$ and $y = [Q_1\ Q_2]^T$).

This equation corresponds to a nonlinear multiple input multiple output (MIMO) system of the general form:

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \tag{18}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the input, $y \in \mathbb{R}^p$ is the output and f, g, h are sufficiently differentiable function vectors.

For system (18), it is possible to define the following vector of output time derivatives:

$$\begin{aligned} V(t) = \Big[& y_1(t)\ \dot{y}_1(t)\ \ldots\ y_1^{(k)}(t)\ \ldots \\ & y_p(t)\ \dot{y}_p(t)\ \ldots\ y_p^{(k)}(t)\ \ldots \Big] \end{aligned} \tag{19}$$

From model (18), $V(t)$ can be expressed as a function of $x, u, \dot{u}, \ldots, u^{(k)}, \ldots$ as

$$V(t) = \Gamma(x, u, \dot{u}, \ldots, u^{(k)}, \ldots) \tag{20}$$

Observability somehow means that this relationship is invertible, and that one can find elements among the components of Γ defining an invertible map with respect to x [19]. Let us denote by $\bar{\Gamma}(x, u, t)$ this map, and consider the vector $\bar{V}(t)$ conformed by n independent elements selected from $V(t)$. Then,

$$x = \bar{\Gamma}^{-1}\left(\bar{V}(t), u, t\right). \tag{21}$$

3.2.1 Observer design for the LDI system

Considering the augmented model in (17) and unidirectional flow (*i.e.* $x_1 > 0$ and $x_3 > 0$), it is easy to check that an invertible map $\bar{\Gamma}$ can be formed with the output time derivatives vector as: $\bar{\Gamma} = \left[y_1(t)\dot{y}_1(t)\ddot{y}_1(t)y_2(t)\dot{y}_2(t)\right]$.

Then in this case by using $f(x)$, $g(x)$ and $h(x)$ one gets: $\bar{\Gamma} = \Big[x_1,\ -\frac{Ag}{x_4}\left(x_2 - u_1\right) - \mu^*\tau(x_1)x_1^2,\ (\mu^*)^2\tau'(x_1)\tau(x_1)x_1^4 + 2(\mu^*)^2\tau^2(x_1)x_1^3 + \frac{b^2}{x_4^2}\left(x_3 - x_1 + x_5\sqrt{x_2}\right) + \frac{Ag}{x_4}\left(\dot{u}_1 + 2\mu^*\tau(x_1)x_1x_2 - 2\mu^*\tau(x_1)x_1u_1 + \mu^*\tau'(x_1)x_1^2x_2 - \mu^*\tau'(x_1)x_1^2u_1\right), x_3,\ \frac{Ag}{L-x_4}(u_2-x_2)-\mu^*x_3^2\Big]^T$.

Therefore, the system state in terms of output derivatives (21), is written as

$$
\begin{aligned}
x_1 &= y_1\\
x_2 &= \frac{\dot{y}_2+\mu y_2^2}{Ag}\left[L-\left(\frac{L\left(\dot{y}_2+\mu y_2^2\right)-Ag\left(u_1-u_2\right)}{\left(\dot{y}_2+\mu y_2^2\right)-\left(\dot{y}_1+\mu y_1^2\right)}\right)\right]+u_2\\
x_3 &= y_2\\
x_4 &= \frac{L\left(\dot{y}_2+\mu y_2^2\right)-Ag\left(u_1-u_2\right)}{\left(\dot{y}_2+\mu y_2^2\right)-\left(\dot{y}_1+\mu y_1^2\right)}\\
x_5 &= \frac{x_4^2}{b^2\sqrt{x_2}}\left(\ddot{y}_1+2\mu y_1\dot{y}_1-\frac{Ag}{x_4}\dot{u}_1\right)+\frac{1}{\sqrt{x_2}}\left(y_1-y_2\right)
\end{aligned}
\tag{22}
$$

Now, noting that the system output time derivative is unknown, the problem reduces to the real time estimation $\hat{\bar{V}}(t)$ of $\bar{V}(t)$ in (21) so that an estimation of the estate can be given by (see [1] for details):

$$\hat{x} = \bar{\Gamma}^{-1}\left(\hat{\bar{V}}(t), u, t\right) \tag{23}$$

IV. EXPERIMENTAL RESULTS

In this section we present experimental results in order to evaluate the performance of the designed LDI scheme. For the present work, we consider the pilot water pipeline built at the Research and Advanced Studies Center in Guadalajara, Mexico (CINVESTAV-Guadalajara) reported in [10], whose main parameters are given in Table I.

