

A Rigorous Numerical Comparison for Parameter Extraction Using Classical and Kullback-Leibler Formulations Illustrated with Full-Wave EM and Equivalent Circuit Models for Microstrip Filters

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Abstract—Numerical circuit parameter extraction (PE) is a key sub-process of space mapping (SM), which is used to efficiently optimize full-wave EM responses of microwave structures exploiting faster but inaccurate physics-based auxiliary models. Any improvement in PE has a positive impact on SM design optimization. In this paper, we apply for the first time a PE formulation based on the Kullback-Leibler distance to microstrip filters using their full-wave EM responses as targets. We perform a rigorous numerical comparison of PE based on the K-L formulation against PE using classical norms. Our results confirm a better PE performance using the Kullback-Leibler formulation than those obtained with traditional PE formulations.

Index Terms—Chebyshev, EM, Euclidean, filter, Kullback-Leibler, Manhattan, microstrip, norm, optimization, parameter extraction, space mapping.

I. INTRODUCTION

Space mapping (SM) is a well-established method to optimize, in a very efficient manner, highly accurate but computationally expensive models, referred as fine models, by exploiting much faster physics-based but inaccurate models [1], referred as coarse models [1]. Parameter extraction (PE) is a critically important sub-process of SM design optimization [2]. Improving the PE sub-process has a positive impact on SM.

Objective functions based on classical norms, also known as l -th norms, have been traditionally employed to do PE [3]. An innovative formulation inspired in Shannon entropy theory was proposed in [4] by exploiting the Kullback-Leibler (K-L) distance [5], demonstrating a promising behavior of the resultant PE objective functions in some 1- and 2-dimensional synthetic examples [4]. In this paper, we use the Kullback-Leibler distance formulation for PE as in [4] and apply it, for the first time, to microstrip low-pass and band-pass filters whose target fine model responses are obtained from accurate full-wave EM simulations, where the corresponding coarse models are fast equivalent circuits implemented in Keysight ADS. We perform a rigorous numerical comparison of this PE using the K-L formulation against PE using classical norms (Manhattan, Euclidean, and Chebyshev norms). We show that the K-L formulation for PE applied to these realistic microstrip filters numerically outperforms the corresponding classical l -th norm PE formulations, confirming the advantages found in [4] using graphical synthetic examples.

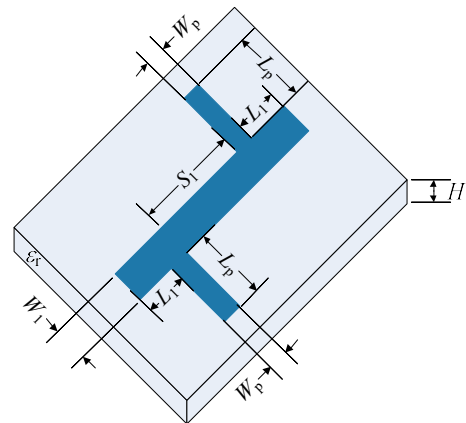


Fig. 1 Low-pass filter in microstrip technology [6].

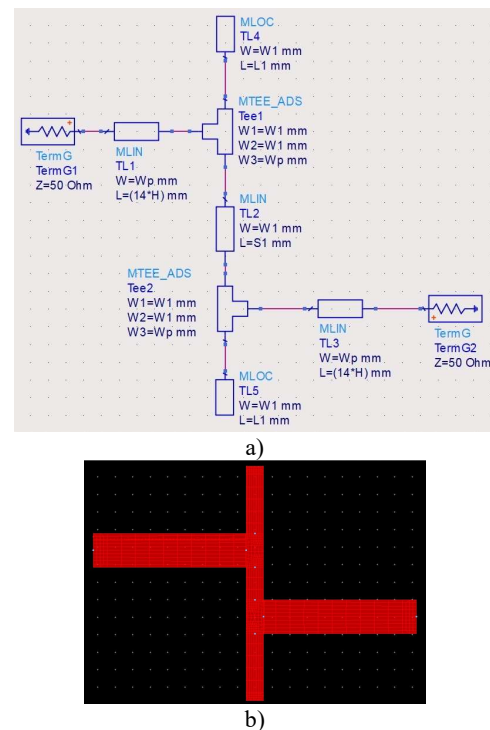


Fig. 2 Low-pass filter in microstrip technology: a) coarse model in ADS, b) fine model in ADS Momentum.