The pipeline is equipped with: two water-flow (FT) and two pressure-head (PT) sensors at inlet and outlet of the pipe; a 5 hp centrifugal pump connected to a variable-frequency driver fixed at 50 Hz (this is in order to experiment flow-rate variation effects over the LDI scheme); and three valves, in order to emulate the effect of a leak, at 16.8, 33.3 and 49.8 meters respectively (more information can be found in [10]).

Table I. Pipeline prototype parameters

Parameter	Symbol	Value
Length between sensors	L_r	68.4 m
Internal diameter	D	0.0654 m
Pressure wave speed	b	376 m/s
Roughness	ε	7×10^{-6} m
Kinematic viscosity	ν	2×10^{-6} m^2/s

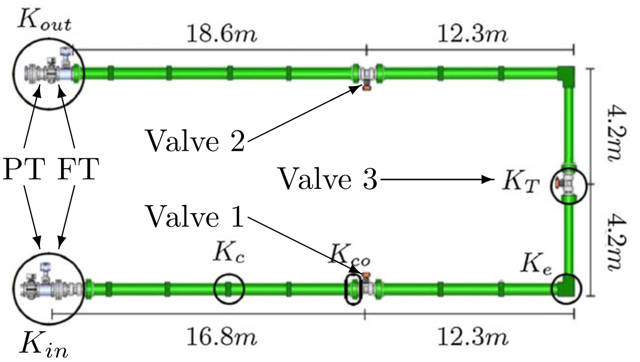

Fig. 3. Fitting diagram.

The fittings are: 12 coupling, 8 connectors, 3 tees, and 2 elbows. Their respective FLC are denoted by K_c, K_{co}, K_T and K_e. The FLC related with the initial and final connections are denoted by K_{in} and K_{out}, respectively. Fig. 3 shows a fitting pipeline diagram. A picture of the pipeline prototype is shown in Fig. 4.

Table II. summarizes the FLC taken from the manufacturer datasheet. In this case, the lower FLC value considered is K_c so the Equation (13) can be written as follows:

$$L_e = L_r + \frac{DK_c\sum_{j=1}^{27} c_j}{f(Q)} \tag{24}$$

with $n = 27$ in model (15). According to Table II, it is possible to determine a proportional factor for the each fitting. In this work we consider that this proportional factor remains constant. The proportional factors for each fitting are summarized in Table III.

It should be also noted that, as we mentioned before, the leak isolation observer uses the ESL so the leak position is estimate in ESL coordinates. Therefore, it is necessary to compute the leak position in original coordinates. We emphasize that the ESL of each fitting can be computed solving l_e from equation (10) as follows:

Fig. 4. Pipeline prototype picture.

Table II. FLC taken from the manufacturer's data-sheet

Parameter	Symbol	Value
FLC for the initial connections	K_{in}	0.40
FLC for a coupling	K_c	0.25
FLC for a connector	K_{co}	0.40
FLC for a Tee	K_T	0.45
FLC for an elbow	K_e	2.00
FLC for the final connections	K_{out}	0.40

Table III. Proportional factor for each fitting

Parameter	Symbol	Value
Coupling proportional factor	$c_1, \cdots, c_{12}$	1
Connector proportional factor	$c_{13}, \cdots, c_{20}$	1.6
Tee proportional factor	c_{21}, c_{22a}, c_{23}	1.8
Elbow proportional factor	c_{24}, c_{25}	8
Initial connection proportional factor	c_{26}	1.6
Final connection proportional factor	c_{27}	1.6

$$l_{e_i} = \frac{DK_i}{f} \tag{25}$$

where the subscript i denotes the ith fitting.

Now, one just needs to find the ESL sum that equals the leak equivalent length position. This sum obviously starts at the beginning of the pipeline. Based on Fig. 3, we can follow the next steps in order to recover the leak position in the original coordinates:

1. Initialize the sum with the ESL of the first fitting, (*i.e.* $l_{e_{sum}} = l_{e_1}$).
2. Is $l_{e_{sum}} \geq z_1$? If yes, it means that the leak is on the first fitting and the leak position in original coordinates is the physical length of the fitting. End the searching. If not, go to the next step.
3. $l_{e_{sum}} = l_{e_1} + l_{e_2}$ (it is due to a connector is the second fitting in the pipeline, Fig. 3).
4. Is $l_{e_{sum}} \geq z_1$? If yes, it means that the leak is on the second fitting and the leak position in the original coordinates is the sum of the physical length of both fittings. End the searching. If not, go to the next step.
5. Now, the third part of the pipeline is a straight pipe, so here the sum is equal to the previous sum plus the length of the pipe in question (*i.e.* $l_{e_{sum}} = l_{e_1} + l_{e_2} + l_1$, l_1 denotes the length of the first pipe).
6. Is $l_{e_{sum}} \geq z_1$? If yes, it means that the leak is on the first straight pipe. In this point, it is possible to know the exact position of the leak just subtracting the z_1 value from the actual sum (*i.e.* $z_{1_{oc}} = l_{e_1} + l_{e_2} + l_1 - z_1$, here $z_{1_{oc}}$ denote the distance from the last fitting to the leak point in original coordinate). End the searching. If not, go to the next step.
7. And so on, until the z_1 value is reached.

It should be pointed out that:

(i) On the one hand, it is easy to assume from equation (25) that without the knowledge of the fitting loss coefficient it is not possible to recover the leak position in original coordinates.

(ii) On the other hand, it is neither possible to obtain a general function to recover the leak position in original coordinates due to the shape and the number of fitting change in each application.

In order to test the previous scheme, a leak located at $z_l = 49.8\ m$ (Valve number 3, Fig. 3) was induced at time 500 s (noted as t_l), approximately. At time $t = 0$ the first observer (Fitting loss coefficient estimator) is started. In the leak occurrence, the ESL is fixed with the fitting loss coefficients estimated by the observer. This parameter (ESL) is fed to the second observer at the time it is started. A simplest leak alarm given by $|Q_{in} - Q_{out}| > \delta$ (Q_{in} represents the measured inflow whereas Q_{out} is the measured outflow) was implemented. At this time, the leak isolation begins. The δ threshold is defined experimentally according to the noise of the system (*i.e.* δ must be at least larger than variance of difference between Q_{in} and Q_{out} in order to avoid false alarms). For this study, $\delta = 8 \times 10^{-5}$.

The initial conditions for the Kalman Filter are fixed as follows: $\hat{Q}_1^0$ and $\hat{Q}_2^0$ are equal to the mean values of the input and output flows in the operating point, $\hat{K}$ is computed with the loss coefficient provided by the manufacturer, $\hat{H}_2^0$ is the pressure head calculated at distance $\hat{z}_1^0$ in absence of leak. $\hat{z}_1^0$ is equal to $L/2$, and $\hat{\lambda}^0 = 0$ since the pipe is not leaking. It is important to note that these initial conditions are kept constant until the leak is detected.

Table IV. Initial conditions sfor the observer

Estimate	Symbol	Value
Inflow	$\hat{Q}_1$	$7.76 \times 10^{-3}\ m^3/s$
Outflow	$\hat{Q}_2$	$7.76 \times 10^{-3}\ m^3/s$
Pressure head	$\hat{H}_2$	$10.8\ m$
Total loss coefficient	$\hat{K}$	12.3 [−]
Leak position	$\hat{z}_1$	$50.1\ m$
Lambda parameter	$\hat{\lambda}$	$0\ m^{\frac{5}{2}}/s$

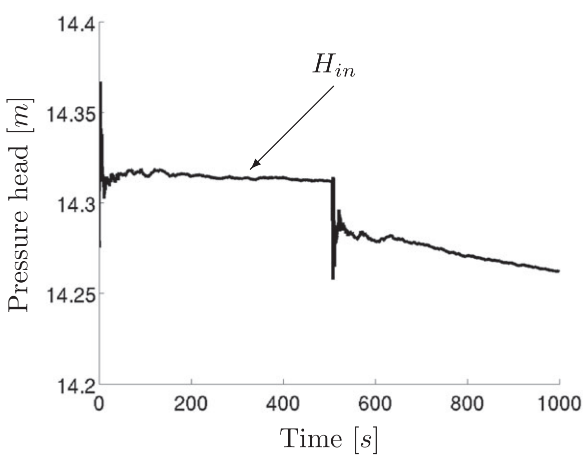

Fig. 5. Pressure head at inlet of the pipe (u_1).