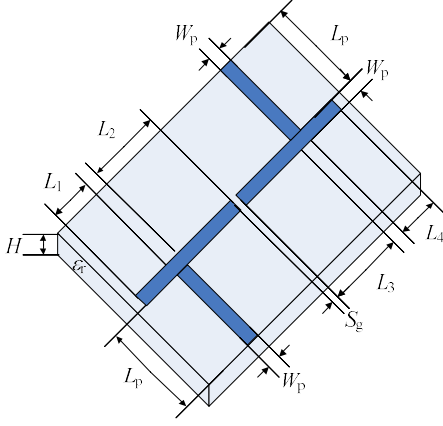


Fig. 3 Band-pass filter in microstrip technology [7].

II. PARAMETER EXTRACTION OBJECTIVE FUNCTIONS

Numerical parameter extraction (PE) can be formulated as the following optimization problem:

$$\mathbf{x}^{\text{PE}} = \arg \min_{\mathbf{x}} U_{\text{PE}}(\mathbf{R}(\mathbf{x}), \mathbf{R}^t) \quad (1)$$

where the n model parameters are in vector $\mathbf{x} \in \mathbb{R}^n$, the r model responses are represented by a multidimensional vector function $\mathbf{R}(\mathbf{x}): \mathbb{R}^n \rightarrow \mathbb{R}^r$, and vector $\mathbf{R}^t \in \mathbb{R}^r$ contains the target response for PE. The difference between $\mathbf{R}(\mathbf{x})$ and $\mathbf{R}^t$ is measured by a multidimensional scalar PE objective function $U_{\text{PE}}: \mathbb{R}^n \rightarrow \mathbb{R}^1$. The purpose of PE, hopefully achieved by solving (1), is to find the optimal model parameters $\mathbf{x}^{\text{PE}} \in \mathbb{R}^n$ that make the model response $\mathbf{R}(\mathbf{x})$ as close as possible to the

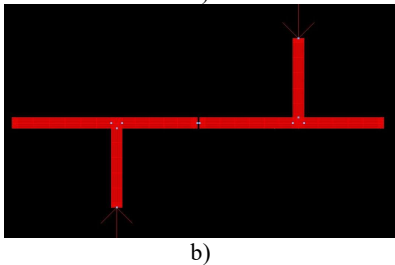
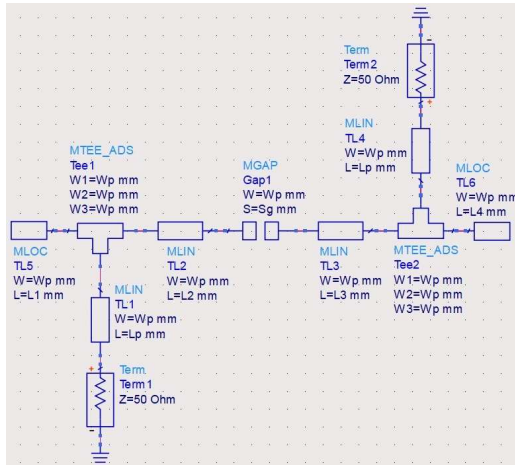


Fig. 4 Band-pass filter in microstrip technology: a) coarse model in ADS, b) fine model in ADS Momentum.