Table IV below summarizes these values. For our setting, the Reynolds Number is in order of 1.69×10^5 (note that it is not a complete turbulence zone) and the sampling time was chosen as $1.43 \times 10^{-2}\ s$.

The process and measurement noise covariance matrices for the Extended Kalman Filter were experimentally tuned with the following values:

$$\mathtt{Q} = \text{diag}\left(1 \times 10^{-6},\ 1 \times 10^{-2},\ 1 \times 10^{-6},\ 1\right)$$
$$\mathtt{R} = \text{diag}\left(1 \times 10^{-8},\ 1 \times 10^{-8}\right)$$

for the first observer. Finally, in order to obtain the algebraic estimation of the derivatives, a moving window of 7.13 seconds was fixed (see [1] for details).

Figs 5 and 6 presents the measured pressure heads at inlet ($H_{in} = H_1$) and outlet ($H_{out} = H_3$) of the pipe

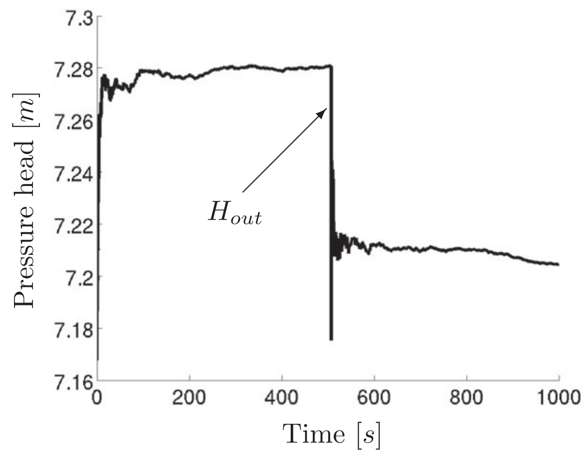

Fig. 6. Pressure head at outlet of the pipe (u_2).

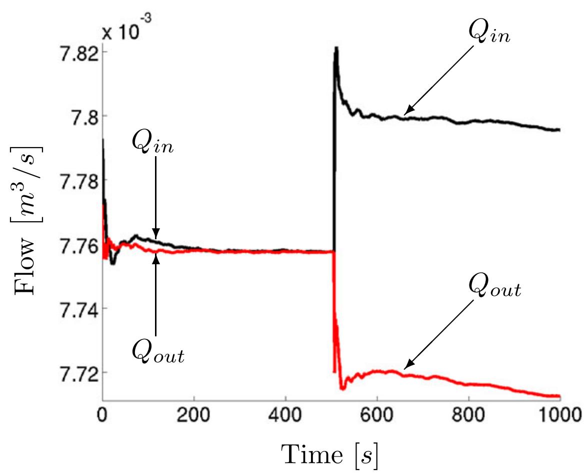

Fig. 7. Flow at inlet and outlet of the pipe ($[\,y_1\ y_2\,]^T$).

respectively (*i.e.* the observer inputs). Fig. 7 depict the evolution of the flow at inlet ($Q_{in} = Q_1$) and outlet ($Q_{out} = Q_2$) of the pipe (*i.e.* the observer outputs). Figs 8 and 9 show the results for the FLC observer. Specifically, Fig. 8 displays the value of $\hat{K}_c \sum_{j=1}^{27} c_i$ (denoted by $\hat{K}_T$) where $\hat{K}_c$ is the FLC estimated by the observer. This figure also shows the sum of all FLC values in the pipeline obtained from the manufacturer data-sheet. Note that this value is larger than the observer value, as expected. In order to verify, in an indirect way, the effectiveness of the observer, the ESL of the pipeline was calculated in three different forms:

(a) By using the Darcy-Weisbach equation (12);

(b) By using equation (24) with the FLC determined from our observer;

(c) By using equation (11) with the FLC from data-sheet pipe manufacturer.

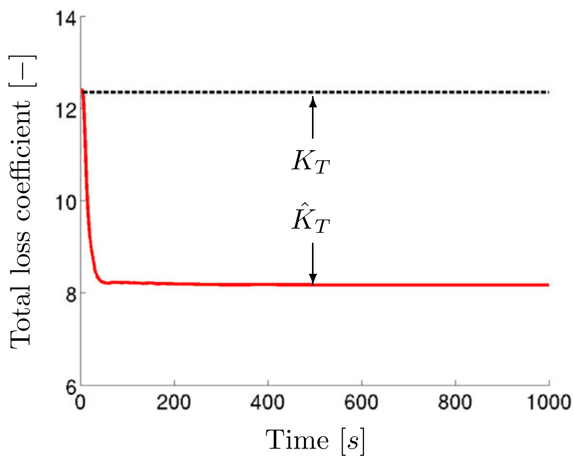

Fig. 8. Total loss coefficient and its estimation.

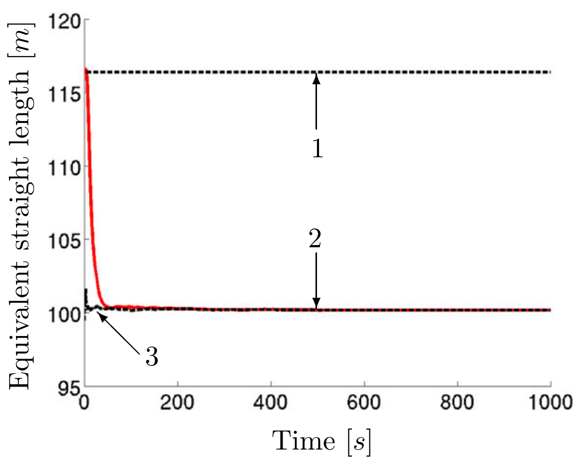

Fig. 9. Equivalent straight length of the pipeline computing by using: (i) the FLC from data-sheet; (ii) the estimated FLC; (iii) the Darcy-Weisbach equation.

The results are shown in Fig. 9. As can be seen in this figure, the ESL obtained by using the procedure in (a) an (b) gives similar values for the ESL, which states that the FLC estimates by our observer are appropriate. It is important to note that the Darcy-Weisbach equation provides a direct way to calculate an appropriate value of the ESL of the pipeline. Nonetheless, this formula does not provide information about the value of the FLC which are needed to recover the original length coordinates of a leak position. In this figure, we also note that the use of the procedure in (c) gives a bad estimation of the ESL, due to the use of the FLC given by the manufacturer which are only approximate values.

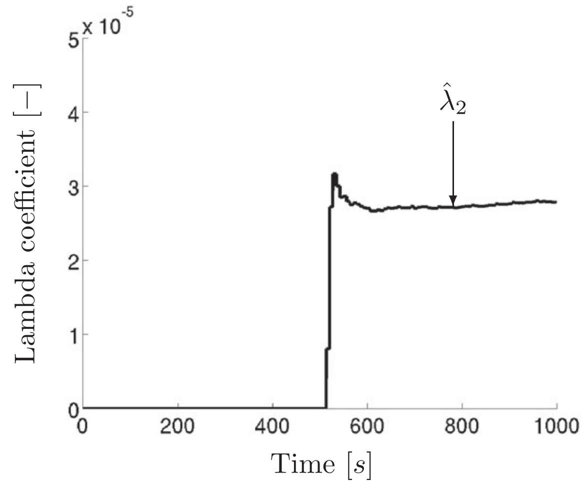

Fig. 10. Lambda parameter estimation.

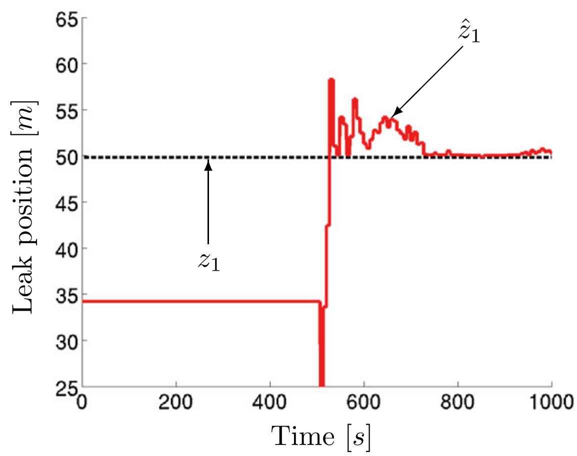

Fig. 11. Leak position and its estimation (real coordinates).