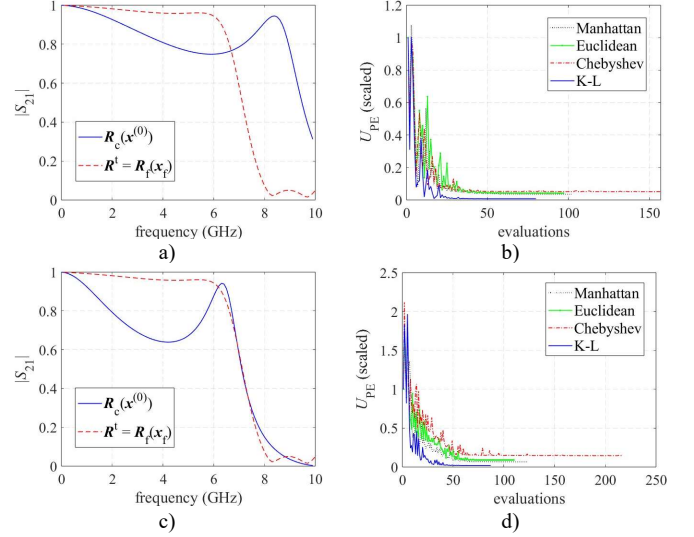


Fig. 5 PE results for the low-pass microstrip filter using two different starting points for the same target. a), c) EM filter response used as target $\mathbf{R}^t$ and initial coarse model response at the starting points, $\mathbf{R}_c(\mathbf{x}^{(0)})$; b), d) evolution of scaled U_{PE} . a) and b) use $\mathbf{x}^{(0)} = [4.0 \ 3.0 \ 2.0]^T$ mm (Case 1); c) and d) use $\mathbf{x}^{(0)} = [6.0 \ 4.0 \ 4.0]^T$ mm (Case 2).

target response $\mathbf{R}^t$. In the SM context, $\mathbf{R}^t$ is obtained from the fine model, while $\mathbf{R}(\mathbf{x})$ is obtained from the coarse model.

The PE objective function using an l -th norm is

$$U_{\text{PE}}(\mathbf{R}(\mathbf{x}), \mathbf{R}^t) = \|\mathbf{e}(\mathbf{x})\|_l \quad (2)$$

where $\mathbf{e}(\mathbf{x}) \in \mathbb{R}^r$ is the error of the model response with respect to the target response, $\mathbf{e}(\mathbf{x}) = \mathbf{R}(\mathbf{x}) - \mathbf{R}^t$. The classical norms employed in (2) are Manhattan ($l = 1$), Euclidean ($l = 2$), and Chebyshev ($l = \infty$), defined respectively as

$$U_{\text{PE}}^{\text{M}} = \sum_{i=1}^r |e_i(\mathbf{x})| \quad (3)$$

$$U_{\text{PE}}^{\text{E}} = \sum_{i=1}^r e_i^2(\mathbf{x}) \quad (4)$$

$$U_{\text{PE}}^{\text{C}} = \max_i \{|e_i(\mathbf{x})|\} \quad (5)$$

Using the Kullback-Leibler distance [4], the PE objective function $U_{\text{PE}}(\mathbf{R}(\mathbf{x}), \mathbf{R}^t)$ can be implemented as

$$U_{\text{PE}}^{\text{KL}} = \sum_{i=1}^r \left\{ R_i(\mathbf{x}) \ln \left| \frac{R_i(\mathbf{x})}{R_i^t} \right| + (1 - R_i(\mathbf{x})) \ln \left| \frac{1 - R_i(\mathbf{x})}{1 - R_i^t} \right| \right\} \quad (6)$$

where R_i is the i -th model response at model parameters $\mathbf{x}$, and R_i^t is the i -th target response.

Since the PE objective function values using the above formulations differ from each other, the following scaled PE objective function U_{PE}^{s} is used to make fair numerical comparisons and to apply the same termination criteria to stop the numerical optimization,

$$U_{\text{PE}}^{\text{s}} = \frac{U_{\text{PE}}(\mathbf{x})}{U_{\text{PE}}(\mathbf{x}^{(0)})} \quad (7)$$

where $\mathbf{x}^{(0)} \in \mathbb{R}^n$ is the starting point for numerical PE.

III. EXAMPLE 1: LOW-PASS MICROSTRIP FILTER

As a first example we use the microstrip low-pass filter shown in Fig. 1 [6]. It has a substrate with a relative dielectric permittivity $\epsilon_r = 2.2$, height $H = 0.794$ mm, and loss tangent $\tan(\delta) = 0.01$. Metal traces and ground plane use copper with thickness $t = 15.24 \mu\text{m}$ and conductivity $\sigma = 5.8 \times 10^7$ S/m. The input-output 50- Ω microstrip lines have a width $W_p = 2.45$ mm

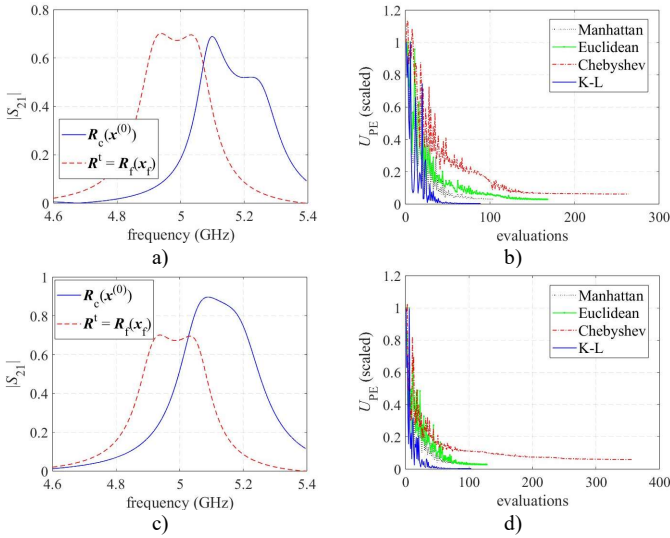


Fig. 6 PE results for the band-pass microstrip filter using two different starting points for the same target. a), c) EM filter response used as target $\mathbf{R}^t$ and initial coarse model response at the starting points, $\mathbf{R}_c(\mathbf{x}^{(0)})$; b), d) evolution of scaled U_{PE} . a) and b) use $\mathbf{x}^{(0)} = [6.2 \ 4.6 \ 5.9 \ 5.1 \ 0.15]^T$ mm (Case 1); c) and d) use $\mathbf{x}^{(0)} = [6.5 \ 4.5 \ 6.0 \ 5.0 \ 0.13]^T$ mm (Case 2).

and a length $L_p = 14H$. They are separated by a distance S_1 with a microstrip line with width W_1 and open stubs with length L_1 . This filter has three design parameters $\mathbf{x} = [W_1 \ L_1 \ S_1]^T$. The pre-assigned parameters are $\mathbf{z} = [H \ \epsilon_r \ W_p \ L_p \ \tan(\delta) \ \sigma \ t]^T$.

The coarse model is the Keysight ADS equivalent circuit shown in Fig. 2a. The full-wave EM fine model is in Keysight ADS Momentum Microwave, as shown in Fig. 2b, using a high-resolution discretization until convergence is reached.

IV. EXAMPLE 2: BAND-PASS MICROSTRIP FILTER

As a second example we use the microstrip band-pass filter shown in Fig. 3 [7]. The dielectric substrate uses alumina with $\epsilon_r = 9$, $H = 0.6$ mm, and $\tan(\delta) = 0.001$. All microstrip lines used in this filter have the same width $W_p = 0.7$ mm. The feeding microstrip lines have a length $L_p = 5$ mm. Metallic strips and ground plane use $t = 15 \ \mu\text{m}$ and $\sigma = 5.8 \times 10^7$ S/m. Main resonators have lengths L_1, L_2, L_3 , and L_4 , with a separating gap S_g . This band-pass filter has five design parameters $\mathbf{x} = [L_1 \ L_2 \ L_3 \ L_4 \ S_g]^T$ and seven pre-assigned parameters $\mathbf{z} = [H \ \epsilon_r \ W_p \ L_p \ \tan(\delta) \ \sigma \ t]^T$.

TABLE I. PE RESULTS FOR THE LOW-PASS FILTER USING THE NELDER-MEAD METHOD FOR EACH PE OBJECTIVE FUNCTION

$\mathbf{x}^{(0)}$	OF	i	$\mathbf{x}^{PE}$	ϵ_{avg}	ϵ_{max}	NMSE
Case 1	M	102	[1.3729 5.3993 2.8353] ^T	0.93	5.40	2.10
	E	97	[1.4973 5.3133 2.9794] ^T	1.02	4.94	2.01
	C	157	[2.2123 4.9795 3.5190] ^T	0.41	4.64	3.65
	K-L	80	[1.5159 5.2744 3.1133] ^T	1.06	4.95	2.06
Case 2	M	123	[1.3722 5.3997 2.8358] ^T	0.93	5.41	2.10
	E	110	[1.4984 5.3126 2.9799] ^T	1.03	4.94	2.01
	C	216	[2.2177 4.9768 3.5229] ^T	0.41	4.64	3.67
	K-L	87	[1.5157 5.2751 3.1106] ^T	1.06	4.95	2.06

OF is the PE objective function: Manhattan (M), Euclidean (E), Chebyshev (C), or Kullback-Leibler (K-L).

i is the total number of OF evaluations (or coarse model simulations) to get $\mathbf{x}^{PE}$. Termination tolerances used for Nelder-Mead: $\Delta\mathbf{x} = 1 \times 10^{-3}$ and $\Delta U_{PE}^S = 1 \times 10^{-3}$.