In Fig. 10, the lambda ($\hat{\lambda}$) parameter estimation is shown. Using (4), it is possible to compute the leak intensity. For this case, the leak intensity is around 2.7×10^{-5} $[m^3/s]$.

Fig. 11 shows the estimated leak position in original coordinates, $\hat{z}_{1_{oz}}$. As we can see in this figure, the leak position is well estimated.

V. CONCLUSIONS

This work has proposed an LDI algorithm which has been divided into two main stages: an FLC observer and a leak position and magnitude estimator.

The first point to underline is that using the FLCs given by the manufacturer can lead to a wrong determination of the ESL and thus lead to poor leak isolation. Additionally, without the knowledge of the fitting loss coefficient it is not possible to recover the leak position in original coordinates. For this reason, and in order to improve leak isolation task, the designer must find more accurate values for the FLC. In this work, we have proposed a possible alternative to finding0 such coefficients by using a Kalman Filter and relations between them.

The second part of this work designs and tests in real-time a leak detection and isolation algorithm based on an algebraic observer to locate a water-leak on a plastic pipeline prototype.

Flow rates have been well estimated in the presence of a leak, which means that the algebraic observer correctly follows the dynamics of the model with a leak. In the same way, the observer has estimated the leak position and its intensity in a very acceptable way, in spite of the noise and uncertainties found in this real application.

The Swamee-Jain equation, implemented to calculate in a more precise form the friction value, has been a key to achieve good estimation. This formula allows to calculate the friction by directly using flow measurements. This makes unnecessary any dynamic estimation of a friction coefficient (by identification or state estimation), consequently avoids many problem related to such a dynamic estimation.

Finally, the acknowledgement of the K values given for the ESL observer leads a better leak position estimation rather than using the K values given by the manufacturing

As a future work, three points should be tackled: (i) extend the approach to locate two leaks; (ii) this algorithm will be refined to achieve better performance; (iii) a more sophisticated leak alarm will be designed in order to start the leak isolation algorithm in a more accurate way.

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Gildas Besançon received an electrical engineering diploma in 1993, and a PHD degree in control in 1996, both in Grenoble, France. After a post-doctoral period at the Dipartimento di Informatica e Sistemistica of Rome University La Sapienza, Italy, he joined the Institut National Polytechnique de Grenoble on a teaching and research position in control. He is currently a full professor at Grenoble INP, and was distinguished as a member of Institut Universitaire de France in 2010 for a five-year position. His research activities are held at the Control Systems Department of Gipsa-lab - where he isthe head of the research group on 'nonlinear and complex systems', and he addresses various topics about such systems, with a special interest in observer issues, and applications in energy, hydraulics or micro/nano-sciences.

Ofelia Begovich was born in Mexico City, Mexico. She obtained the M. Sc. in Electrical Engineering from Advanced Studies and Research Center (CINVESTAV), Mexico City, in 1987, and the Ph.D. degree in Electrical Engineering from the University of Rennes I, Rennes, France in 1992. Since 1992, she has been a Professor of Electrical Engineering of graduate program in CINVESTAV, Guadalajara, Mexico. Her major research fields are the control of processes, leak diagnosis, and control and monitoring of Greenhouses and heuristic optimization applied to parameter estimation.

Adrián Navarro Díaz received his bachelor degree in Communication and Electronic Engineer from University of Guadalajara, Mexico, in 2002. He obtained the MSc and PhD degree in electrical engineering from the Advanced Studies and Research Center of the National Polytechnic Institute (CINVESTAV del IPN), Guadalajara, Mexico, in 2006 and 2012, respectively. In 2014, he joined the Department of Electronics, Systems and Computing of Technological Institute of Higher Education on the West (ITESO).His main research interest are leak diagnosis, nonlinear observer, neural network and data mining methods.

Juan Diego Sánchez-Torres received his control engineering degree from the Universidad Nacional de Colombia in 2008, the M.Sc. and PhD degrees in electrical engineering from the Advanced Studies and Research Center of the National Polytechnic Institute -CINVESTAV-, Campus Guadalajara, Mexico in 2011 and 2015, respectively. After finishing his doctoral studies, he joined the Department of Mathematics and Physics of ITESO University, Guadalajara. His research interests include sliding mode algorithms and their applications to dynamical systems control, estimation and optimization with applications in smart grids, electrical machines, mechanical systems and chemical processes.