TABLE II. PE RESULTS FOR THE BAND-PASS FILTER USING THE NELDER-MEAD METHOD FOR EACH PE OBJECTIVE FUNCTION

$\mathbf{x}^{(0)}$	OF	i	$\mathbf{x}^{PE}$	ϵ_{avg}	ϵ_{max}	NMSE
Case 1	M	105	[6.6199 4.7714 5.8767 5.3965 0.1338] ^T	1.23	4.27	3.06
	E	168	[6.6440 4.7414 5.8708 5.4011 0.1300] ^T	1.24	2.58	2.77
	C	264	[6.3725 4.9211 6.2208 5.2004 0.1386] ^T	0.16	5.49	6.82
	K-L	89	[6.6094 4.7837 5.8934 5.3795 0.1380] ^T	1.56	7.92	4.57
Case 2	M	110	[6.6489 4.7292 5.8664 5.4033 0.1237] ^T	1.32	2.8	2.86
	E	128	[6.6553 4.7236 5.8654 5.4040 0.1242] ^T	1.32	2.43	2.83
	C	358	[6.3755 4.9193 6.2568 5.1737 0.1375] ^T	0.14	5.94	7.63
	K-L	104	[6.6215 4.7509 5.8855 5.3873 0.1246] ^T	1.53	6.11	4.31

OF and i are defined as in Table I.

Termination tolerances used for Nelder-Mead: $\Delta\mathbf{x} = 1 \times 10^{-2}$ and $\Delta U_{PE}^S = 1 \times 10^{-4}$.

$\tan(\delta) \ \sigma \ t]^T$.

The coarse model is the Keysight ADS equivalent circuit shown in Fig. 4a. The full-wave EM fine model is in Keysight ADS Momentum Microwave, as shown in Fig. 4b.

V. PE RESULTS AND NUMERICAL COMPARISONS

The following error metrics are used for numerical comparison between all the PE formulations: the average absolute relative error (ϵ_{avg}), the maximum absolute relative error (ϵ_{max}), and the normalized mean square error (NMSE), as

$$\epsilon_{avg} = \frac{100 \|e_{PE}\|_1}{\|\mathbf{R}^t\|_\infty} (\%) \quad (8)$$

$$\epsilon_{max} = \frac{100 \|e_{PE}\|_\infty}{\|\mathbf{R}^t\|_\infty} (\%) \quad (9)$$

$$NMSE = \frac{100 \|e_{PE}\|_2}{\|\mathbf{R}^t\|_2} (\%) \quad (10)$$

where e_{PE} is the error vector after PE, $e_{PE} = \mathbf{R}(\mathbf{x}^{PE}) - \mathbf{R}^t$.

The target fine model response $\mathbf{R}^t$ for PE is obtained from the EM simulation at reference design $\mathbf{x}_f = [1.270 \ 4.880 \ 2.365]^T$ (mm) for the low-pass filter, and $\mathbf{x}_f = [6.275 \ 4.75 \ 5.90 \ 5.05 \ 0.15]^T$ (mm) for the band-pass filter. We consider two cases for PE for each filter, using two starting points $\mathbf{x}^{(0)}$. The results are shown in Fig. 5 and Table I for the low-pass filter, and in Fig. 6 and Table II for the band-pass filter. In both examples we use the same optimization method (Nelder-Mead) with the same termination criteria: tolerances employed for extracted variables ($\Delta\mathbf{x}$) and objective function (ΔU_{PE}^S) are indicated in Tables I and II.

It is seen from Fig. 5 and Fig. 6 that, for both examples and both starting points, the K-L formulation requires less evaluations of the PE objective function than those needed by l -th norms, arriving to a smaller value of the scaled PE objective function. Table I and Table II reconfirm those results, and additionally indicate that K-L yields similar response matching errors than those obtained with l -th norm formulations.

VI. CONCLUSION

A parameter extraction objective function using a Kullback-Leibler formulation intended for space mapping was applied for the first time to realistic microstrip filters whose full-wave EM responses are used as targets, making a rigorous numerical comparison with the corresponding classical l -th norms. Our results indicate that the promising behavior seen in [4] for some 1- and 2-dimensional synthetic examples remain in more

realistic structures, confirming that Kullback-Leibler requires less coarse model evaluations for parameter extraction with practically equivalent matching errors than classical l -th norms.

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